

Sound Waves II

Noise, Complex Sound Waves, Production of Sound Waves, Music and Harmonic Structure

- Sounds that are considered noise generally have little to no harmonic content. Musical sounds have a highly refined harmonic structure that is pleasing to the ear. Many sounds have harmonic content but are not, themselves, harmonic (e.g. speech)
- Pure tones are uncommon in nature. Most systems vibrate in more than one mode (simultaneously). Generally the fundamental or first harmonic is the dominant tone though this is not always the case.
- Superposition of fundamental frequencies and harmonics results in a complex sound waveform.
- Harmonic overtones are related to the fundamental frequency mathematically (as integer multiples). The second harmonic is always twice the frequency of the fundamental tone. The third harmonic is three times the fundamental tone, etc.
- An octave is double the frequency of a tone. The first harmonic of a tone is 1 octave up.
- The fundamental frequency and related harmonic series of overtones are responsible for the characteristic sound of musical instruments.
- The sound that we hear from an instrument playing a note is a complex waveform containing all of the various frequencies superimposed into one complex wave.
- The same note played on different instruments sounds different because of the harmonic structure of the individual instruments. The reason that middle C (about 262 Hz) played on a piano sounds different than played on a guitar is that the ratios of harmonic intensities are different.

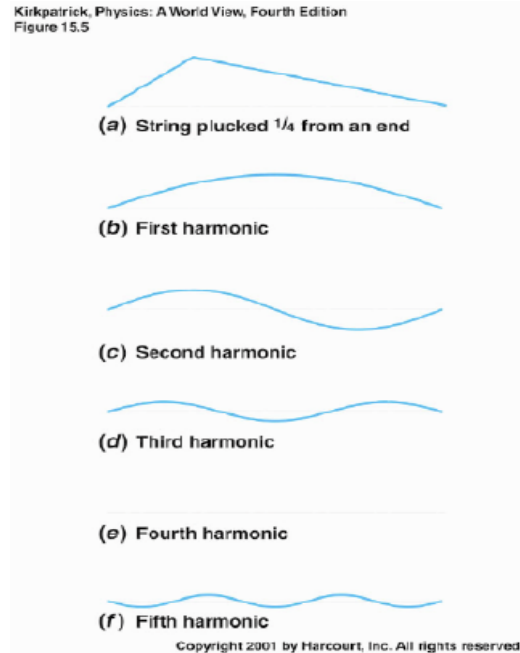
- In contemporary western music, scales are constructed in such a manner as to divide each octave into 12 equal semitones (seven notes with five sharps and flats). This is known as the evenly (or equally) tempered scale and is widely used to aid key transposition and ensemble playing. Most of the world uses equally tempered scales because every key sounds the same and all of the octaves are the same - just higher or lower. No key sounds bad but no key is pure (Pythagorean temperament) either.
- Scale temperament may be Pythagorean temperament (the original form of temperament), Well temperament or Equal temperament (modern form).
- Pythagorean temperament uses pure fifths and pure octaves to construct a scale. Thirds are not pure and one out of every 12 fifths (a wolf) is very impure.
- The well-tempered scale uses pure octaves. It is not necessarily the scale most pleasing to the ear in all cases. Different keys sounded different. Some were warm and some were harsh. Music could be written to the "color" of the scale. Most baroque music was well-tempered
- The range of human hearing is 20 - 20,000 Hz in infants (about 10 octaves).
- The shape, construction, mass, density and size of a musical instrument all contribute to its tonal quality (harmonic spectrum), as does how it is played.
- Percussion instruments produce harmonics that are not related to the fundamental tone of the instrument in the same mathematic sense as harmonic instruments, i.e., they have non-integer harmonics.

