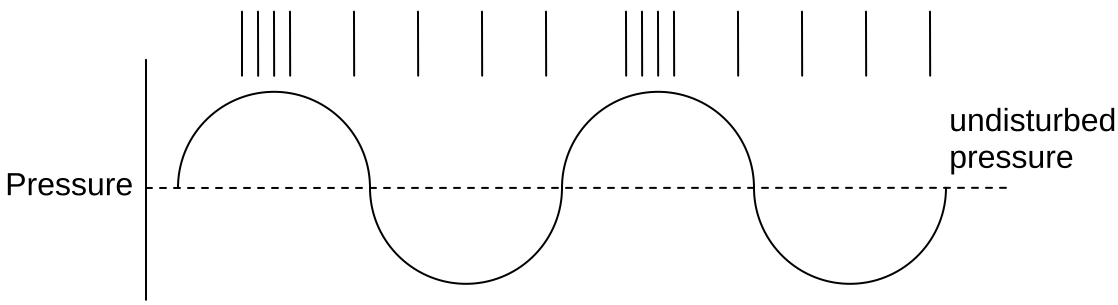
Sound Waves I

Sound Pressure Level

Sound or Acoustic waves are longitudinal waves.



Greater Amplitude → Greater SPL

- The loudness of a sound wave is related to its amplitude.

- In acoustic waves amplitude may be though of as either a fluctuation in pressure above and below nominal pressure or a displacement of molecules from some undisturbed position.

- The pressure fluctuations are *not* in phase with the displacement of the air molecules (they are approximately 90° out of phase due to the Bernoulli effect).

- When we speak of amplitude in a sound wave in air we are generally speaking of *pressure amplitude* rather than the displacement amplitude of the air molecules.

- Since the fluctuations in pressure that constitute acoustic waves tend to be very small, the *change* in pressure amplitude rather than absolute pressure amplitude is most significant.

- Δp_m, the maximum increase or decrease in pressure, is defined as the pressure amplitude of an acoustic wave.

- Pressure amplitude (*A*) is related to displacement amplitude. In air $\Delta p_m = v\rho 2\pi fA$, where ρ the density of air and v is the speed of sound in air and f is the frequency of the wave (typically taken to be 1kHz).

- The maximum pressure amplitude Δp_m that the human ear can accommodate without temporary damage is about 30 Pascals (around 123 dB SPL) and is many orders of magnitude less than static standard atmospheric pressure which is about 10^5 Pa.

- The displacement amplitude of air molecules corresponding to an acoustic wave producing 123 dB SPL at 1kHz is:

$$A = \frac{\Delta p_m}{v\rho 2\pi f} = \frac{30Pa}{(343m/s)(1.21kg/m^3)(2\pi)(1000Hz)} = 1.1\times 10^{-5} m = 11\mu m$$

- Notice that the actual displacement of air molecules for even the loudest sound that the ear can tolerate is quite small - about 1/7 the thickness of the paper upon which this page is printed.

- The pressure amplitude for the faintest sound (0 dB SPL – by definition) that the ear can detect at 1 kHz is about 2.8×10^{-5} Pa.

- The faintest sound the human ear can detect corresponds to a displacement of air molecules of through a distance of about 1.1×10^{-11} m, or 11 picometers – about 1/10 the radius of a mid-sized atom.

- The human ear is a remarkable detector of acoustic radiation. Not only can the human ear detect vibrations with a sensitivity that spans six orders of magnitude, it can also detect sounds across nearly a 10 octave range of frequencies.

Sound Wave Intensity

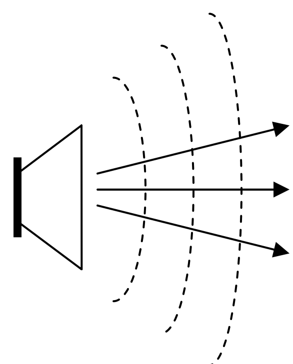
- Energy per unit time is power.

- The intensity (I) of an acoustic wave is the amount of power the wave transfers through some area or imparts onto some surface.
- The units of intensity are power per unit area or watts/m^2, i.e., $I = \frac{P}{A}$.
- Intensity and pressure amplitude are related by: $I = \frac{(\Delta P_m)^2}{2\rho v}$.
- Intensity and displacement amplitude are related by $I = \frac{1}{2}\rho v(2\pi f)^2 A^2$.

