Atwood's Machine

Apparatus: Two wooden blocks, string, Smart Pulley, computer, PASCO Precision Timer and series 6500 interface, rods, clamps and other hardware.

Objective: The purpose of this procedure is to study Atwood's machine, a commonly used device consisting of two blocks and a light pulley that may be used to demonstrate the consistency of Newton's Laws with energy conservation and the Work-Energy relationship.

Physics Theory: Atwood's machine is shown schematically in Figure 1. This device consists of two blocks connected by a light string running over a massless pulley suspended some distance above a floor or table. If one of the masses is greater than the other (e.g., $m_1 > m_2$ in figure 1), the system moves as shown. If one assumes a light (i.e., massless) pulley and light strings that do not stretch, this system lends itself to analysis using either Newton's Laws, the Work-Energy principle or with Conservation of Energy. In this procedure you will analyze Atwood's machine using all three methods listed above and compare theoretical values derived from an idealized Atwood's machine with empirical values obtained using the real McCoy.

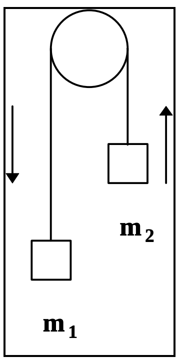
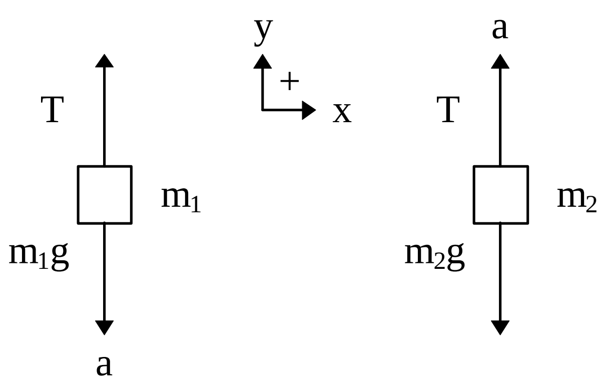


Figure 1.

Analysis I. Newton's Laws

Let's construct free body diagrams for each of the masses in figure 1. Since we are assuming a "light" pulley, we don't have to worry about its inertia, therefore we have:



Assuming that up and right are positive directions, using Newton's second law we can write:

$$\Sigma F = ma$$
$$T - m_1 g = -m_1 a$$
$$T - m_2 g = m_2 a$$

Notice that the only effect that the pulley has on this system is to redirect T from one block to the next. Since the magnitude and sign of T are unaffected:

$$m_1 g - m_1 a = m_2 g + m_2 a$$
or
$$m_1 g - m_2 g = m_1 a + m_2 a$$
$$(m_1 - m_2)g = (m_1 + m_2)a$$
$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

If one is able to measure the distance that either mass moves during the operation of this mechanism, then the final velocity of the system (that is, the velocity of the both blocks, which are the same, just before m_1 hits the floor) may be calculated as a function of acceleration and distance:

$$v^2 = v_0{}^2 + 2a(y - y_0)$$
or
$$v = \sqrt{v_0{}^2 + 2\frac{(m_1 - m_2)g}{m_1 + m_2}(y - y_0)}$$

$$v = \sqrt{2g(y - y_0)\frac{(m_1 - m_2)}{m_1 + m_2}}$$

Analysis II. Work-Energy Principle

The translational kinetic energy of this system (i.e., the kinetic energy of the moving blocks) changes as the speed of the moving blocks increases. The *work-energy principal* states that *the net work done on an object is equal to its change in kinetic energy*, or: $W_{net} = \Delta_{KE}$. In order to apply the work-energy principal to Atwood's machine we must compute the work done by the net force acting on each block as the blocks move.

$$\text{Net Work} = \Delta\text{KE} = \tfrac{1}{2}(m_{1+} m_2)v_{\text{final}}{}^2 - \tfrac{1}{2}(m_{1+} m_2)v_{\text{init}}{}^2$$

Let's assume that the blocks both move some distance h during this process starting from rest. Since the blocks are attached to each other by a cord they move at the same rate of speed. The velocity of the blocks just before block 1 hits the floor may then be computed by using the work-energy principal, recalling that work is also defined as being equal to Force × Distance.

