Resonance in Resistive, Capacitive, Inductive AC Circuits

Objectives:
- To study the phenomena of resonance in AC circuits.
- To observe the effects of capacitors and inductors in AC circuits.
- To analyze AC series RLC circuits.

Equipment: 486DX2 Computer, Electronics Workbench®.

Discussion

You have previously examined the phenomena of resonance in your investigation of the speed of sound in air in last semester's lab. All electrical and mechanical systems have certain normal modes of vibration. Last semester you matched these modes in a column of air to the vibrations of a tuning fork to send the column of air into resonance. When a periodic force, such as a pulse or tap, is applied to system, it vibrates with a frequency equal to that of the force. This type of vibration is knows as a *forced oscillation*. Normally the amplitude of a forced oscillation is quite small, but if the frequency of forced oscillations approaches the frequency of one of the normal modes of the system, the amplitude of the oscillations grows quickly. Resonance occurs whenever the two frequencies match. Theoretically, the amplitude of vibrations at resonance should approach infinity. In mechanical systems friction limits the growth of the amplitude of the oscillations. In electrical systems, resistance plays the same role. In this exercise you will study resonance in alternating current *RLC* (resistive, inductive, capacitive) circuits.

Radios use resonance to tune the receiving circuitry to the broadcast frequency of the station being tuned in. Each radio station broadcasts at a precise carrier frequency. When your receiver is in resonance at this frequency you are in tune with that station. Tuning, in most radios, is accomplished by changing the capacitance of the receiving circuit at a fixed inductance.

Electronics Workbench® computer software will be used to simulate the circuits used in this procedure. In the following exercise *it is extremely important that you not save any changes that you make in the circuits you load from your computer's hard disk*. If you do this you will create problems for whoever follows you. If at any point during the simulation your circuit becomes scrambled, REVERT TO SAVED in the file menu will restore the circuit to its original form.

PROCEDURE

Resonance in AC Circuits

Start Electronics Workbench® and load circuit **`rci1`**. This is a RLC series circuit with an oscilloscope being used to measure the signal produced by the circuit. The AC input is being supplied by a signal generator operating at 100 Hz. Move the cursor to the switch icon on the bar at the top right of the display. This switch is known as the **GO** switch and starts the simulation. Click the **GO** switch on its right side now and see what happens. A trace of an AC waveform should be visible on the oscilloscope screen.