  Note that intensity goes as the square of amplitude.

- The faintest sound the human ear can detect has an intensity of 10^{-12} w/m^2. The loudest sound the human ear can tolerate without damage has an intensity of 1 w/m^2.

- Recall that sound waves radiating isotropically in a free field obey the inverse square law, i.e., their intensity diminishes inversely with increasing distance from their source because a fixed amount of energy is being radiated through successively larger areas.



- *Inverse square* implies that this decrease in intensity occurs as a function of $\frac{1}{r^2}$.
- In sound waves radiating isotropically in a free field the inverse square law is written: $I = \frac{P}{4\pi r^2}$. This expression is often rearranged to yield a ratio

  for intensities at two distances from a source as $\frac{I_1}{I_2} = \frac{r_2^{\ 2}}{r_1^{\ 2}}$

- It is important to note that sound waves are usually produced by non-isotropic sources that beam acoustic energy in preferred directions, and that unrestricted free-field conditions rarely exist (even outdoors the free field is restricted to a hemisphere for a source located at ground level). Nonetheless, the inverse square law is a useful rule of thumb in estimating the intensity of acoustic radiation at a given distance from a source.

The Speed of Sound

In general sound travels slowest in gasses, faster in liquids, fastest in solids. Dense materials generally have higher speeds of sound. The speed of sound is faster in materials that are stiff (like steel) and slower materials that are soft (like rubber).

For gasses: $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{m}}$

For liquids: $v = \sqrt{\frac{B}{\rho}}$

For Solids: $v = \sqrt{\frac{Y}{\rho}}$

Where:

R = Boltzmann's constant
T = Kelvin temperature
m = molecular mass
P = pressure
B = bulk modulus
ρ = density
B = bulk modulus
$\gamma = C_p/C_v$ (adiabatic constant)
Y = Young's modulus

For longitudinal waves traveling in a compressible medium the Bulk Modulus, β, determines the speed of sound waves.

- The bulk modulus describes both the stiffness and resistance to shear in a medium. It is a measure of the elasticity or "springiness" of the medium.

$$v = \sqrt{\frac{\gamma RT}{m}} = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\beta}{\rho}} \leftarrow \frac{(elastic)}{(inertial)}$$

Note: β always +

- For the values for air, γ = 1.4, *R* = 8.314 J/mol, *m* = 0.02895 kg/mol and T = temperature in K:

$$v = \sqrt{\frac{1.4(8.314 J \cdot mol^{-1})}{.02895 kg \cdot mol^{-1}}}\sqrt{T} = 20\sqrt{T} m \cdot s^{-1}$$

For room temperature (around 295K) this yields a value of 344 m/s at sea level.

- The speed of sound in air increases with increasing temperature.
- The speed of sound in air increases with decreasing density
- The speed of sound in air is nearly the same for all frequencies and amplitudes.

The Decibel (dB) Scale for Comparing Loudness

We have seen how the human ear is capable of detecting movements of air molecules from about 10^{-5} to 10^{-11} meters, a range of 10^6. Since the intensity of a sound wave varies as the *square* of its amplitude ($I = \frac{1}{2}\rho v(2\pi f)^2 A^2$), the range of intensities between the loudest and faintest sounds the ear can detect is 10^{12}.

- Twelve orders of magnitude is an enormous range of intensities. A convenient way of dealing with such a large range of values is with logarithms.

- Consider the log equation $y = \log_{10} x$. The log function works in such a manner that if *x* is multiplied by 10 (10^1), *y* increases by 1. If *x* is multiplied by 10^{12}, *y* increases by 12.

- Because of the compression of scale afforded by logarithms, SPL (sound pressure level), β, is used to compare the loudness of acoustic waves rather than intensity. β is defined as: $\beta = 10\log_{10}\frac{I}{I_0}$.
- I_0 is a standard reference intensity of 10^{-12} w/m^2, chosen because it is near the lower threshold of hearing for the human auditory system.

- Note that for $I = I_0$ the value obtained is 0 dB.

