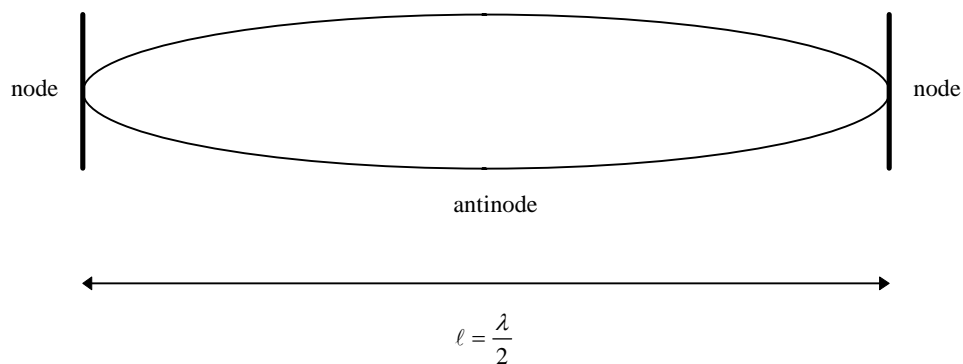


# Standing Waves On a String

**Apparatus:** 9753 mechanical wave driver, sine wave generator, string, mass set.

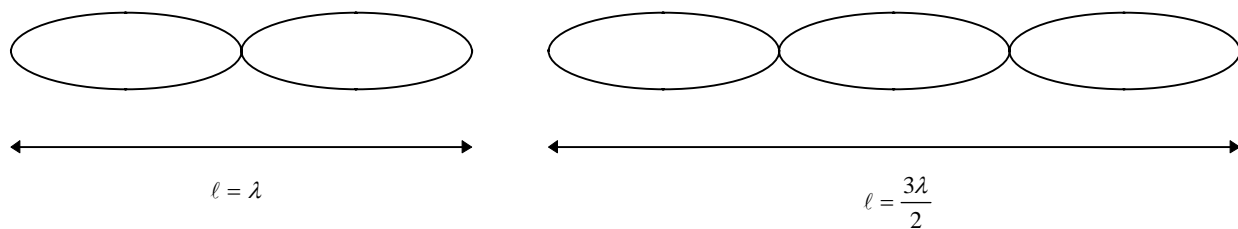
**Objective:** The purpose of this procedure is to study an example of a simple oscillating system, a standing wave produced on a stretched string.

**Physics Theory:** When a string stretched between two fixed points is plucked, it vibrates in its *fundamental mode* (Figure 1), also known as the *first harmonic*. If the string is forced to oscillate at



**Figure 1. Fundamental mode of a wave (first harmonic).**

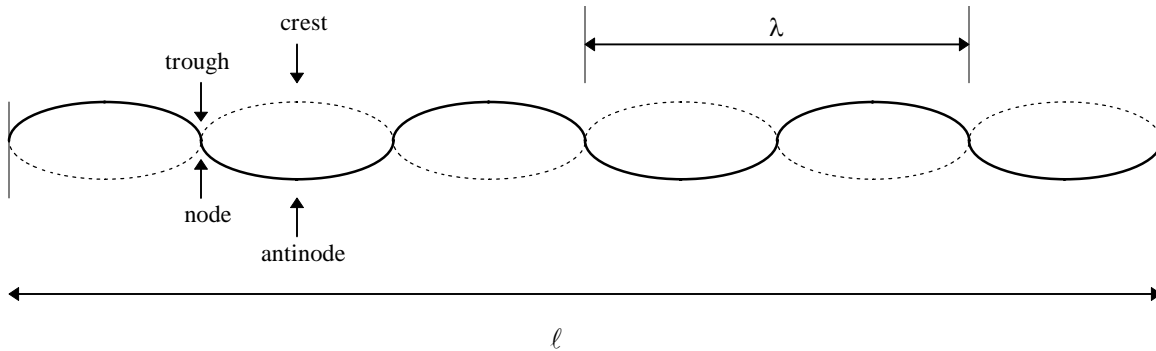
this fundamental frequency by some sort of mechanical driver, a *standing wave* will form. A standing wave is a stable wave pattern consisting of an incident and a reflected wave in which the *crests* and *troughs* of both waves maintain fixed positions. Standing waves will also be formed if the string is driven at frequencies that are integer (whole number) multiples of the fundamental frequency (Figure 2). Notice that the number of “loops” or segments in each of the first three figures



**Figure 2. Second and third harmonics.**

corresponds to the harmonic number of the standing wave pattern. For a given harmonic, the wavelength,  $\lambda$ , may be written:  $\lambda = \frac{2\ell}{n}$ , where  $\ell$  is the length of the string between the fixed endpoints and  $n$  is the number of loops in the string.