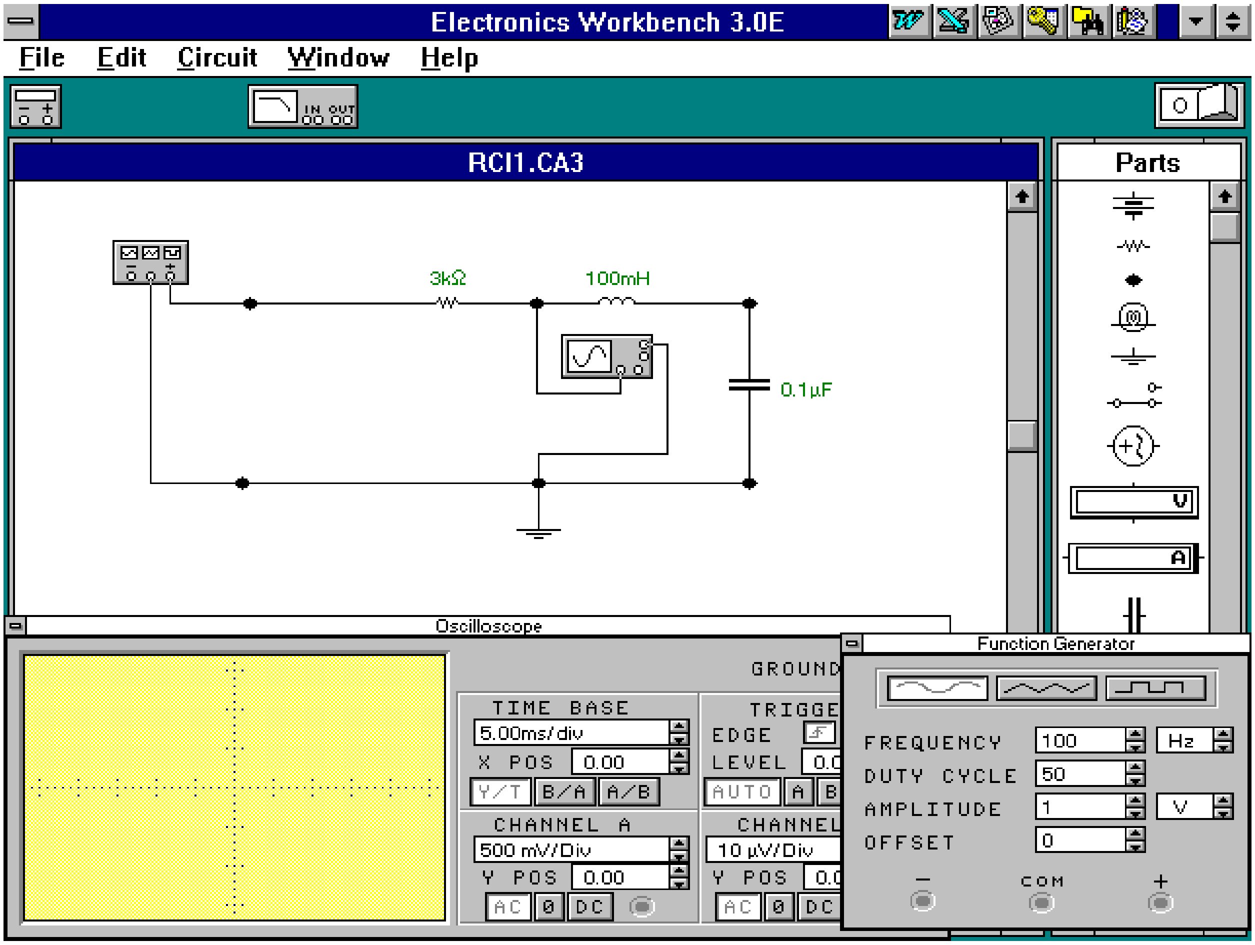

Figure 1.

Change the both the TIME BASE and CHANNEL A settings on the oscilloscope to a setting where a good sine wave can be seen. Vary the frequency produced by the Function Generator both up and down to determine which direction decreases the amplitude (vertical height) of the wave pattern. It may be necessary to make changes in the scope setting to study the pattern when these frequency changes are made.

The resonant frequency of an *RLC* series circuit is given by:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where L is the value of the inductor and C is the value of the capacitor. Does your recorded value match this predicted value?

Series and Parallel Components

Return to the main menu (NEW in the file menu). Load the circuit **rci2**. This is the same circuit as before with an additional coil added in series with the other one. Repeat the steps to determine the resonant frequency of this new circuit.

Return to the main menu and load **rci3**. This circuit is the same has the first with an additional coil added in parallel to the other coil. Find the resonance frequency of this circuit. Be sure you understand how to calculate the impedance of coils in parallel (similar to resistors in parallel)

Load **rci4**. This circuit is the same has the first with an additional capacitor added in parallel with the other capacitor. Find the resonance frequency of this circuit.

Load circuit **rci5**. Again this is like the preceding circuit but, this time the added capacitor is now in series with the other. Find the resonance frequency of this circuit.

Analyzing a Series *RLC* Circuit

Load circuit **rci6**. This circuit is driven by a 150 VAC, 60 Hz source. Recall that the relationship between angular and linear frequency is $\omega = 2\pi f$. What is the angular frequency of this circuit? What is the peak value for the voltage in this circuit? Use the multimeter to record the values of the voltage drops across the inductor, capacitor, and resistor, individually, then record the voltage drop across the entire circuit. How do they compare?