- Because of the logarithmic nature of SPL's, the sound pressure level, β, of an acoustic wave increases by 10 dB every time the intensity increases 10 fold. Thus β = 40 corresponds to an intensity that is 100,000 times the reference level.

- The inverse square law is normally written in terms of intensity (W/m^2). But because SPL's are more commonly used with sound waves it is convenient to express the inverse square law in terms of decibels. A restatement of the inverse square law yields the *inverse distance law* and is written:
$$difference = 20\log\frac{d_1}{d_2}.$$
- If, for example, d_2 is double the distance from a source of d_1, the difference in SPL is: $difference = 20\log(0.5) = -6.0dB$. This is the origin of the 6dB doubling rule of thumb – halving the distance to a source results in a 6dB increase in SPL. This rule of thumb assumes isotropic radiation in a free field.

Addition of Logarithms/SPL's.

Suppose that one wishes to add the contributions of two sound sources to compute the total SPL. Two loudspeakers in the same room or the sum of direct to reflected sound in a reverberant room. Let's say that one measures independently the SPL in the direct field of a loudspeaker to be 90dB and the reflections coming off the walls in the room to be 80dB. Algebraic addition yields 170dB!

Because decibels are based on logarithms they cannot be added in a simple algebraic fashion. Instead, they must first be converted back to their original power then added as follows:

$$dB_1 + dB_2 + ...dB_n = 10\log\left(10^{\frac{dB_1}{10}} + 10^{\frac{dB_2}{10}} + ...10^{\frac{dB_n}{10}}\right)$$

- In our example 80dB +90dB = $10\,log\left(10^{\frac{80dB}{10}} + 10^{\frac{90dB}{10}}\right)$ = 90.4dB.

- This calculation has a wide range of applications and implications. What SPL, for instance, results when two loudspeakers capable of producing SPL's of 100 dB are placed side by side? (103.01dB).

- Based on this calculation, does the act of adding more loudspeakers to an existing system make it a lot louder all by itself?

- It is often convenient to be able to compute the dB level of individual sources from their combined measurement, e.g.,

$$dB_1 = 10\log\left(10^{\frac{dBcombined_1}{10}} - 10^{\frac{dB_2}{10}}\right)$$

The Decibel Scale – Typical SPL's of Common Sources

160 dB SPL	immediate permanent ear damage
150 dB SPL	jet airplane takeoff
140 dB SPL	large caliber pistol shot (10 meters)
130 dB SPL	threshold of pain
120 dB SPL	loud rock concert
110 dB SPL	typical large club
100 dB SPL	loud theater
90 dB SPL	threshold of comfort
80 dB SPL	neighbors complain about you stereo
70 dB SPL	conversational level (1 meter)
60 dB SPL	quiet conversation
50 dB SPL	average home interior at night
40 dB SPL	quiet auditorium
30 dB SPL	whisper
20 dB SPL	quiet studio
10 dB SPL	gentle rustle of leaves
0 dB SPL	threshold of hearing

SPL Changes

dB	Power ratio	Perception of change
1 dB	1.25:1	barely perceptible
3 dB	2:1	noticeable
6 dB	4:1	same as the loss or gain when the distance from a source is doubled or halved (goal for most system changes)
10 dB	10:1	twice as loud

- Doubling power or intensity does not double loudness.
- 1 dB is the smallest change in intensity that is perceptible to the ear
- 3 dB change in loudness results from a doubling of power
- 6 dB change in loudness results for halving distance

Sources/Detection of Acoustic Waves

We have shown how harmonic waveforms are produced but how, exactly, are these sounds converted to longitudinal waveforms and transmitted through the air?

- Consider the sounds produced by your voice. When you force your vocal cords to oscillate, they bump into air molecules surrounding them.

- Almost immediately these air molecules begin to oscillate at the same frequency as your vocal cords. In this manner a transverse wave (the vibration of your vocal cords) is converted into longitudinal acoustic wave.

- This process, by itself, is inefficient and produced sound waves of very small amplitude (not very loud). But the air confined to your chest and larynx form a large resonant chamber that helps to amplify the sound produced by your vocal cords by increasing the amount of air set into motion.

