Waves - General

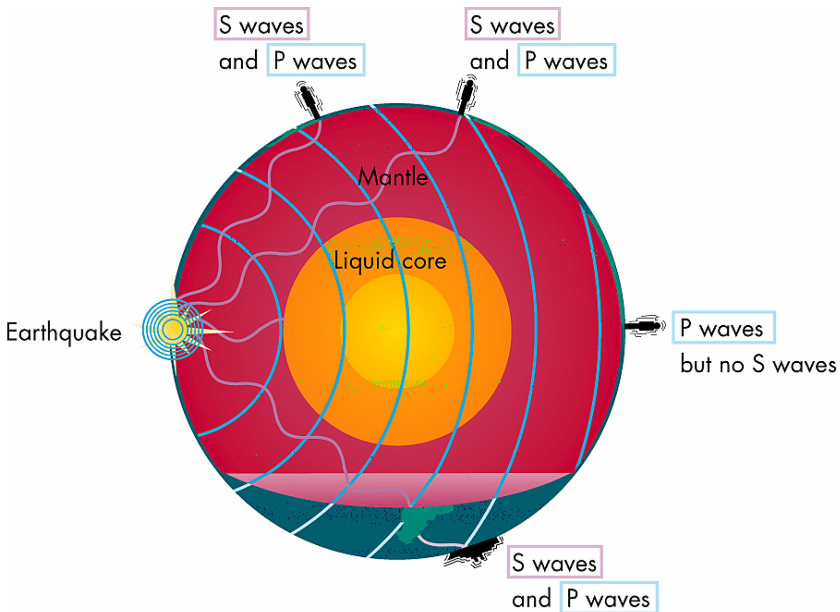
A *wave* may be defined as a periodic disturbance in a medium that carries energy from one point to another.

- All waves require a *source* and a *medium* through which to propogate.
- Waves normally radiate *isotropically*, i.e., in all directions away from sources.
- *Plane waves* move only in a single direction as they move away from a source.
- If one is far enough from an isotropic source the waves sweeping past one's point of observation behave like plane waves.
- Waves transport energy. As a wave moves away from its source energy is conserved - unless the wave is attenuated but some non-conservative property of the medium through which it is moving.
- The energy of all waves is proportional to wave amplitude.
- The rate of energy transfer or *power* is a useful way of describing wave energy.
- Though the power (joules/sec or watts) of any non-attenuated wave remains constant as the wave moves away from its source, the intensity (power/area) does not (unless the wave is a plane wave) because the waves spread out.
- Waves that radiate isotropically from a source diminish in intensity due to the fact that the energy they carry is spread out over an increasingly large area as they move away from the point of origin.
- The relationship that describes intensity vs. distance from a source for an isotropic wave is known as the *inverse square law*.

Physics, A World View, 4th ed. Kirkpatrick

Transverse and Longitudinal Waves

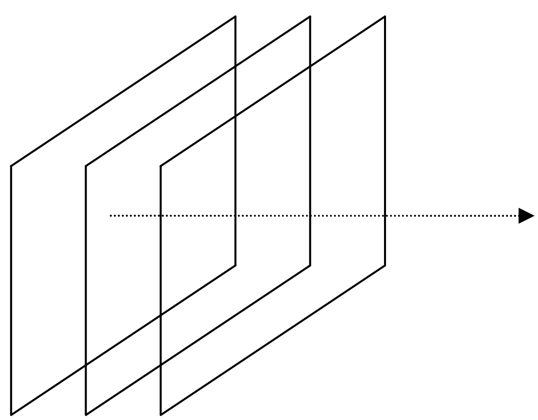
- There are two general types of waves: *Longitudinal* and *Transverse*.
- Sound is an example of a longitudinal wave.
- Light is an example of a transverse wave.
- In the natural world many waves are neither purely transverse nor longitudinal. Ocean waves for instance are partially each. Earthquake waves come in two forms, one of each.



