

Freely Falling Body/Spreadsheet Physics

Objective: The purpose of this experiment is to measure the position, velocity and acceleration of an object in free fall, the latter quantity being known in physics as the ubiquitous constant g .

Equipment: A spark tape, a meter stick, a PC with Microsoft Excel.

Physics Theory: Recall the relationship for distance in terms of time and initial velocity for a freely falling body:

$$y - y_0 = v_0 t + \frac{1}{2} g t^2$$

For an object in free fall starting from rest at height $y_0 = 0$, this relationship may be written:

$$y = \frac{1}{2} g t^2$$

where $g = -9.80\text{m/s}^2$ close to the earth's surface (if one ignores the effects of air friction).

Experimental: In this experiment you will use a tape created by a spark apparatus. This device consists of a metal plummet suspended between two long wires. One of the wires is connected to a high voltage spark generator while the other is connected to ground. When the plummet is released it falls between the two wires to the ground. If a paper tape is placed between the plummet and one of the wires, a spark will travel from the high potential wire, through the tape and plummet, to the low potential wire leaving a small hole in the tape. If the spark generator is set up to produce sparks at regular time intervals, a series of small holes will be produced in the tape at increasing distance intervals.

Without synchronizing the spark generator with the release of the plummet, we do not have enough information make a quantitative statement about exactly when $t = 0$. As it turns out, we do not know exactly where $y = 0$ either. It is possible, however, to determine the velocity and acceleration of the plummet as functions of *position* by measuring the discrete distance between spark holes (Δy) at a fixed time interval (Δt).

You will be provided with a tape that has already had the spark holes burned into it by the plummet apparatus. You are to guess the distance from the first hole (y_1) to the point y_0 , and mark this point on the tape. You will then measure, as accurately as possible, the distance from the point y_0 to each successive spark hole in the tape.

Data Analysis: Let's define y_i as the distance to spark hole i from some arbitrary point $y = 0$ (y_0), approximately near where $y = 0$ should be, along the same line as the other spark holes. The time from release of the plummet to the i^{th} spark hole is $t_i = t_0 + i\Delta t$, where we do not know t_0 . The average velocity between spark hole i and spark hole $i + 1$ is $v_i = \Delta y_i / \Delta t$ where $\Delta y_i = y_{i+1} - y_i$. The average acceleration is then $a_i = \Delta v_i / \Delta t$ where $\Delta v_i = v_{i+1} - v_i$.

It should be obvious by now that this experiment lends itself to spreadsheet manipulation due to the number simple, repetitive calculations required. An Excel spreadsheet has been prepared for your use. A copy of it is in the /excel/lab223 directory on the computer at your table (Figure 1). All that is necessary is for you to enter your measurements of y_i in the spaces provided. You will need to enter twelve values for y_i into the spreadsheet, starting with $i = 0$ at $y_i = 0$.

Difference Table for Freely Falling Body Engineering Physics Laboratory PHYS 223						
		$\Delta t =$	0.05			
i	y_i	Δy_i	v_i	$\Delta^2 y_i$	a_i	$\Delta^3 y_i$
0	0					
		0				
1		0	0	0		
		0			0	0.0000
2		0	0	0		
		0			0	0.0000
3		0	0	0		
		0			0	0.0000
4		0	0	0		
		0			0	0.0000
5		0	0	0		
		0			0	0.0000
6		0	0	0		
		0			0	0.0000
7		0	0	0		
		0			0	0.0000
8		0	0	0		
		0			0	0.0000
9		0	0	0		
		0			0	0.0000
10		0	0	0		
		0			0	0.0000
11		0	0	0		
		0			0	0.0000
12		0				
					0	mean
					0	std

Figure 1. Spreadsheet used to compute the acceleration due to gravity of an object in free fall.

When you examine the data you will notice that the values in the y_i column increase with each data point, showing the total distance the plummet has moved. You will also notice that the *first difference* Δy_i , which is proportional to the plummet's velocity, increases with each data point due to the increasing distance the plummet falls during each successive fixed interval of Δt . The *second difference*, $\Delta^2 y_i$ ($\Delta y_{i+1} - \Delta y_i$), is proportional to acceleration and should remain fairly constant. The acceleration of the plummet itself is given in the next column. Since the purpose of this procedure is to determine the acceleration due to gravity of an object in free fall, a mean and standard deviation are computed by the spreadsheet for this column. Finally, a *third difference* column, $\Delta^3 y_i$ ($\Delta^2 y_{i+1} - \Delta^2 y_i$), has been included that should change both values (slightly) and sign rather randomly.

From your calculus background you should recognize that the discrete expressions we have used for velocity and acceleration may be written:

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} = v \quad \lim_{\Delta t \rightarrow 0} \frac{\Delta^2 y}{\Delta t^2} = \frac{d^2 y}{dt^2} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = a$$

and are actually time derivatives of the displacement of the plummet.

Questions:

1. How do you know, from the spreadsheet data, that the velocity is increasing in this experiment?
2. In the spreadsheet, which columns indicate that acceleration is uniform. Why?
3. What is the significance of the *third difference* ($\Delta^3 y_i$) in the spreadsheet? What is implied by the random nature of the sign change of this quantity?
4. Make a plot of v vs. y from your data. What does this plot tell you?
5. Make a plot of a vs. y from your data. What does this plot tell you?