- For the same reason, an acoustic guitar with its resonant chamber of air produced a much louder sound than a smaller guitar.

- Once the air molecules around your vocal cords begin to oscillate them, in turn, bump into the air molecules surrounding them.

- Because the molecules are moving back and forth sympathetically with your vocal cords, a pattern is established where air molecules periodically rush first toward, then away from each other. This establishes a disturbance consisting of a series of compressions (air molecules close to each other) and rarefactions (air molecules far apart), that moves through the air at a speed of about 344 meters/sec (1130 ft/sec) at sea level pressure at a temperature of 60°F.

- In denser (less compressible) media sound waves travel much faster.

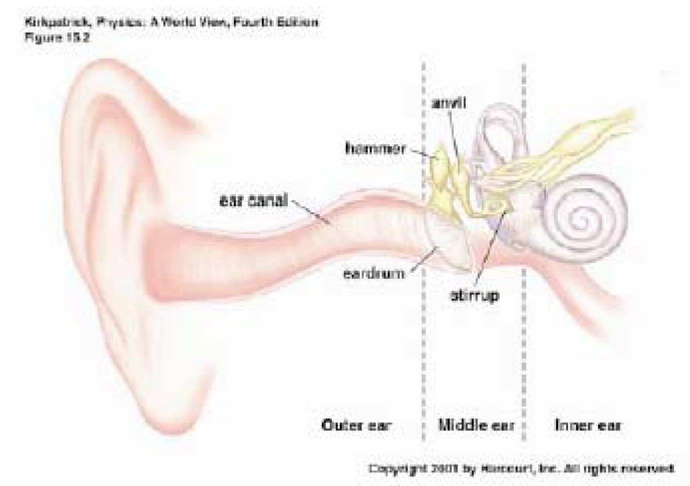
So far we have considered the source and medium of propagation of acoustic waves, but how are they detected?

- Just as the air around a vibrating object oscillates sympathetically with the object, an object or surface that is impacted by a vibrating mass of air will oscillate sympathetically with the air. This is how acoustic waves transfer energy from one region to another through the air.

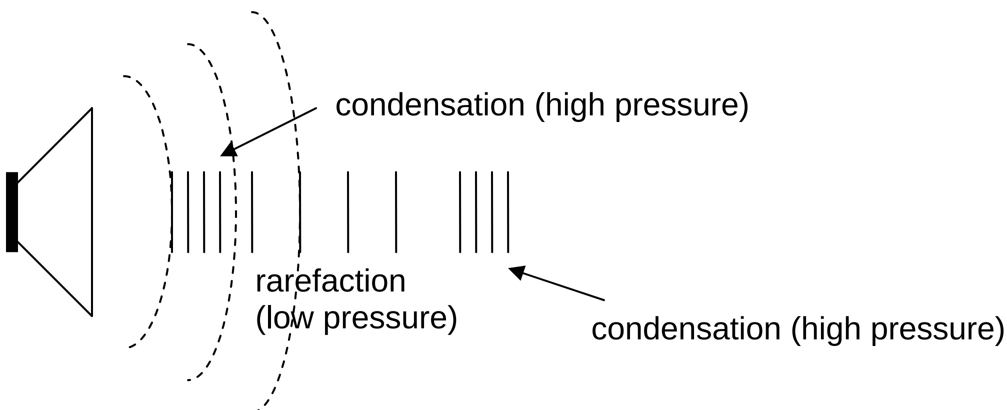
- The human auditory system is designed to focus sound waves on a portion of the inner ear known as the eardrum that vibrates at the same frequency as the incoming waves.

- Microphones contain diaphragms that work the same way.

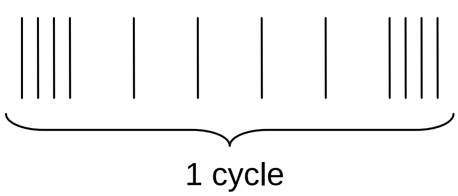
- All detectors of acoustic waves operate on much the same principle.