How Harmonic Series are Produced

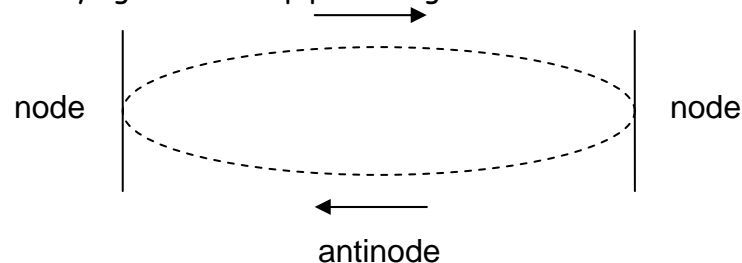
The type of harmonic series produced in any medium depends on the boundary conditions that exist at the edges of the medium where the waves are reflected.

Standing Waves on a String (Transverse Standing Waves)

- When a string stretched between two fixed points is plucked it vibrates in its *fundamental* mode of vibration (also known as the *first harmonic*), but more than one mode of oscillation is always present at the same time.

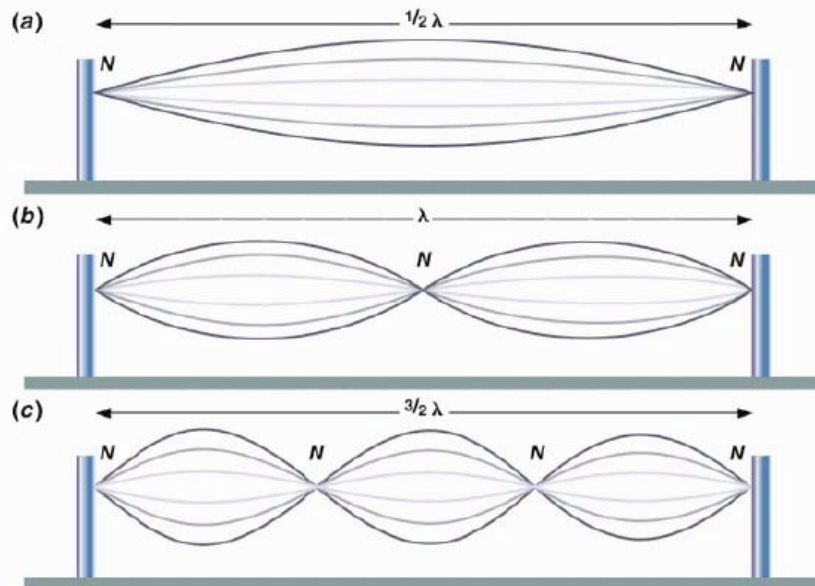


- The other simultaneous modes (overtones) are mathematically related to the fundamental tone (as integer multiples of the fundamental).
- The incident and reflected waves traveling back and forth along the string produce a standing wave pattern.
- If the string oscillates harmonically under the influence of some mechanical driver a stable *standing wave* pattern will form.
- Standing wave patterns are characterized by very large amplitude oscillations - a condition known as *resonance*. At resonance the wave will produce a loud sound.
- A standing wave pattern consists of incident and reflected waves in a spatially unvarying relationship producing a series of nodes and antinodes.



- Tension in the string and driving frequency affect standing wave formation.

Kirkpatrick, Physics: A World View, Fourth Edition
Figure 15.6

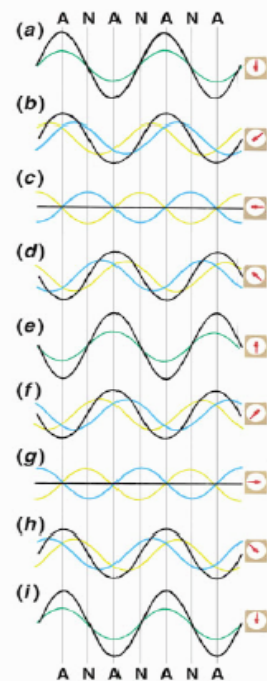


Copyright 2001 by Harcourt, Inc. All rights reserved.