The *velocity* of a wave may be expressed as the product of its wavelength and frequency:  $v = \lambda f$ . The velocity of a wave is the speed at which a fixed point on the wave propagates through the medium in which the wave is traveling. For a wave traveling down a string the velocity may be expressed:  $v = \frac{2\ell f}{n}$ . A standing wave occurs when there is exactly an integral number of half waves ( $\lambda/2$ ) between the ends of the string. In order to produce a standing wave pattern, the velocity of the incident and reflected waves must be such that  $\ell$  is an integral multiple of half wave lengths, or:

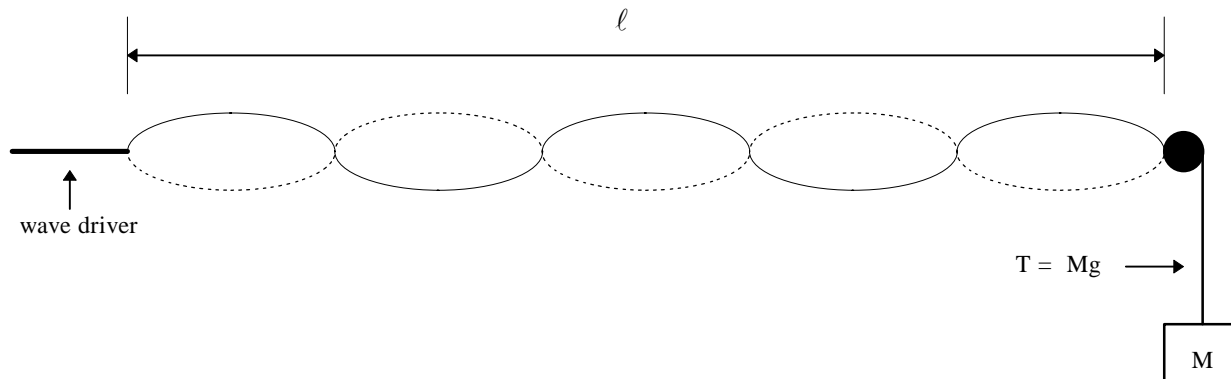
$$\ell = n \frac{\lambda}{2}.$$


**Figure 3. A standing wave on a string.**

In the case of strings on musical instruments, such guitars, violins, or pianos, it is often convenient to express the velocity of a wave as a function of string tension and linear mass density:  $v = \sqrt{\frac{T}{\mu}}$ , where  $T$  is the tension in the string as  $\mu$  is the mass per unit length of the string. Setting this expression for velocity equal to the former expression for velocity and solving for tension yields:  $T = \frac{(4\mu\ell^2 f^2)}{n^2}$ . If the tension in the string is varied while the length and frequency are held constant, a plot of  $T$  vs.  $\frac{1}{n^2}$  will yield a straight line with a slope of  $4\mu\ell^2 f^2$ . So by computing the slope one may determine the linear mass density of a string. Alternatively, one may rearrange the equation for tension and solve for frequency:  $f = \sqrt{\frac{T}{4\ell^2 \mu}} n$ . Now if the frequency is varied while the tension and length are held constant, a plot of  $f$  vs.  $n$  will yield another straight line with a slope of  $\sqrt{\frac{T}{4\ell^2 \mu}}$  that may be used to compute the linear mass density of the string.

**Experimental:** A string, wave generator and wave driver have been provided for you. Your lab instructor will assist you in making sure that this device is correctly set up.

**(Part I)** We will begin by examining the effect of varying frequencies on standing wave patterns. Place a *total* of 500 grams on the mass hangar. Turn on the wave generator and set the frequency to 40 Hz. Adjust the amplify knob to about halfway.



**Figure 4. Standing wave apparatus.**

Vary the frequency by use of the frequency dial on the left of the wave generator until the string is vibrating in its fundamental mode (largest displacement in the center of the string). Once the fundamental resonance has been reached, fine tune the frequency by use of the middle dial on the generator until the nodes at each end of the string are clearly defined (i.e., sharp instead of fuzzy). Mark this frequency as the fundamental by pushing and holding the store button. Press the increment up to double the driving frequency. What phenomenon do you observe? If necessary, fine tune the frequency to achieve the best standing wave pattern. The frequency will increase by the fundamental frequency every time the increment up is pushed. Observe all of the first seven harmonics and be sure to record each of the driving frequencies. **Note:** the 5th, 6th, and 7th harmonics may be difficult to observe. Record their frequencies anyway. To stop the wave driver at any time, turn down the “amplify” knob.

**(Part II)** Now that you have had the opportunity to observe the variation of standing wave patterns with driving frequency, the next step is to examine the variation with string tension. Compute and record the tension in the string created by 500 grams of mass, and measure the length of the string between the post on the wave driver and the pulley ( $\ell$ ).

Return the driver to its fundamental frequency. Now instead of varying the driver frequency to reach the higher harmonics, change the tension in the string by changing the mass on the hanger. Notice that changing the mass by as little as a single gram may have a profound effect on the standing wave pattern. Record the mass and tension associated with each harmonic through the first seven. Be sure to turn off the sine wave generator when you are finished.

## Data Analysis:

- Weigh a known length (at least 10 meters) of the same string used in the experimental setup and calculate its linear mass density. Record this value in your lab notebook as  $\mu_{\text{direct}}$ .
- Plot the frequency vs.  $n$  data from part I. Determine the best straight line that fits these points and find the slope of the line. Using the slope of the line, compute the linear mass density of the string. Record this value as  $\mu_{\text{frequency}}$ .
- Plot the tension vs.  $1/n^2$  data from part II. Determine the best straight line that fits these points and find the slope of the line. Using the slope of the line, compute the linear mass density of the string. Record this value as  $\mu_{\text{tension}}$ .
- Compute the mean and standard deviation of all three computations for the linear mass density of the string. Are they statistically the same? Compute a % error for these values if appropriate.

## Questions:

1. What effect does increasing frequency have on the harmonics of a standing wave?
2. What effect does decreasing tension have on the harmonics of a standing wave?
3. What effect does increasing  $T$  have on  $v$ ?
4. What effect does increasing  $f$  have on  $v$ ?
5. Show that  $\frac{(4\mu\ell^2 f^2)}{n^2}$  has units of force.
6. Show that  $\sqrt{\frac{T}{4\ell^2 \mu}} n$  has units of frequency.