Explorations, 4th ed. Arny

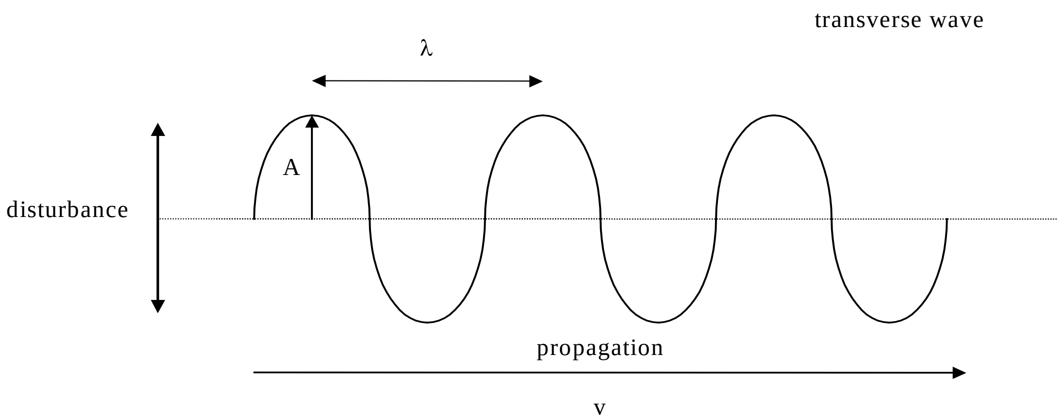
- Transverse waves behave in much the same manner as longitudinal waves with respect to reflection, refraction, diffraction, diffusion, and dispersion. A significant difference between the two lies in a property known as polarization.
- Polarization is a property unique to transverse waves. A polarized wave vibrates in a single plane is space. Since longitudinal waves vibrate along their axis of propagation, it is not possible to polarize a longitudinal wave.
- It is interesting to note that while the behavior of both acoustic and light waves under reflection, for instance, is the same – though the physical cause is quite different in each case.
- All periodic waves may be described in terms of wavelength (λ), frequency (f), amplitude (A) and wave speed (v).
- Wavelength is the distance between the same points on successive waves, i.e., the spacing between similar points on successive waves.

- Frequency is the number of times per second that the wave completes one full cycle.
- In *transverse* waves a cycle consists of one complete oscillation of the wave medium from its undisturbed position.
- In *longitudinal* waves a cycle consists of one complete pressure fluctuation above and below nominal pressure.
- Wave *amplitude* is the maximum displacement of a wave media from its undisturbed position.
- In transverse waves, the amplitude is the maximum distance the disturbance moves from the undisturbed position.
- In longitudinal waves the amplitude is related to the rise and fall of nominal pressure within the wave medium. The higher the pressure change, the greater the amplitude of the wave.
- In light waves greater amplitudes correspond to brighter light.
- In sound waves greater amplitudes correspond to louder sounds.
- Frequency, wave speed and wavelength are related mathematically by the expression: $v = \lambda f$.
- The velocity a wave in a given media is fixed and is related to the physical characteristics (temperature, density, etc.) of the media. Frequency and wavelength, therefore, are inversely proportional to each other, i.e.; higher frequencies correspond to shorter wavelengths.
- A series of parallel waves all in phase is known as coherent. Similar points on adjacent coherent waves are linked by *wavefronts*.



Transverse Waves

- Transverse waves transfer energy in a direction perpendicular to the direction of the disturbance in the medium.
- A vibrating string is an example of a transverse wave. Although all points on the string itself are constrained to move only up and down, wave pulses move perpendicularly along the length of the string.
- The *wave speed* is the speed with which a pulse moves along the string.