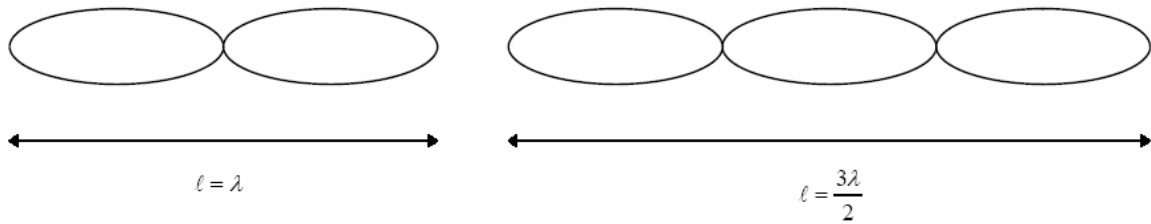
A standing wave is a stable wave pattern consisting of an incident and a reflected wave in which the *crests* and *troughs* of both waves maintain fixed positions.

- Incident (blue) and reflected (yellow) waves combine to form a standing wave pattern (black). The green wave represents points in the pattern where the blue and yellow waves are *in phase*.
- The black line is the result of the superposition of the yellow and blue waves. What wave(s) would you see if you were looking at this string?
- Nodes and antinodes are points along the string where it is either not moving (node) or at its maximum displacement (antinode).
- For a stable standing wave this pattern is fixed, i.e., the position of nodes and antinodes do not change with time.

Kirkpatrick, Physics: A World View, Fourth Edition
Figure 14.17



Copyright 2001 by Harcourt, Inc. All rights reserved.



Standing Wave Schematic

- The number of loops (or segments) corresponds to the harmonic number of the standing wave pattern, e.g., the 2nd and 3rd harmonics are shown above (what would the *fundamental* or first harmonic look like?).
- All harmonics (i.e., 1st, 2nd, 3rd, etc.) are possible on a string fixed at both ends.
- For a given harmonic, the wavelength, λ , may be written: $\lambda = \frac{2\ell}{n}$, where ℓ is the length of the string between the fixed endpoints and n is the number of loops in the string.
- The *velocity* of a wave on a string may be expressed in terms of frequency and wavelength:

$$v = \lambda f$$

Recall that this velocity is the speed at which energy is transported down the string. This may alternatively be expressed:

$$v = \frac{2\ell f}{n}$$

- A standing wave occurs when an integral number of half waves ($\lambda/2$) are present between the ends of the string.
- In order to produce a standing wave pattern, the velocity of the incident and reflected waves must be such that ℓ (length of the string between the fixed boundaries) is an integral multiple of half wave length, or: $\ell = n \frac{\lambda}{2}$.

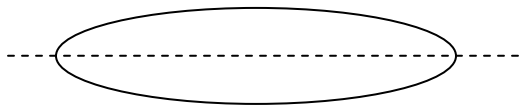
- For musical instruments with strings such as guitars, violins, or pianos it is convenient to express the velocity of a wave as a function of string tension and linear mass density:

$$v = \sqrt{\frac{T}{\mu}}$$

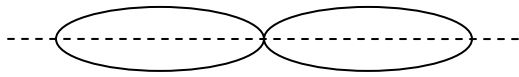
where T is the tension in the string and μ is the mass per unit length of the string.

- Not all of the possible vibrating modes of any medium are harmonic in nature (harmonics must be integer multiples of the fundamental). The non-integer multiple frequencies are known as partials or *inharmonics*. Many of these are actually unpleasant sounding,

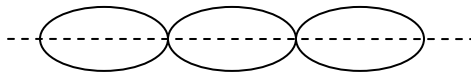
Harmonic Series for Standing Waves on a String



