

Special Relativity

Einstein's Postulates:

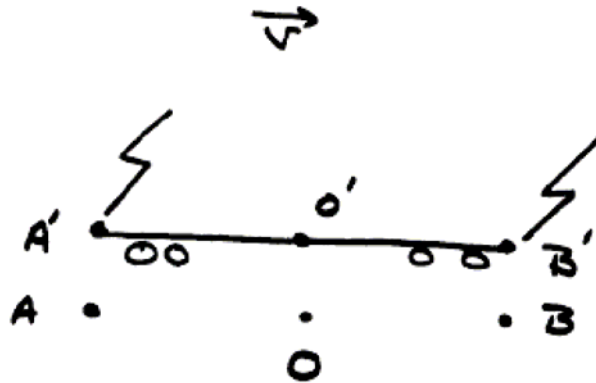
- The laws of physics are the same in all inertial frames of reference. There is no absolute or preferred frame of reference. There is no way to detect absolute motion.
- Observers in all inertial systems measure the same value for the speed of light in a vacuum.

An *inertial frame* is one in which a free body experiences no acceleration. As a consequence:

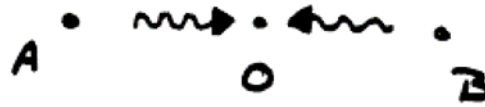
- An experiment done on a platform moving at constant velocity will yield the same results as one performed in a laboratory.
- All motion is relative

Simultaneity

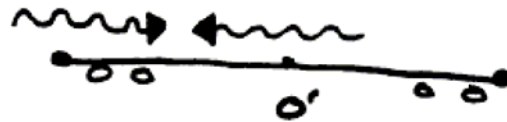
Consider a railroad flat car traveling along a set of tracks when it is struck at both ends at the same time by identical lightning bolts. Exactly how this might occur isn't important but the effect of such an occurrence is.



The bolts will leave scorch marks on both ends of the rail car at points A' and B' and on the ground at points A and B beneath the ends of the rail car.



Notice that as a consequence of the motion of the rail car an observer on the ground at point O (midway between the lightning bolts) sees a different sequence of events than an observer in the middle of the rail car at O' .



- The observer at O sees both events occur simultaneously.
- The observer at O' sees light from the front lightning bolt before the back and concludes front of the car is struck first.

So the question is who is right? The answer is *both*.

Two events that are simultaneous in one reference frame are not necessarily simultaneous in another reference frame moving with respect to the first.

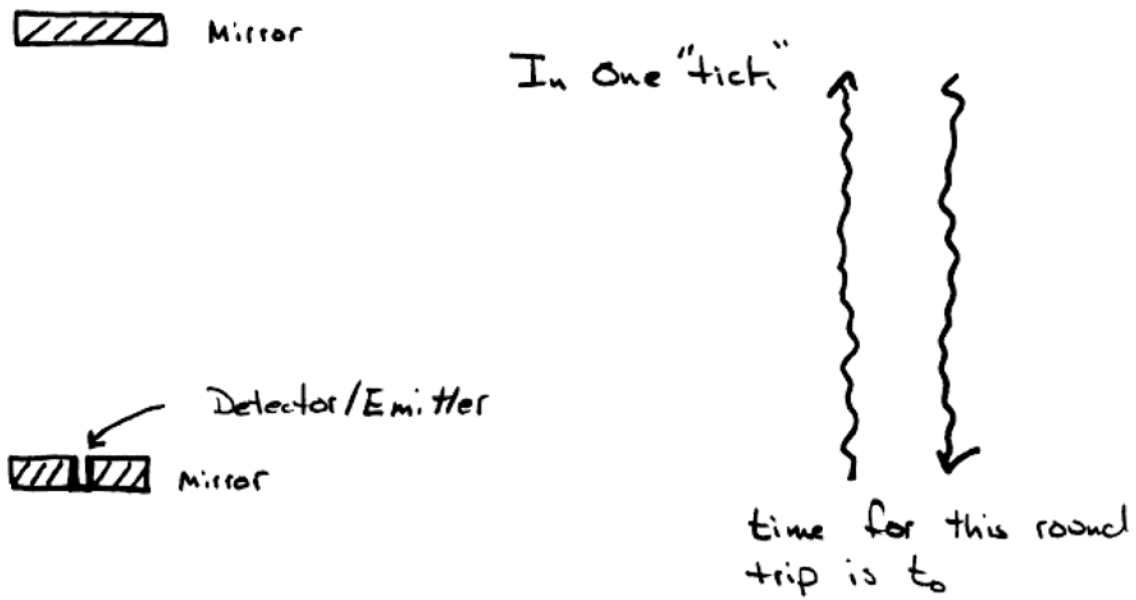
Notice that there is no inference here that the events can change order (travel backwards in time) only that they can occur simultaneously or one always precedes the other.