In a purely resistive AC circuit the resistor acts in exactly the same manner as in a DC circuit. The voltage across the resistor and current through the resistor obey Ohm's Law at all times. The instantaneous voltage may be written:

$$v_R = V_{\max} \sin \omega t = I_{\max} R \sin \omega t$$

where the sine term has been introduced to account for the time varying nature of the voltage. The voltage and current increase and decrease at the same time in a resistive circuit and are said to be *in phase*. You should be familiar by now with rms values for voltage and current which *do not* vary with time. The voltage across the resistor you have measured with the multimeter is a rms value. Compute the rms and peak current values, and the peak voltage for the resistive portion of this circuit.

Capacitors behave differently in AC and DC circuits. In an AC circuit the capacitor *impedes* or resists the changing current much like a resistor in a DC circuit. This occurs due to the fact that the current is at a maximum when the capacitor first begins to charge and the potential between the plates is zero. Therefore the voltage and current, which are in phase for a resistor, are 90^0 out of phase for a capacitor. *The current leads the voltage by 90^0 in a purely capacitive AC circuit.* The values for both voltage and current increase and decrease, but with a phase difference of 90^0.

The peak or maximum value of the current in a capacitive AC circuit is:

$$I_{\max} = \omega C V_{\max} = \frac{V_{\max}}{X_c}$$

where X_c is known as the *capacitive reactance* of the circuit and is equal to:

$$X_C = \frac{1}{\omega C}$$

(in ohms). The rms current is given by a similar expression with V_{rms} replacing V_{max}. The instantaneous voltage drop across the capacitor is:

$$v_c = -V_{\max}\cos\omega t = -I_{\max} X_c \cos\omega t$$

where the (-) cosine term has been used to indicate that the voltage lags the current by 90^0 ($-\cos\omega t$ is 90^0 behind $\sin\omega t$). Compute the capacitive reactance, the peak and rms current values, and the peak voltage for the capacitive portion of this circuit.

Inductors also behave differently in AC and DC circuits. As you have seen, inductors act like resistors in AC circuits. Since the voltage across an inductor is proportional to the change in current, the voltage attains its maximum value when the current is changing most rapidly. Since *i* vs *t* is sinusoidal, the maximum rate of change (maximum slope) occurs when the curve goes through zero, i.e., when the current has a value of zero. The current and the voltage are therefore 90^0 out of phase for an inductor. But unlike a capacitor *the current lags the voltage by* 90^0 *in a purely inductive circuit.* The peak or maximum value of the current in an inductive AC circuit is:

$$I_{\max} = \frac{V_{\max}}{\omega L} = \frac{V_{\max}}{X_L}$$

where X_L is known as the *inductive reactance* of the circuit and is equal to $X_L = \omega L$ (in ohms). The rms current is given by a similar expression with V_{rms} replacing V_{max}. The instantaneous voltage drop across the inductor is:

$$v_L = V_{\max}\cos\omega t = I_{\max} X_L \cos\omega t$$

where the (+) cosine term has been used to indicate that the voltage leads the current by 90^0 ($\cos\omega t$ is 90^0 ahead of $\sin\omega t$). Compute the inductive reactance, the peak and rms current values, and the peak voltage for the inductive portion of this circuit.

What is the relationship between the peak and rms currents for each of the three components? Could this result have been predicted? Since the components are in series, *the AC current at all points in the circuit has the same amplitude and phase*. The voltage across each element will have different amplitudes and phases as summarized in Figure 2. State the relationship between peak, rms, and instantaneous voltage and current values for each component in this circuit.