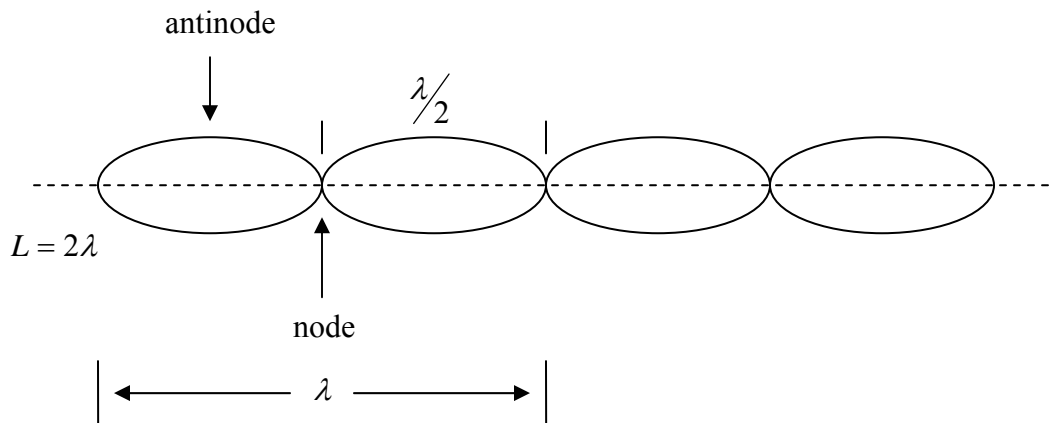
Fundamental Frequency
1st Harmonic



2nd Harmonic 1st over-tone
(2 × fundamental)



3rd Harmonic 2nd over-tone
(3 × fundamental)



$$L = n \frac{\lambda}{2} \rightarrow \frac{2L}{n} = \lambda$$

$$v = \lambda f \rightarrow f = \frac{v}{\lambda}$$

The condition for a standing wave:

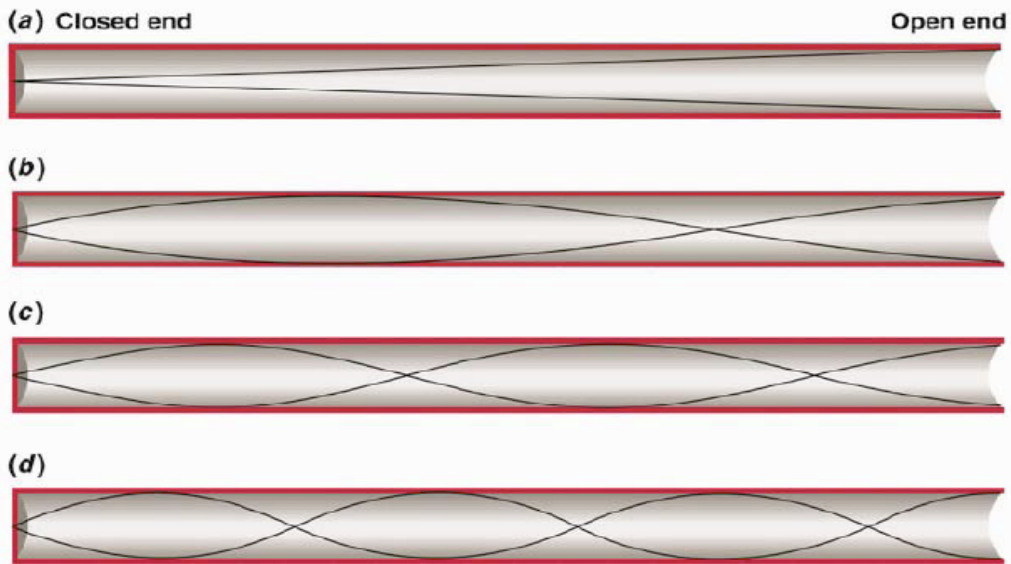
$$f_n = n \frac{v}{2L} \quad n = 1, 2, 3, \dots$$

Since n may have any integer value, all harmonics are present.

Standing Waves in Columns of Air - Longitudinal Standing Waves

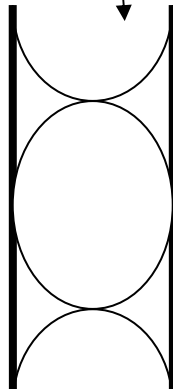
Waves traveling in enclosed columns of air behave in much the same manner as waves traveling down strings except that the waves are longitudinal rather than transverse

Kirkpatrick, Physics: A World View, Fourth Edition
Figure 15.9



Copyright 2001 by Harcourt, Inc. All rights reserved.

pressure node (low)
displacement antinode (high)



A displacement antinode occurs with the greatest vibration of air molecules. Bernoulli's Principle states that high velocities correspond to low pressures.