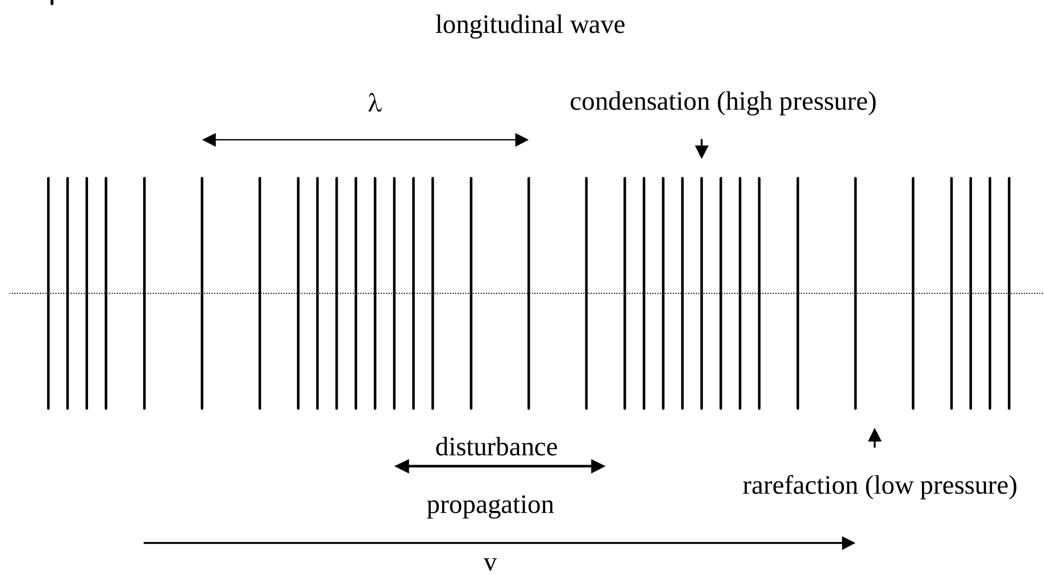


Transverse Wave Train

- A transverse wave may consist of more than one pulse.
- Transverse waves consisting of many pulses occur when the wave source oscillates for long periods of time. Under such conditions an initial pulse is followed immediately by another pulse of opposite displacement.
- A series of such alternating pulses is known as a *wave train*.
- If the oscillations are steady the resulting waveform is periodic.
- If the oscillations are *harmonic* in the series of pulses is known as a *harmonic wave train*.
- All of the electromagnetic spectrum, including light, consists of transverse waves that result from harmonic oscillations in molecular, atomic or nuclear structures, or the oscillations of charged particles in large structures (e.g. dipole radiation).

Longitudinal Waves

- Longitudinal waves are waves that transfer energy in the same direction as the disturbance in the medium of propagation.
- Like transverse waves, longitudinal waves may consist of single or multiple pulses.
- All longitudinal waves consist of regions of high and low density known as condensations and rarefactions that oscillate around local positions parallel to the path of energy transfer.
- If the oscillations are steady the resulting waveform is periodic.
- If the oscillations are *harmonic* in the series of pulses is known as a *harmonic wave train*.
- Sound waves are longitudinal waves. Sound waves propagating through air consist of a series of pressure fluctuations above and below atmospheric pressure.



A longitudinal waveform. In acoustic waves areas of high pressure (pressure nodes) correspond to condensed regions of low molecular displacement (displacement antinodes).

Speed/Velocity of a Wave

Longitudinal (acoustic) and transverse (light) waves have different physical characteristics but they both may be described mathematically by the same set of equations.

- Wave velocity is defined as how fast the disturbance moves through the disturbed medium. Wave velocity is generally constant and points in the direction of energy transport.
- Wave speed is determined by the physical properties of various media through which waves move. In general: $v = \lambda f$
- Transverse waves on a string (all other factors being equal):
    - Large tension $\rightarrow$ higher speed
    - Smaller mass per unit length, $\mu = \frac{m}{\ell}$, $\rightarrow$ higher speed
    - $v = \lambda f = \sqrt{\frac{F}{\mu}}$
- Longitudinal waves in air (sound) – wave speed depends on the density and shear properties of the medium Sound waves move at different speeds through air and water, for instance.

Particle Velocity vs. Wave Velocity

- Particle velocity is non-constant and perpendicular to wave velocity for transverse waves, parallel for longitudinal waves
- Since particle velocity is non-constant its expression must contain information that states position as a function of time.
- For harmonic waves position as a function of time may be easily expressed in terms of sines and cosines.