Relativity - Time Dilation and Length Contraction

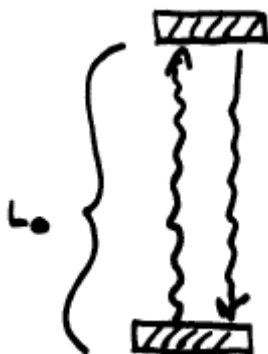
Time Dilation - A clock in motion with respect to an observer appears to tick more slowly than one at rest.

Convention: t_0 is the frame of the observer.
 t - time in the moving frame.

Now consider a clock that keeps track of time with "ticks" that involve the transit of light between two mirrors.



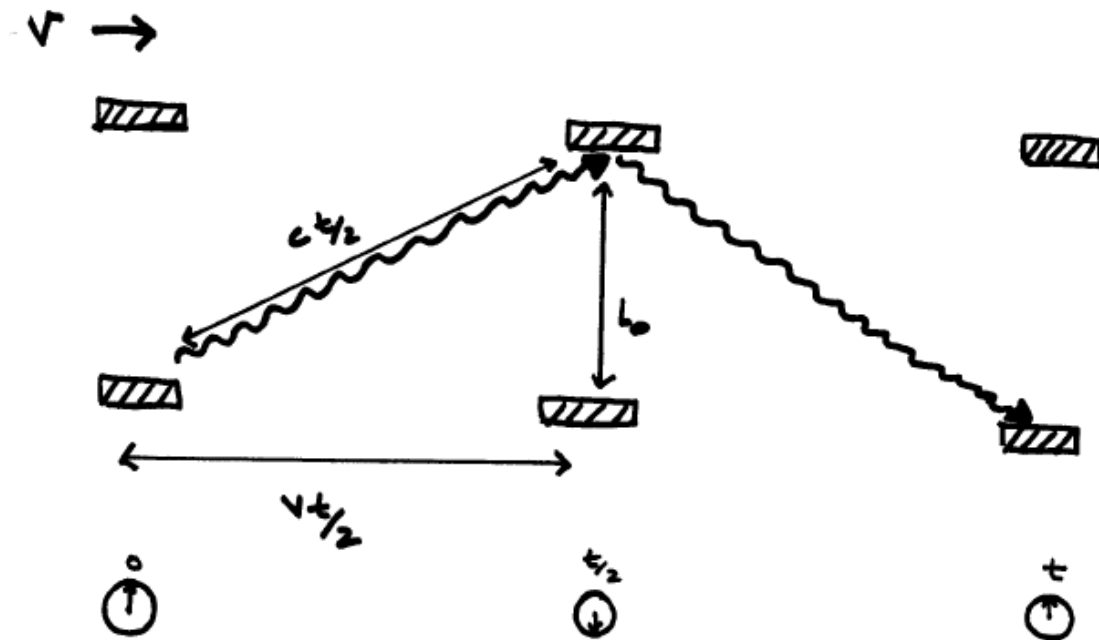
In a lab with such a clock at rest with respect to an observer:



- The time required to reach upper mirror is $\frac{t_0}{2}$
- The time for a round trip between the mirrors is t_0

$$\text{Since } d = vt, \quad L_0 = c \frac{t_0}{2} \rightarrow t_0 = \frac{2L_0}{c}$$

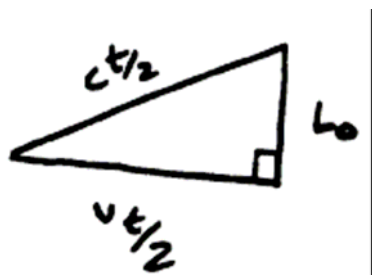
Now consider an identical clock in motion with respect to the observer in the lab:



The light "ticks" now appear to follow a diagonal path:

- In time $\frac{t}{2}$ the horizontal distances the clock travels is $v\frac{t}{2}$
- In time $\frac{t}{2}$ the distance the beam of light travels is $c\frac{t}{2}$

Let's examine the geometry of this situation and see if we can use it to relate v , c , and t .



$$a^2 + b^2 = c^2$$

$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2$$

$$\frac{c^2 t^2}{4} = L_0^2 + \frac{v^2 t^2}{4}$$

$$\frac{t^2}{4}(c^2 - v^2) = L_0^2$$

$$t^2 = \frac{4L_o^2}{c^2 - v^2}$$

$$t^2 = \frac{(2L_o)^2}{c^2 \left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{\frac{2L_o}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Recall that for a stationary clock we obtained a relationship between t_o and L_o that applies here as well: $t_o = \frac{2L_o}{c}$, and substituting this into the numerator of the equation above we obtain:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- t - time on moving clock
- t_o - time on clock at rest
- v - relative speed
- c - speed of light

Note that according to this relationship, t is always $> t_o$. This relationship is known as relativistic *time dilation*.

In relativistic mechanics

- the sequence of events is unchanged
- one cannot peer into the future
- time does not run backwards

Example 1 What is the speed of a spacecraft whose clock runs 2 second slower per hour than the clock of an observer on earth?

t = time observer on clock in spacecraft

t_0 = time observed on clock on earth

We solve for v :

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{t_0}{t}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$1 - \frac{t_0^2}{t^2} = \frac{v^2}{c^2}$$

$$v = c\sqrt{1 - \frac{t_0^2}{t^2}}$$

$$\approx 3.33\%c$$

Evidence for special relativity - muon (μ^\pm) production

- Muons are created by collisions with cosmic rays (γ rays) and atomic nuclei high in the atmosphere.
- A muon has a mass of 207 times that of an electron, has the charge of $\pm e$, and decays into an electron or a positron after a mean lifetime of $2\mu\text{s}$ before decaying into other particles.
- Muons have a speed of $.998c$ ($2.994 \times 10^8 \text{ m/s}$) and are found in profusion at the earth's surface.

Classically there should not be any muons detected at the earth's surface since

$$\text{muon lifetime (2ms)} \times \text{muon speed (.998c)} = 600 \text{ m}$$

which means that muons decay after traveling about 600 meters from their point of origin. Since muons are created at heights of 8000m and above none should reach the earth's surface.

Now consider relativity. Since muons travel at relativistic speeds we must use special relativity to compute the lifetime of a muon in the frame of reference of an observer on earth. Given $t_o = 2 \times 10^{-6} \text{ s}$, $v = .998c$:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2 \times 10^{-6} \text{ s}}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}}$$

$$t = 31.6 \mu\text{s}$$

In this amount of time, $d = vt = (2.994 \times 10^8 \text{ m} \cdot \text{s}^{-1})(31.6 \times 10^{-6} \text{ s}) = 9500 \text{ m}$

So a ground-based, stationary observer records the lifetime of a muon as $31.6\mu\text{s}$ and in this amount of time a muon travels 9500 meters which is why they are found in abundance at Earth's surface.

The corollary to time dilation is *length contraction*. To an observer traveling with the muon the path traveled is shortened by virtue of motion. This length contraction occurs only along the direction of motion. The contraction is by the

factor $\sqrt{1 - \frac{v^2}{c^2}}$.

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = 9500m \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = 600m$$

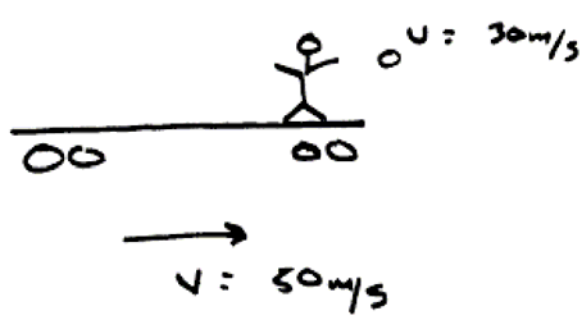
So an observer moving with the muon measures a path of 600 meters before decay.