← Pressure antinode
displacement node

pressure node
displacement antinode

Notice that pressure amplitude and displacement amplitude are $\frac{1}{4}$ of a cycle out of phase for a sound wave

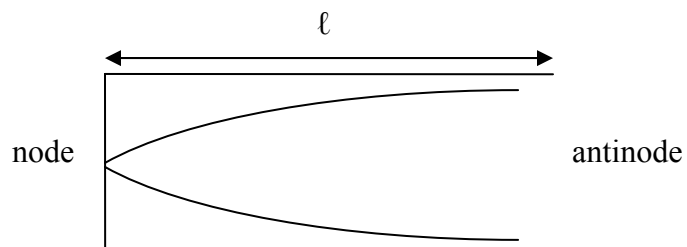
Harmonic Series for Acoustic Waves in a Tube Open at One End

Recall that condition for transverse standing wave formation on a string was

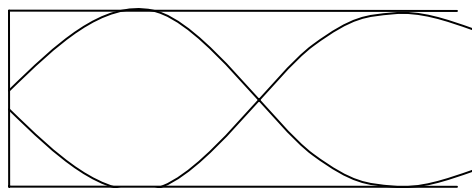
$v = \frac{2\ell f}{n}$ or $f = n \frac{v}{2\ell}$ ($n = 1, 2, 3, \dots$). For longitudinal waves, this relationship still

holds.

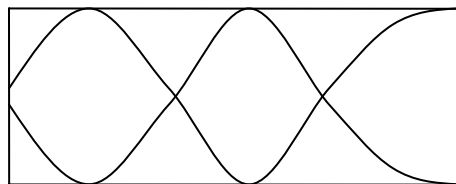
- In the case of sound waves, f is the frequency at which the air vibrates in the tube (perhaps driven by a tuning fork), ℓ is the length of the column of air, n is the number of half wave lengths, $\frac{\lambda}{2}$, and v is the speed of sound in air.
- If a tuning fork is held over the open end of a tube closed at the other end and struck it excites the air molecules in the tube and causes a sound wave to propagate down the length of the tube to the closed end where it is reflected back up the length of the tube to the open end (where it is reflected again at the mouth of the tube).
- In tubes, pipes, or columns open at one end and closed at the other a standing wave pattern requires that a displacement antinode exist at the open end and a displacement node at the closed end of the tube. This means that the fundamental (first harmonic) standing wave such a tube occurs when the column of air is of length $\frac{\lambda}{4}$.
- The series of nodes and antinodes in a tube open at one end and closed at the other form an *odd harmonic series* (in tubes open at both ends or closed at both ends all harmonics are possible). Thus the condition for a standing wave to form in a tube closed at one end is $f = n \frac{v}{4\ell}$ or $\lambda = \frac{4\ell}{n}$ ($n = 1, 3, 5, \dots$).
- A stable standing wave pattern will form if such a system is driven, e.g., by something like a tuning fork.



$$\ell = \frac{\lambda}{4} \text{ Fundamental, 1}^{\text{st}} \text{ Harmonic } (f)$$



$$\ell = \frac{3\lambda}{4} \text{ 1}^{\text{st}} \text{ Overtone, 3}^{\text{rd}} \text{ Harmonic } (3f)$$



$$\ell = \frac{5\lambda}{4} \text{ 2}^{\text{nd}} \text{ Overtone, 5}^{\text{th}} \text{ Harmonic } (5f)$$