Harmonic Waves

When a harmonic wave passes through a medium it disturbs the particles in the medium by displacing them from their undisturbed positions in harmonic fashion, i.e., in an elastic manner described by Hooke's Law.

- This disturbance is either parallel to or perpendicular to the direction of energy transport, depending on the type of wave.
- Most sound waves and all light waves originate from essentially the same underlying physical process: that of *simple harmonic motion*.
- Not all sound waves are harmonic in nature. Nonetheless, it convenient and essentially correct to assume that most sound waves act harmonically most of the time.
- Devices that display simple harmonic motion are known as simple harmonic oscillators.
- For a harmonic wave traveling right along the *x*-axis of a coordinate system, assuming that *y* = 0 when *x* = 0 and *t* = 0, the displacement of the medium with respect to time may be expressed as:

$$y = A\sin\left(-x\frac{2\pi}{\lambda} + 2\pi ft\right)$$

For a wave traveling to the left:

$$y = A\sin\left(x\frac{2\pi}{\lambda} + 2\pi ft\right)$$

If $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$ then: $y = A\sin(kx \pm \omega t)$ - where the + sign indicates the wave is moving to the left.

Does this equation remind you of the equations of harmonic motion? Why or why not?

- Note that this expression yields displacement of the medium in a Cartesian plane in the y direction as a function of both time and distance from the origin along the x-axis.
- This represents an extension of our previous displacement as a function of time expression for a harmonic oscillator $x = A\cos(\omega t)$.

In general the displacement of the medium is represented by the general coordinate, ψ, so that:

$$\psi = \psi_0 \sin(kx \pm \omega t) \text{ - where: } k = \frac{2\pi}{\lambda},\ \omega = \frac{2\pi}{T},\ f = \frac{1}{T},\ v = \lambda f = \frac{\omega}{k}$$

This is known as *a traveling wave.*

- For harmonic traveling waves $\psi = \psi_0 \sin(kx \pm \omega t)$ describes the displacement of the medium with respect to the (temporal and spatial) coordinates *t* and *x*.
- ψ is the instantaneous amplitude (height) of the wave and ψ_0 is the maximum height of the wave (displacement of the medium).
- $k = \frac{2\pi}{\lambda}$ is known as the wave number
- $\omega = \frac{2\pi}{T}$ is the angular frequency
- $f = \frac{1}{T}$ is the linear frequency
- $v = \lambda f = \frac{\omega}{k}$ is the wave speed
- It may be shown that the energy transported through a medium by a harmonic wave is given by:

  $$\Delta E = \frac{1}{2}\Delta m \omega^2 A^2 = \frac{1}{2}\mu \Delta x \omega^2 A^2$$

  and that the power generated by such a wave is:

  $$P = \frac{1}{2}\mu v \omega^2 A^2$$

Example 1 A wave moves to the right with the following characteristics: v = 7.1 m/s, A = 0.15 m, λ = 0.28 m. What is a valid expression of a harmonic wave equation for this wave?

$$y = A\sin(kx - \omega t) \rightarrow y = 0.15m\sin\left(\frac{2\pi}{\lambda}x - 2\pi\frac{v}{\lambda}t\right) \rightarrow y = 0.15m\sin(22.4x - 160t)$$

Example 2 Given the wave function $\psi = 2.0cm\sin(2.11rad \cdot m^{-1}x - 3.62rad \cdot s^{-1}t)$
Determine:

• Amplitude	A = 2.0 cm (0.02 m)
• Wave Number	k = 2.11 rad/m
• Wavelength	$\lambda = 2\pi/k$ = 2.98 m
• Angular Frequency	ω = 3.62 rad/s
• Wavespeed	$v = \omega/k = 3.62\ s^{-1}/2.11\ m^{-1}$ = 1.72 m/s (→)
• Linear Frequency	$f = \omega/2\pi$ = 0.58 Hz

Example 3 Write a wave function for a harmonic wave traveling to the left with the following characteristics:

- ψ_0 = 80 cm
- λ = 8 cm
- $f = 3\ s^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{6.28}{0.08m} = 78.5m^{-1}$$

$$\omega = 2\pi f = 6\pi s^{-1}$$

Now $\psi = \psi_0\sin(kx - \omega t)$ so for a wave moving left $\psi = 0.8m\sin(78.5m^{-1}x + 6\pi s^{-1}t)$

The Linear Wave Equation

For 1 dimensional traveling waves it may be shown that the wave equation is:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Solutions to this equation include:

$\psi = A\sin(\pm kx + \omega t)$
$\psi = \psi_0 \sin(kx \pm \omega t)$
$\psi = A\sin(kx)B\cos(\omega t)$

Example 4 Show that $\psi = A\sin(kx)B\cos(\omega t)$ is a solution to the 1-dimensional traveling wave equation $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$.

Taking the partial derivatives:

$$\frac{\partial\psi}{\partial x} = \frac{d\psi}{dx} = Ak\cos kxB\cos\omega t$$
$$\frac{\partial^2\psi}{\partial x^2} = \frac{d^2\psi}{dx^2} = -Ak^2\sin kxB\cos\omega t = -ABk^2\sin kx\cos\omega t$$

$$\frac{\partial\psi}{\partial t} = \frac{d\psi}{dt} = -\omega B\sin\omega tA\sin kx$$
$$\frac{\partial^2\psi}{\partial t^2} = \frac{d^2\psi}{dt^2} = -\omega^2 B\cos\omega tA\sin kx = -AB\omega^2\sin kx\cos\omega t$$

substituting back into the wave equation:

$$-ABk^2\sin kx\cos\omega t = \left(\frac{1}{v^2}\right) - AB\omega^2\sin kx\cos\omega t \rightarrow k^2 = \frac{\omega^2}{v^2} \therefore v = \frac{\omega}{k} \quad Q.E.D.$$

Example 5 Consider two harmonic traveling waves given by:

$$\psi_1 = \psi_0 \sin(kx - \omega t)$$
$$\psi_2 = \psi_0 \sin(kx + \omega t)$$

Show that:

$$\psi_{1+2} = 2\psi_0 \sin(kx)\cos(\omega t)$$

From the CRC handbook:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\sin(\alpha) + \sin(\beta) = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

So $\psi_1 + \psi_2 = \psi_0\left[(\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)) + (\sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t))\right]$

Noting that the 2nd and 4th terms cancel

$$\psi = 2\psi_0 \sin(kx)\cos(\omega t)$$

This is known as the principle of *linear superposition*. If one superimposes two harmonic waves a valid solution for the resultant wave is the superposition of solutions for the individual waves. The principle of superposition is very important in studying a property of wave interaction known as *interference*.

What is the instantaneous amplitude of the resultant wave?

$$\psi = 2\psi_0 \sin kx$$

Describe $\psi_1 + \psi_2$. What are their amplitudes, directions and angular frequencies?

$\psi_1 \rightarrow$
$\psi_2 \leftarrow$
ψ_0
ω

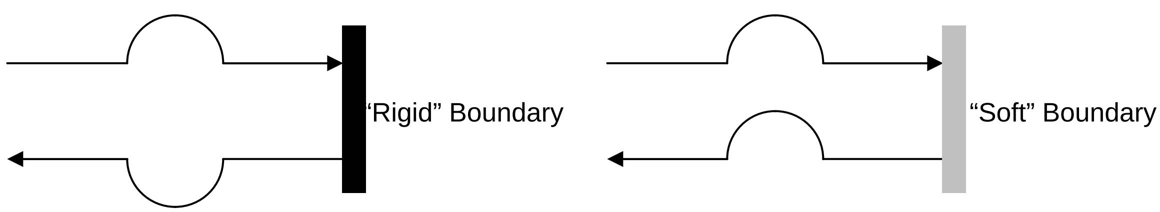
Wave Interactions and Their Qualitative Treatment

Although there are many interactions betweens waves and matter (and waves and other waves), we will concentrate, for the moment, on reflection, refraction, interference and diffraction.

