Projectile Motion

Apparatus: Ballistic pendulum/projectile launcher, inclined plane, 2-meter sticks, meter stick, carbon paper, protractor, wooden blocks, clamps.

Objective: The purpose of this experiment is to study parabolic motion in a plane, i.e., motion in two dimensions under the influence of a single force (gravity).

Physics Theory: Consider a system such as that shown in Figure 1., where a projectile is fired horizontally with an initial velocity of v_0.

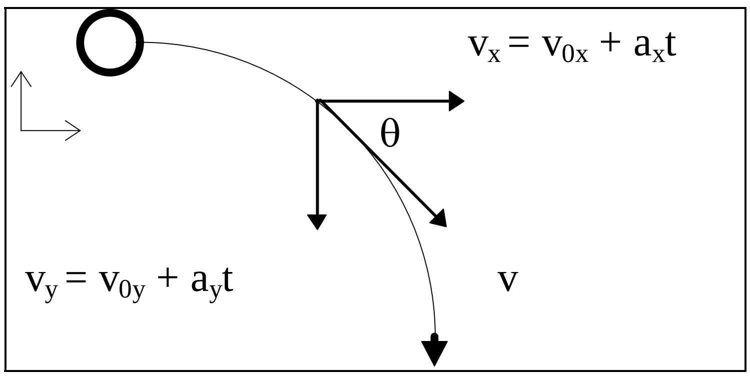

Figure 1.

The instantaneous velocity at any point along the path of the projectile may be expressed as a vector, which may be written in terms of its components:

$$\vec{v}_x = \vec{v}_{0x} + a_x t \quad (1)$$
$$\vec{v}_y = \vec{v}_{0y} + a_y t \quad (2)$$

Where $v_{0x}= v_0\cos\theta$ and $v_{0y}= v_0\sin\theta$. Since the launch angle is zero the initial *y* velocity is also zero. The acceleration in the *x* direction is zero (why?) and the acceleration in the *y* direction is -9.8 m/s^2. We have, therefore:

$$\vec{v}_x = \vec{v}_{0x} \quad (3)$$
$$\vec{v}_y = -(9.8 m/s^2)t \quad (4)$$

In this special case where the projectile is launched horizontally, the components of the velocity vector depend only upon initial velocity, the acceleration due to gravity, and time. In the more general case where the launch angle may vary:

$$\vec{v}_x = \vec{v}_0 \cos\theta \quad (5)$$

$$\vec{v}_y = \vec{v}_0 \sin\theta - (9.8 m/s^2)t \quad (6)$$

the components of velocity depend upon initial velocity, acceleration due to gravity, launch angle, and time.

We may also write equations that relate the components of velocity to displacement. Using the kinematic relationship for displacement in terms of velocity, time and acceleration: $x - x_0 = v_0 t + \frac{1}{2} a t^2$ we find that:

$$x - x_0 = (v_0 \cos\theta)t \quad (7)$$

$$y - y_0 = v_0 (\sin\theta)t - \frac{1}{2} g t^2 \quad (8)$$

These equations may be combined by solving the first equation for *t* and substituting it into the second equation (try it!). If we assume that $x_0 = 0$, the result is:

$$y - y_0 = x \tan\theta - \frac{x^2 g}{[2(v_0 \cos\theta)^2]} \quad (9)$$

This equation may be rearranged to solve for *x*. One may then predict the horizontal distance a projectile will travel before striking the floor, given the height of the launcher above the floor, the launch angle, and the initial velocity of the projectile.

$$\frac{x^2 g}{[2(v_0 \cos\theta)^2]} - x \tan\theta + (y - y_0) = 0 \quad (10)$$

Notice that this is a quadratic equation in *x*. Does this surprise you? What is the shape of the graph of a quadratic equation? What is the shape of a projectile's path?

Experimental: Part I - Calibration

The first step in the experimental portion of this procedure is to calibrate the projectile launcher. Set up the device near the end of your bench making sure that it is level. Load a ball onto the launcher and measure the distance from the bottom of the ball to the floor. Next fire the ball several times (how many times are enough to insure good statistics?) carefully measuring the range, or the distance from the point where the ball leaves the launcher to where it lands for each shot (Note: It will be left up to you to determine an effective method for doing this but you lab instructor will be glad to advise you on some commonly used strategies for accomplishing this). Compute the mean and standard deviation of this data set.

Now that you have a good value for the average range for the projectile launcher when fired from a horizontal position at a set height, you must devise a means for determining the initial velocity of the projectile, v_0. One may compute this directly using equation (9). Another method is to compute the time of flight of the projectile. Since the time of flight is independent of the motion in the x direction, one may use equation (8) to solve for time, then use this value to solve for v_0 in equation (7). Once you have obtained a value for v_0, you have finished the calibration phase of this procedure.

Part II - Investigation of Range as a Function of Launch Angle

Place the projectile launcher on the floor. Fire the projectile at angles of 20^0, 30^0, 40^0, 50^0, 60^0. Make enough shots at each angle to acquire good statistics. Then make a plot of average range vs. launch angle (be sure to include error bars in this plot). Next compute the theoretical range for the projectile at each launch angle using your calibrated value of v_0. Superimpose a plot of the theoretical range values over the experimental values on the same graph. How do the two curves compare? What are the comparative angles for which one achieves maximum range in this experiment?

Note: A spreadsheet (sophl1pm.xls) has been prepared to aid you with computation in this procedure

Questions:

1. Compare the two curves generated in this procedure. Is this experiment well designed? Why or why not?

2. How do the ranges at each angle compare with the ranges at each complementary angle?

3. Why was the acceleration in the x direction always zero in this exercise?

4. Why is time of flight independent of motion in the x direction? Does this make intuitive sense? Does this mean that if one fires a bullet from a rifle and drops another bullet at the same moment they will both hit the ground at the same time?