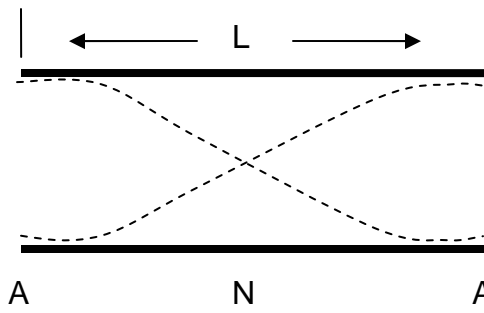
$$\lambda = \frac{4\ell}{n} \quad n = 1, 3, 5, \dots$$

$$f = \frac{nv}{4\ell} \quad n = 1, 3, 5, \dots$$

Condition for a standing wave: $f = \frac{nv}{4\ell}$, $n = 1, 3, 5, \dots$, so only odd harmonics exist.

Harmonic Series for Acoustic Waves in a Tube Open at Both Ends

- In tubes, pipes, or columns open at one end and closed at the other a stable standing wave pattern requires that a displacement antinode exists at each open end of the tube.
- This means that the fundamental (first harmonic) standing wave in such a tube occurs when the column of air is of length $\frac{\lambda}{2}$.
- The series of nodes and antinodes in a tube open at both ends forms a complete harmonic series, $f = \frac{v}{\lambda} = \frac{nv}{2L}$ $n = 1, 2, 3, \dots$ just like a standing wave on a string.
- A tube open at both ends must be twice as long as a tube open at one end only to produce the same fundamental.

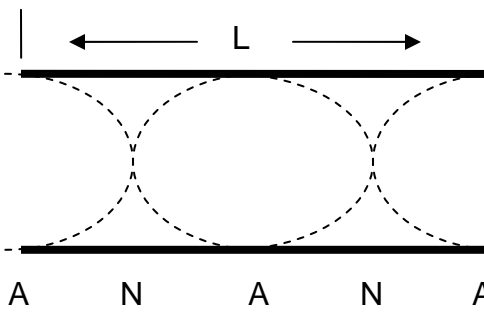


Fundamental

1st Harmonic

$$\frac{\lambda}{2} = L$$

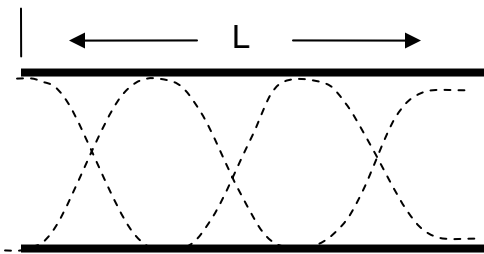
$$\lambda = 2L$$



2nd Harmonic

(1st Overtone)

$$\lambda = L$$



3rd Harmonic

(2nd Overtone)

$$\frac{3}{2}\lambda = L$$

$$\lambda = \frac{2}{3}L$$

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, \dots$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad n = 1, 2, 3, \dots$$

Condition for a standing wave: $f_n = n \frac{v}{2L}$ $n = 1, 2, 3, \dots$, so all harmonics are possible

Beats

What happens when waves of different frequencies overlap?

- The beat frequency is the composite that occurs when two waves of differing frequency overlap.
- Beat Frequency = $F_A - F_B$
- Players of stringed instruments frequently use beats to tune strings to the proper relative pitch. As two strings are tuned closer to the same frequency the beat frequency slows and then disappears.

Example 1. A six string electric guitar is tuned as follows: E2, A2, D3, G3, B3, E4

E2	82.4069 Hz
A2	110.0000 Hz
D3	146.8324 Hz
G3	195.9978 Hz
B3	246.9417 Hz
E4	329.6277 Hz

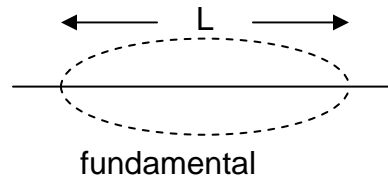
How many harmonics are theoretically heard within the audible range for the B3 string assuming that the audible range is 20 - 16,000 Hz? The strings are .63m long (Les Paul scale - Stratocaster scale strings are about .65 meters long. Most electric guitars have string lengths somewhere between these extremes).