$$W_{net} = \Delta KE = \tfrac{1}{2}(m_1 + m_2)v_{final}^2 - \tfrac{1}{2}(m_1 + m_2)\, v_{init}^2 = (F_{net})(h)$$

Recall from analysis I that the net force acting on the system is equal to $m_1 g - m_2 g$. Hence:

$$W_{net} = \Delta KE = \tfrac{1}{2}(m_1 + m_2)v_{final}^2 - \tfrac{1}{2}(m_1 + m_2)\,(0)^2 = (m_1 - m_2)(g)(h)$$

or

$$v_{final} = \sqrt{2gh\frac{(m_1 - m_2)}{(m_1 + m_2)}}$$

Analysis III. Conservation of Energy

When there are no non-conservative forces (e.g., friction) acting on a system the total mechanical energy of the system remains constant in any process. Anytime a quantity remains constant as a system changes it is said to be *conserved*. Our simple Atwood's machine is an example of such a system. Since the total kinetic and potential energy remains constant:

$$\Delta KE + \Delta PE = 0$$

or

$$KE_{init} + PE_{init} = KE_{final} + PE_{final}$$

or

$$\tfrac{1}{2}(m_1 + m_2)v_{init}^2 + m_1 g h_{init} + m_2 g h_{init} = \tfrac{1}{2}(m_1 + m_2)v_{final}^2 + m_1 g h_{final} + m_2 g h_{final}$$

Since the blocks are connected by a cord they will travel at the same speed. If we assume that the blocks begin at rest, with block 2 resting on the floor, that the height of the floor is zero, and that the height of block 1 just before it hits the floor is essentially zero, the speed at the moment before block 1 hits the floor is:

$$\tfrac{1}{2}(m_1 + m_2)(0)^2 + m_1 g h_{init} + m_2 g(0) = \tfrac{1}{2}(m_1 + m_2)v_{final}^2 + m_1 g(0) + m_2 g h_{final}$$

or

$$m_1 g h_{init} = \tfrac{1}{2}(m_1 + m_2)v_{final}^2 + m_2 g h_{final}$$

or

$$v = \sqrt{2gh\frac{(m_1 - m_2)}{(m_1 + m_2)}}$$

In all three methods of analysis we have assumed $m_1 > m_2$. How would each analysis change if this were reversed, i.e., $m_2 > m_1$? Before proceeding to the experimental part of this procedure you are to analyze Atwood's machine by the same three methods used before except assuming $m_2 > m_1$.

Experimental: In this experiment you will use the PASCO 6500 Interface, the Precision Timer program, and a Smart Pulley. The Smart Pulley uses a photogate to convert analog data (the speed and number of rotations of the pulley) to a digital form that may be processed by a computer. Your lab instructor will help you set up the equipment you need for this experiment.

- Turn on the computer, enter the WINDOWS environment, click the 213/214 icon and run the Precision Timer Program. Hit [ENTER] twice to get to the main menu.

- Since $m_2 > m_1$ for each set of blocks, you will begin by holding block 1 down on the table.

  [M] - **Motion Timer**; [ENTER]

- This step begins timing. Release block 1. Hit [ENTER] again as soon as block 2 hits the table. Hit [ENTER] again to return from the data display to a menu.

  [G] - **Graph Data**; [ENTER]
  [V] - **Velocity vs. Time**; [ENTER]
  [A] - **Smart Pulley**; [ENTER] (10 spoke string in groove)
  On - [R] - **Regression Line**; [ENTER]
  On - [S] - **Statistics**; [ENTER]
  X axis autoscaling (axis starts at 0)
  Y axis autoscaling (axis starts at 0)

Observe the shape of the velocity vs. time graph and sketch it in your lab notebook.

[T] - Display Table of Values

The velocity should increase until the moment just after block 2 hits the table. Throw out any data points after this and use the highest velocity in the table. Repeat the steps above for enough trials to insure good statistics. Compute the mean and standard deviation of the final velocity for this system.

Using *your* formula derived from Conservation of Energy (i.e., the formula you derived assuming $m_2 > m_1$), determine the theoretical value for the final velocity of your Atwood's machine. What is your percent error?

Questions:

1. The percent error you arrived at should be very small. What factors influence this?

2. Is Atwood's machine a conservative or a non-conservative system?

3. How would you redesign this experiment to provide greater accuracy?

4. Explain the shape of the velocity vs. time graphs you observed during your trials.