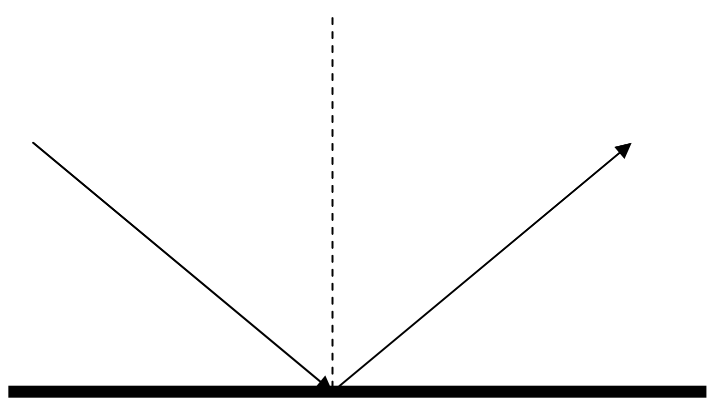
Reflection of Waves

Whenever a wave of any type is incident on a boundary between two media the behavior of the wave at the boundary may be very complex and depends on the wave itself, the angle of incidence, and the properties of both media.

- In general some of the wave is reflected at the boundary some is transmitted across the boundary and enters the new medium.
- If we restrict ourselves for the time being to the case of normal incidence, total or near total reflection in transverse or acoustic waves in solids or fluids, the behavior is much simpler and consists of two general cases, both involving a phase change:

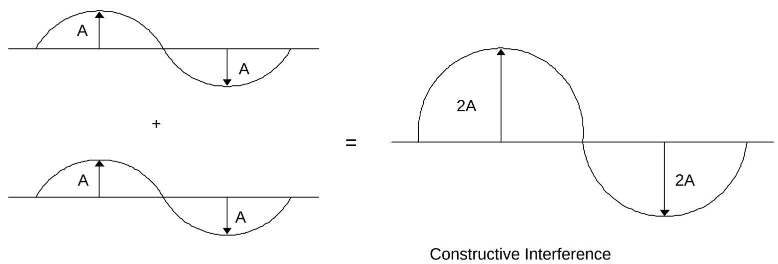


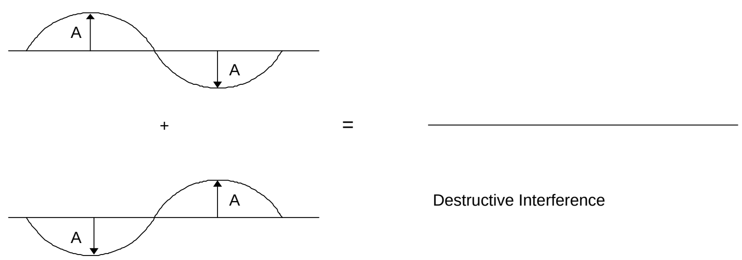
- A rigid boundary is one in which the wave has a difficult time disturbing the new medium. A soft boundary is one in which the wave can disturb the new medium with relative ease. For the case of a boundary of intermediate "hardness" either case is possible.
- For cases of non-normal incidence waves obey the geometric law of reflection: angle of incidence = angle of reflection, coplanar.