Longitudinal sound waves created by vibrating objects



Causes a sympathetic vibration of the ear drum



Frequency and Wavelength of Sounds Audible to the Human Ear

- The range of human hearing is about 20 – 20,000 Hz.
- Ultrasonic sounds are above 20,000 Hz (bat clicks)
- Infrasonic sounds are below 20 Hz (earthquakes, wind)
- The ear does not have linear response across the spectrum. It is most sensitive to sounds around 1 kHz.
- Pitch – a subjective measure of frequency.
- The human ear can distinguish about 1400 pitches.

Comparison of wavelengths: $\frac{v}{f} = \lambda$

20 Hz	17.2 meters
100 Hz	3.44 meters
1000Hz	0.34 meters
5000Hz	0.069 meters
10,000Hz	0.034 meters
20,000Hz	0.017 meters

Locating the Source of Sound Based on Wavelength

- Phase differences (due to path length differences) are one way we localize sounds. This method is effective for wavelengths greater than 2 head widths.

- Sound waves diffract easily at wavelengths larger than the diameter of the human head (around 500 Hz wavelength equals 69 cm). At higher frequencies the head casts a "shadow". Sounds in one ear will be louder than the other.
- Diffraction from sources is also useful in locating the origin of a sound.

Summary of Harmonic Sound Waves

$$\psi(x,t) = \psi_0 \cos(kx \pm \omega t)$$

↗

Maximum displacement from equilibrium

$$\Delta P = \Delta P_m \sin(kx \pm \omega t)$$

↗

Maximum pressure amplitude

$\Delta P_m = \rho v \omega S_m$ where maximum amplitude, A = S_m

Sound Energy and Intensity

$$\Delta E = \frac{1}{2}\Delta m \omega^2 S_m{}^2$$

$$\Delta E = \frac{1}{2}\rho \underbrace{A \Delta x}_{\text{Volume of Medium}} \omega^2 S_m{}^2$$

Volume of Medium

Power = Energy/Time

$$\rightarrow Power = \frac{1}{2}\rho A \frac{\Delta x}{\Delta t}\omega^2 S_m{}^2 = \frac{1}{2}\rho A v \omega^2 S_m{}^2$$

Intensity = Power/Area

$$\rightarrow I = \frac{1}{2}\rho v \omega^2 S_m{}^2$$

Recall: $\Delta P_m = \rho v \omega S_m$

Alternatively: $I = \dfrac{\Delta P_m{}^2}{2\rho v}$

Example 1. Sound system A produces an intensity level of 107dB. Sound system B produces an intensity level of 110dB. Compute the ratio of intensity for the two sound systems.

$$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$$

$$\frac{3}{10} = \log\left(\frac{I_2}{I_1}\right)$$

$$10^{0.30} = \frac{I_2}{I_1} = 2$$

Example 2. Calculate the pressure amplitude corresponding to a sound pressure level of 120dB, in air. (Note that 120 dB is a reference level and corresponds to an intensity of 1 w/m^2).

$$\beta = 10\log\frac{I}{I_0}$$

$$120dB = 10\log\frac{1w\cdot m^{-2}}{10^{-12}\,w\cdot m^{-2}} \therefore I = 1w\cdot m^{-2}$$

$$I = \frac{\Delta P_m{}^2}{2\rho v} \rightarrow \Delta P_m = \sqrt{I2\rho v}$$

$$\Delta P_m = \sqrt{\left(1w\cdot m^{-2}\right)\left(2\right)\left(1.2kg\cdot m^{-3}\right)\left(343m\cdot s^{-1}\right)} = 28.7 \text{ Pascals}$$

Example 3. A six string electric guitar is plugged into an amp and the A2 and D3 strings are played simultaneously. The sound produced has an intensity level of 120dB. Assuming that all of the strings contribute equally to sound intensities, what would the intensity level be if all six strings were played simultaneously?

If 2 strings are sounded at 120 dB the corresponding intensity is 1 W/m^2 so the intensity of 6 strings = 3 W/m^2.

$$\beta = 10\log\frac{I}{I_0}$$

$$= 10\log\frac{3w\cdot m^{-2}}{10^{-12}w\cdot m^{-2}} = 125dB$$

Example 4. Acoustic waves do not experience a tremendous amount of damping as they travel through many materials, yet loudness generally diminishes significantly with increasing distance from the source (the 6 dB rule). Why is this so?