For a vibrating string, $f_n = n \frac{v}{2L}$ and if fixed at both ends all harmonics are possible, i.e., $n = 1, 2, 3, \dots$

$$v = \lambda f$$

$$v = (1.26m)(246.9417Hz)$$

$$v = 311.1m \cdot s^{-1}$$



$$\lambda = 2L$$

$$f_1 = \frac{(1)(311.1m \cdot s^{-1})}{1.26m} = 246.9417Hz$$

$$f_{64} = \frac{(64)(311.1m \cdot s^{-1})}{1.26m} = 15801.9Hz$$

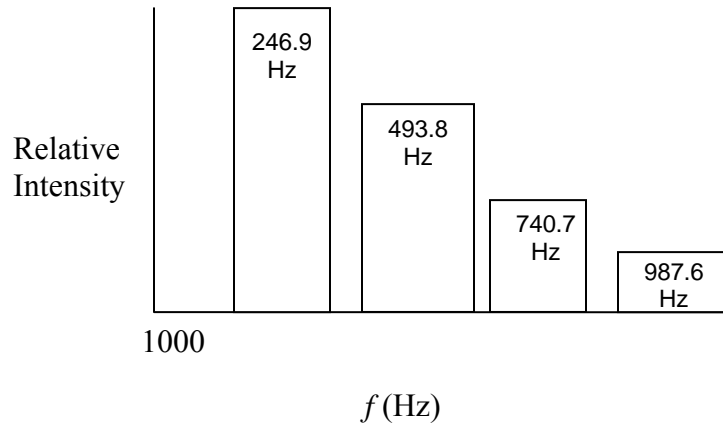
$$f_{65} = \frac{(65)(311.1m \cdot s^{-1})}{1.26m} = 16048.8Hz$$

alternatively (and much less work):

$$16,000Hz = n \frac{311.1m \cdot s^{-1}}{1.26m} \quad n \approx 64$$

Note: Many of these harmonics (especially the higher order ones) are at very low SPL's and are not easily distinguishable.

Example 2. Assuming that the amplitude for each successive harmonic is $\frac{1}{2}$ the value of the amplitude of the preceding harmonic, sketch a sound spectrum for the 1st four harmonics for the B3 string for the guitar in previous examples.



Example 3. What is the string tension in the B3 string if the mass per unit length is $0.708 \times 10^{-3} \text{ kg/m}$ (.013/.330 gauge)?

$$v = \lambda f = \sqrt{\frac{F}{\mu}} \rightarrow v^2 \mu = F = (311.1 \text{ m} \cdot \text{s}^{-1})^2 (0.708 \times 10^{-3} \text{ kg} \cdot \text{m}^{-1}) = 68.5 \text{ N}$$

Note: The tension in all of the strings of any guitar is about the same. The following chart is representative of most electric guitars.

note	gauge	f (hz)	μ (kg/m)	T (N)
E4	.010/.254	329.63	0.401×10^{-3}	69.6
B3	.013/.330	246.94	0.708×10^{-3}	68.5
G3	.017/.432	196.00	1.140×10^{-3}	69.5
D3	.026/.660 (.14)	146.82	2.333×10^{-3}	79.9
A2	.036/.914 (.15)	110.00	4.466×10^{-3}	85.5
E2	.046/1.168 (.16)	82.407	6.790×10^{-3}	73.2
Total				446.2 N

In wound strings (D3, A2, E2) the tension is applied primarily to the core with very little applied to the winding.

Heavier strings require greater tension to achieve a particular pitch. This increased tension makes the string stiffer. A stiffer string enhances the upper harmonics. Experienced guitarists know that by experimenting with different string gauges they may achieve a different harmonic structure to their instrument.

Acoustic guitars with medium gauge strings have almost double the string tension of electrics with .010 -.046 strings. Most acoustic guitars have forces on their necks equivalent to the weight of an average adult male.