Kirchhoff's Rules

Objectives: To analyze resistive, direct current circuits with Kirchhoff's rules and Ohm's law.

Equipment: Computer, Electronics Workbench®

Discussion

For very simple series and parallel combinations of resistors where it is possible to reduce the circuit to a single equivalent loop, Ohm's law works quite well as a tool for analysis. It is not always possible, however, to reduce a circuit to a single loop. For more complicated circuits Kirchhoff's rules may be used simplify circuit analysis. In this exercise you will study several resistive DC circuits more complicated than those you have examined in previous labs. A computer will be used in this exercise to model a number of different test circuits and instruments.

Kirchhoff's Rules:

The sum of the currents entering any junction must equal the sum of the currents leaving that junction.

The algebraic sum of the changes in potential across all of the elements around any closed circuit loop must be zero.

A *junction* is any point in a circuit where the current has a choice about which way to go. The first rule, also known as the point rule, is a statement of *conservation of charge*. If current splits at a junction in a circuit, the sum of the currents leaving the junction must be the same as the current entering the junction.

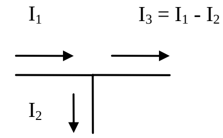
The second rule, also known as the loop rule, is a statement of *conservation of energy*. Recall that although charge is not "used up" as current flows through resistors in a circuit, potential is. As current flows through each resistor of a resistive circuit the potential drops. The sum of the potential drops must be the same as the applied potential.

In the following exercises *it is extremely important that you not save any changes that you make in the circuits you load from your computer's hard disk*. If at any point during the simulation your circuit becomes scrambled, REVERT TO SAVED in the file menu will restore the circuit to its original form.

In general, to apply Kirchhoff'slaws, one proceeds as follows:

1) Identify all of the junctions or branch points in the circuit.

2) Use the point rule to express the unknown currents as few terms as possible, e.g.:



In this case, I_3 may be expressed as the difference of I_1 and I_2. All three currents may then be expressed in terms of I_1 and I_2.

When applying the point rule it is necessary to assume a direction of current flow in each branch of the circuit. Label the terminals of each resistor with a + or - sign depending upon the direction of current flow (recall that we are using the positive test charge model of current , so current flows from the (+) to the (-) end of a resistor). Label each source of EMF as well. If your guess is incorrect for a particular branch, the value that you will eventually obtain for the current in that branch will contain a minus sign. A negative value for current is OK, and should be kept for any subsequent calculations.

3) Identify the current loops that exist in the circuit. Choose any loop and apply the loop rule. Remember that (+) to (-) is a potential drop while (-) to (+) is a potential gain. Write equations for each loop. Remember that the sums of the potential drops and gains must be zero.

4) Reapply the loop rule as needed. For each unknown current you will need to write an equation. The fewer terms in which you express unknown currents, the fewer equations you have to write.

5) Solve the equations to determine the unknown currents.

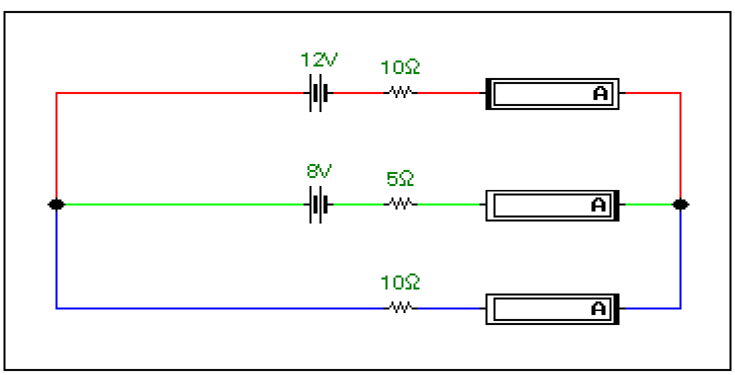
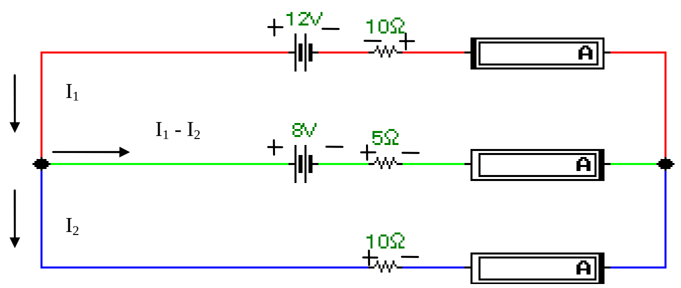


Figure 1.