The factor $\sqrt{1 - \frac{v^2}{c^2}}$, which appears often in relativistic equations is often referred to as γ .

Velocity Addition

When two objects are in motion with respect to a stationary observer, under relativistic conditions, adding their velocities is not trivial.

Consider a platform moving to the right as shown below. Since the velocities involved are *non-relativistic*,



the total velocity of the ball with respect to an observer standing on the ground is $50 \text{ m/s} + 30 \text{ m/s} = 80 \text{ m/s}$

But if one substitutes a flashlight beam for the ball and uses the same classical velocity addition $v_{total} = v + u$ which is greater than c . This violates the second postulate of special relativity.

Relativistic velocity addition is accomplished with the Lorentz velocity transformations:

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

Primed variables are in the *moving* frame with respect to an observer.

Example 2 Assume in the example given above that the rail car is moving with a velocity of $0.8c$ and the ball is released with a velocity relative to the person on the *flatcar* of $0.6c$. What velocity does a stationary observer on the ground observe?

We want the velocity in the stationary frame u_x .

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

- $u'_x \equiv$ the velocity of the second object in motion with respect to the first object in motion.
- $v \equiv$ the velocity of the first object in motion with respect to the observer at rest.
- The signs of these velocities matter only to the extent that objects moving in opposite directions should have an opposite sign.

$$u_x = \frac{0.6c + 0.8c}{1 + \frac{(0.6c)(0.8c)}{c^2}} = .946c$$

Try this with $u'_x = c$ and notice that the result is $u_x = c$.

Example 3 Two spacecraft, A and B, move toward each other with velocities measured on earth of $A = 0.75c$ and $B = 0.85c$. Find the velocity of B with respect to A.

Assume that A is in motion with respect to earth and B is in motion with respect to A.

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

We want to find the velocity of B with respect to A so we employ the primed transformation where:

$u_x \equiv$ motion of B with respect to earth

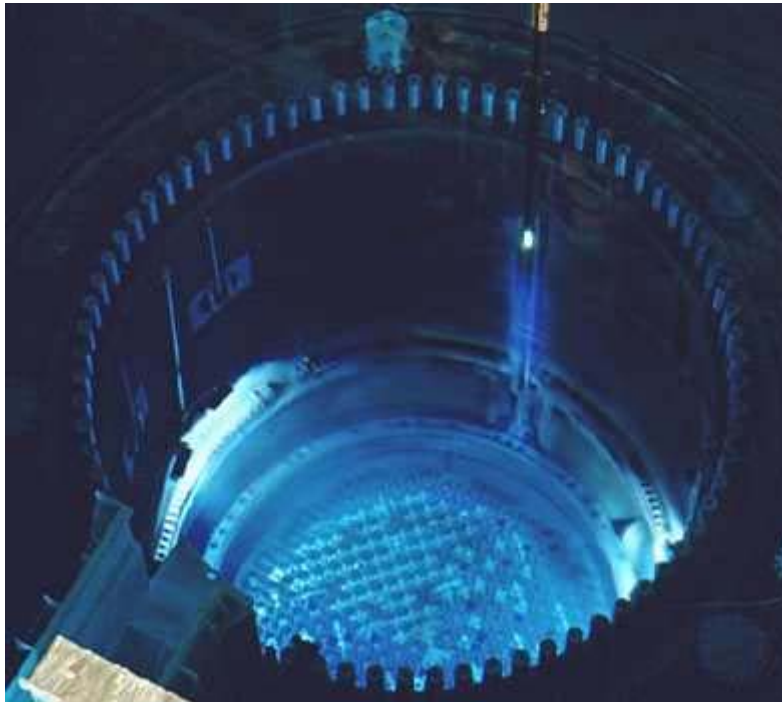
$v \equiv$ motion of A with respect to earth

$$u'_x = \frac{-0.85c - 0.75c}{1 - \frac{(-0.85c)(0.75c)}{c^2}}$$

$$= -0.98c$$

Cerenkov Radiation

- The speed of light in free space, c , is inviolate.
- No massive particle can travel at the speed of light
- Nothing can travel faster than the speed of light
- In dense material light slows down, i.e., $v < c$, and some particles are able to move faster than light in such media.
- When charged particles move faster than light in a dense media light waves similar to the bow waves created by a canoe moving down a stream faster than the velocity of the stream itself are created.
- These light waves are known as Cerenkov radiation and are useful for measuring the speeds of such particles.



