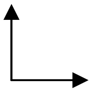
More Vertical Kinematics

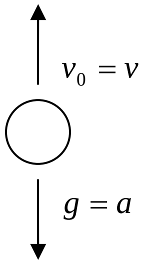
- Acceleration is constant and equal to -9.8m/s^2
- No horizontal vectors
- Remember to keep track of vector signs



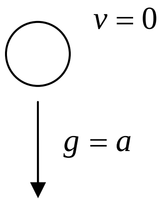
Example 1 Tossing a ball into the air

Consider a ball tossed vertically upward at the three positions shown below. What should the velocity and acceleration vectors look like at each position?

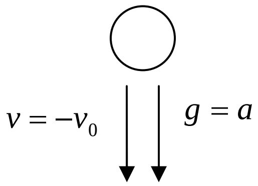
① Instant after release



② Top of arc



③ Same level as release



Example 2 A Parlor Trick

A dollar bill is held centered between the fingers of a person who gets to keep it if they can catch it without moving their hand downward. Who would you bet on?

Bet with the house!

Reaction Time (very good) is about 0.2s

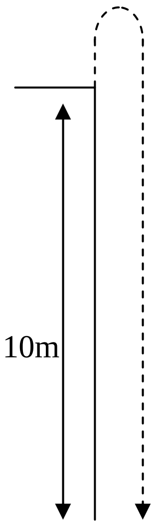
$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$y - y_0 = 0 - (4.9 m \cdot s^{-2})(0.2 s)^2$$

$$y - y_0 = -20 cm$$

So the dollar bill falls about 20 cm in this time. A dollar bill is about 8 cm from center to edge. Professional athletes and racers often have reaction times 0.15 seconds or less.

Example 3 A ball is tossed vertically upward with a velocity of 9.9 m/s as shown below. Find the time of flight (total time in the air), the maximum height the ball achieves, and it's velocity just before hitting the ground.



There are a couple of ways of approaching this problem. One is to break it into two intervals, the other is to take advantage of the continuous nature of the vector equations over this interval.

Method 1: Compute height to which ball rises and time of flight to that point. This is the first interval.

Note: $v = 0$, $v_0 = +9.9m \cdot s^{-1}$, $a = -9.8m \cdot s^{-2}$

To compute the maximum height or distance to the top of the arc:

$$v^2 = {v_0}^2 + 2a(y - y_0)$$

$$\frac{v^2 - {v_0}^2}{2a} = y - y_0$$

$$\frac{-(9.9m \cdot s^{-1})^2}{-19.6m \cdot s^{-2}} = +5m$$

To compute the time of flight: to the top of the arc:

$$v = v_0 + at$$

$$\frac{v - v_0}{a} = t = \frac{-9.9m \cdot s^{-1}}{-9.8m \cdot s^{-2}} = 1.01s$$

The second interval involves an object in free fall.

$$y - y_0 = v_0 t + \frac{1}{2}at^2$$

$$-15m = 0 - \left(4.9m \cdot s^{-2}\right)t^2$$

$$t = \sqrt{\frac{-15m}{-4.9m \cdot s^{-2}}} = 1.75s$$

The total time of flight: may now be found by adding the time from both intervals

1.01s + 1.75s = 2.76s

Method 2 uses the vector nature of the kinematic equations and the continuous shape of the functions of displacement, velocity and acceleration over the interval of this problem.

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$-10m = \left(+9.9m \cdot s^{-1}\right) t - \left(4.9m \cdot s^{-2}\right) t^2$$

This yields a quadratic in *t* with *a* = -4.9, *b* = +9.9, *c* = +10

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: $\sqrt{b^2 - 4ac} = 17.15$

Solving this quadratic yields the roots:

t = -0.41s, *t* = +2.76s

The second root is the one we seek. What is the physical interpretation of the first root?