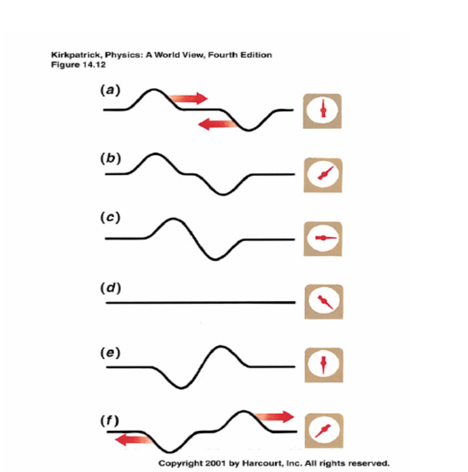


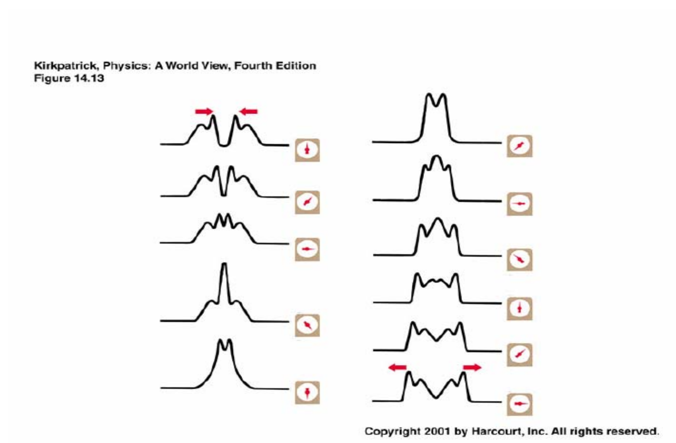
Interference

Interference occurs when two waves are found in the same area of space at the same time. The mathematical treatment of anything other than the simplest case of totally constructive or totally destructive interference of identical waves becomes very complex. We will restrict ourselves, for the present, to these two simple cases.

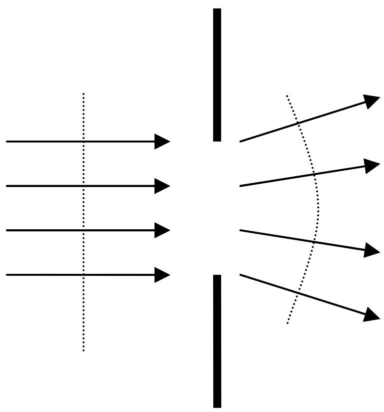


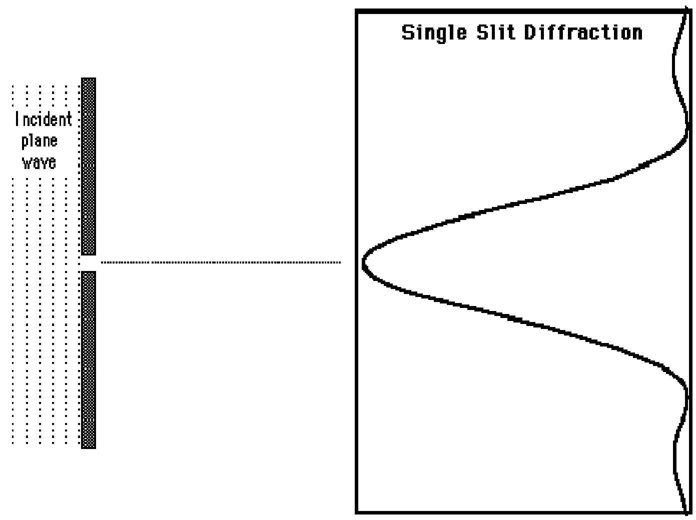






Diffraction - is a spreading of a wavefront that occurs as it passes through a narrow opening or around a sharp corner

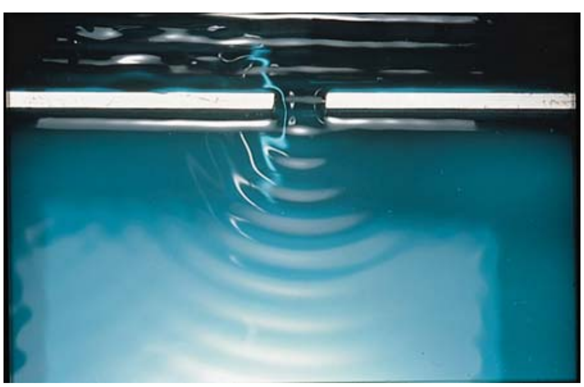




(Courtesy of Hyperphysics)

- For a single opening of width *D*, the bending of the waves produces an interference pattern such that a maxima (due to constructive interference) is found along an axis with the center of the opening and minima due to destructive interference are found on the outside of the central maxima.
- Maxima correspond to bright light or loud sound.
- Minima correspond to dim or no light or sound.