Start Electronics Workbench and open the file **kir1**. This is a DC resistive circuit consisting of two sources of EMF (*e*lectro*m*otive *f*orce), or electrical potential, and three resistors in parallel. A schematic of this circuit is shown in Figure 1.



Let's apply these six steps to the circuit in Figure 1. The branch points are identified by large dots. The point rule has been used to express the three independent currents in terms of two quantities, I_1 and I_2. Given the polarity of the power supply, let's assume that the current flows from right to left in the top branch (red on the computer screen), and from left to right in the middle (green) and bottom (blue) branches. The resistors have been labeled accordingly (Note: we are ignoring the extremely small resistance of the ammeters). To apply the loop rule we may choose from three current loops: one that includes the upper (red) and middle (green) branches, one that includes the middle (green) and lower (blue) branches, and one that includes the upper (red) and lower (blue) branches. Because the three independent currents have been expressed in terms of two unknowns (I_1 and I_2), the loop rule only needs to be applied twice. Any two of the three loops may be chosen to generate the two necessary equations. Applying the loop rule to the red-green loop (counterclockwise):

$$(1) \quad 12V - 8V - 5\Omega(I_1 - I_2) - 10\Omega(I_1) = 0$$

The first term (12 V) is positive because the current flows from low to high potential. Applying the loop rule to the red-blue loop (counterclockwise):

$$(2) \quad 12V - 10\Omega(I_2) - 10\Omega(I_1) = 0$$

There are several methods that may be used to solve these two equations. Perhaps the most straightforward is to simply combine the equations in such a manner as to eliminate one of the unknowns. The first step in this process is to simplify both equations:

$$(3) \quad 4V - 15\Omega(I_1) + 5\Omega(I_2) = 0$$
$$(4) \quad 12V - 10\Omega(I_1) - 10\Omega(I_2) = 0$$

If the first equation is multiplied by 2, the third term in the first equation cancels the third term in the second:

$$(5) \quad 8V - 30\Omega(I_1) + 10\Omega(I_2) = 0$$
$$(6) \quad 12V - 10\Omega(I_1) - 10\Omega(I_2) = 0$$

Now if these two equations are added together:

$$20V - 40\Omega(I_1) = 0$$

Or I_1 = .5 amperes (500 mA). We can plug the value of I_1 into either equation 5 or 6 and solve for I_2. Using equation 6:

$$12V - 5V - 10\Omega(I_2) = 0$$

or I_2 = .7 amperes (700 mA). Using the point rule, I_3 is equal to I_1 - I_2 or -.2 amperes. Click the GO switch on the Electronics Workbench computer screen. Are these values verified? (Note: the shaded end of each ammeter is the negative (-) or low potential terminal)

It should be obvious by now that there are several ways to apply Kirchhoff's rules to a circuit such as the one we have examined. Even though choosing different current loops, or assuming different directions for current flow will result in different equations, the values obtained for I_1, I_2 and I_3 should be the same no matter how one works the problem. To verify this apply the loop rule to the same current loops as we used in the previous example but do so in a clockwise manner. What values do you obtain?

You will be responsible for analyzing two more circuits: **kir2** and **kir3.** In each case you are to follow the steps outlined above and solve for the current in each branch of the circuit. You may then activate the circuit and check your solutions.

Questions

1. Why does the point rule arise as a consequence of conservation of charge?
2. Why does the loop rule arise as a consequence of conservation of energy?
3. What is the relationship between the number of branch points and the number of independent currents in a circuit?
4. Why is Ohm's law not an effective method for solving circuits such as those studied in this procedure?