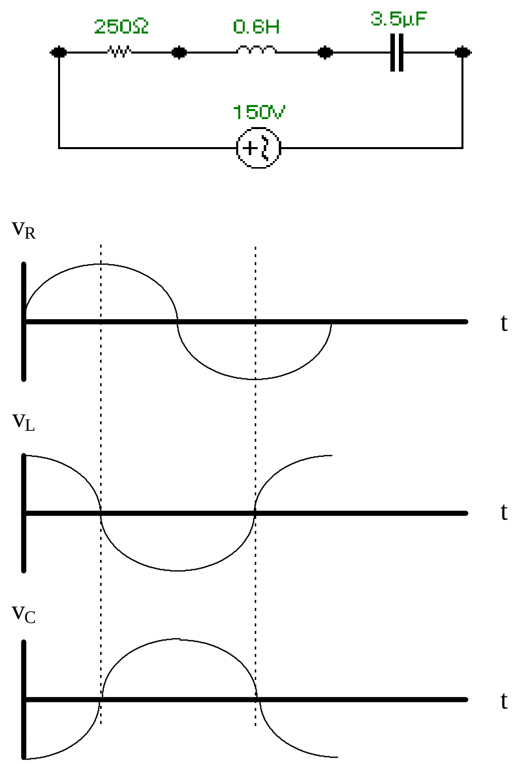

Figure 2. Phase relations in a series RLC circuit.

The peak voltages for a series RLC circuit are given by:

$$V_R = I_{max} R$$
$$V_L = I_{max} X_L$$
$$V_C = I_{max} X_C$$

Since the peak voltage across each of these elements occurs at a different time, these values are of little use without some factor that takes into account the phase differences between these quantities. This factor is known as the *impedance* Z of the circuit. The sum of the voltage drops across the circuit may be written:

$$V_M = \sqrt{V_R^{\ 2} + (V_L - V_C)^2}$$
$$V_M = \sqrt{(I_M R)^2 + (I_M X_L - I_M X_C)^2}$$
$$V_M = I_M \sqrt{R^2 + (X_L - X_C)^2} = I_M Z$$

The impedance of the circuit (in ohms) may then be written $Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$. By using impedance we may write a form of Ohm's law that takes into account the time varying nature of AC circuits: $V_M = I_M Z$. The phase angle ϕ between the current and the voltage for the entire circuit is:

$$\tan\phi = \frac{X_L - X_C}{R}$$

Compute the impedance and phase angle for this circuit. Is the circuit predominantly inductive or capacitive?

When $X_L > X_C$ (typically at high frequencies), the phase angle is positive, meaning that the current lags behind the applied voltage. When $X_L < X_C$, the phase angle is negative and the current leads the applied voltage. When $X_L = X_C$, the phase angle is zero. In this case the impedance matches the resistance and the current is at its peak value. The frequency for which $X_L = X_C$ is the resonant frequency of the circuit. To compute the resonant frequency:

$$\omega L = \frac{1}{\omega C} \therefore \omega = \frac{1}{\sqrt{LC}} \therefore f = \frac{1}{2\pi\sqrt{LC}}$$

which is the same expression given earlier. Compute resonant frequency of this circuit.

Exercises

1. Explain the phase difference between voltage and current in a capacitor. Why does this phase difference occur? Plot v and i vs t to show the phase relationship.

2. Explain the phase difference between voltage and current in an inductor. Why does this phase difference occur? Plot v and i vs t to show the phase relationship.

3. In a series RLC circuit, what determines whether the inductive or capacitive behavior dominates?

4. How does one add inductors in series in an AC circuit? In parallel?

5. Why does the amplitude of oscillations become smaller on an oscilloscope when a series RLC circuit is in resonance?

6. An RLC circuit is used in a radio to tune into a an FM station broadcasting at 99.7 Mhz. The resistance in the circuit is 12Ω and the inductance is 1.40μH. What capacitance should be used?

7. (214) A series RLC circuit has the following values: L = 20mH, C = 100nF, R = 20Ω, and V_{app} = 100 volts. Find the resonant frequency of this circuit, the amplitude of the current at the resonant frequency, and the amplitude of the voltage across the inductor at resonance.