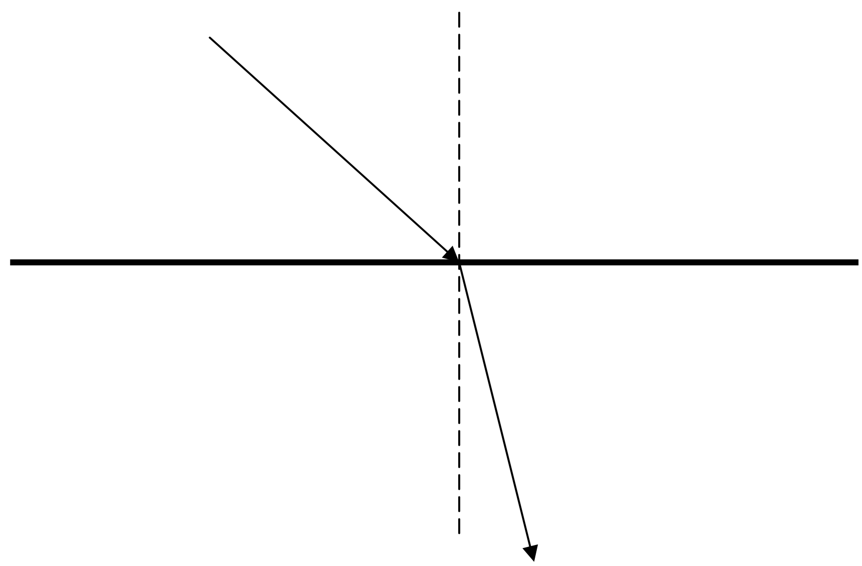
http://hyperphysics.phy-str.gsu.edu/hbase/phyopt/mulslid.html



From *Physics, A World View*, 4th ed. Kirkpatrick

Refraction

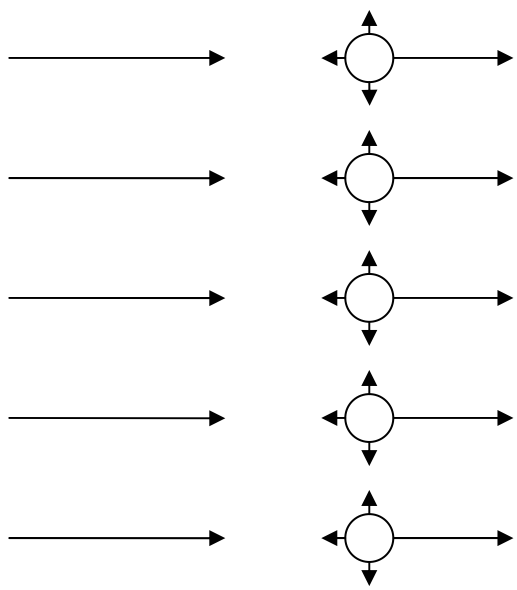
Refraction is a bending or change in direction of a wave as it goes from one medium to another.



- The bending of the wave occurs because the speed of the wave changes as it changes media.
- In the case above the wave has traveled from a less dense to a more dense medium.
- In this case the wave slowed and was bent towards the normal (dashed line)
- Refraction occurs in both transverse and longitudinal waves

Why Waves Reflect, Refract and Diffract

Waves propagate through any dense medium by disturbing the medium. This disturbance takes the form of particles in the medium transferring energy from one particle to the next - generally in the direction in which the wave is moving.



Any point along a wavefront is capable of acting as a new source of the wave. Geometrically it is easy to show that this results in reflection, refraction and diffraction.

How Waves May Be Used to Determine Distance and Speed: Doppler Shift

Any relative motion between an observer and a source of light or any form of e/m radiation results in a *Doppler Shift*, i.e., a shifting of spectral lines toward either shorter or longer wavelengths. Objects moving towards an observer undergo a blue shift and objects moving away from an observer undergo a red shift.