Cerenkov radiation in the core of the nuclear reactor Photo by Charles Bell
Courtesy of <http://www.physlink.com/Education/AskExperts/ae219.cfm>

The Twin Paradox

Consider two twins, J and R, whose lifespans constitute biological clocks. J enters the space program and is sent on a mission to explore a nearby star system while R stays behind observing the voyage from mission control. .

- J and R are both 20 when J blasts off into space. If we assume that J's spaceship is capable of making the voyage at a speed of about $0.8c$ (ignoring the acceleration at liftoff) then everything that J does appears to take longer in R's frame of reference by a factor of:

$$\sqrt{1 - \frac{v^2}{c^2}} \rightarrow \sqrt{1 - \frac{(0.8c)^2}{c^2}} = 0.6 = 60\%$$

- This means that everything J does, breathe, walk and age seems to take longer by a factor of $3/5$. For every three beats of J's heart, for instance, R's heart beats five times.
- Finally J returns to earth 50 years later according to R and mission control. In J's frame of reference the same journey has taken only 30 years so upon their reunion J is 50 years old and R is 70 years old.

As much as this runs against intuition this is not the paradox. The difference in ages is simply a consequence of special relativity and is a function of the way in which our Universe operates. Although relativistic effects have been measured with great accuracy they are not apparent without velocities approaching the speed of light. Our everyday experiences have not equipped us to notice relativistic effects.

- The "paradox" occurs because neither frame of reference in this problem can be considered to be the preferred frame.
- For this reason J should be able to apply the same relativistic calculations to R assuming that the earth is moving away at a speed of $0.8c$.
- Given this apparent symmetry why should either twin be older than the other?
- The solution to the paradox lies in the fact that the two situations are not symmetric.
- When J arrives at the distant star system then turns around to come back he switches inertial frames while R remains in the same inertial frame throughout. R is entitled to apply time dilation while J is not.

- J also moves along a length contracted path along his voyage:

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}} \rightarrow 20ly \sqrt{1 - \frac{(0.8c)^2}{c^2}} = 12ly$$

- In J's inertial frame time goes by at the usual rate but his voyage to the star takes

$$\frac{L}{v} = \frac{12ly}{0.8c} = 15 \text{ years}$$

- while the return voyage takes another 15 years for a total of 30 years.
- So although J's lifetime seems longer to R it does not seem so to J.

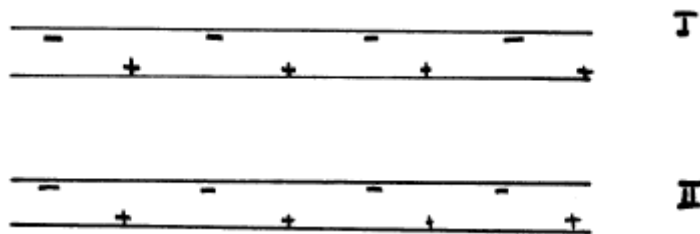
Electricity and Magnetism

- Relativity connects electricity and magnetism and is a triumph of special relativity.
- Because drift velocities in current carriers are small ($\ll 1$ mm per second), it is not immediately apparent that electrodynamics should be based on relativistic effects.
- The magnitude of the Coulomb force yields some light on this.
- In the hydrogen atom the magnitude of the Coulomb attraction between the electron and the proton is 10^{39} times greater than the gravitational attraction.
- A small perturbation in the Coulomb force due to relative motion (which is what gives rise to magnetic fields) may have large consequences.
- The sheer effect of 10^{20} or more moving electrons in the typical current-carrying elements makes any small relativistic effect quite pronounced.

In order to examine, in detail, the relationship between electric and magnetic fields much mathematical sophistication is required. We will consider a fairly simple example that lends itself to a more intuitive evaluation.

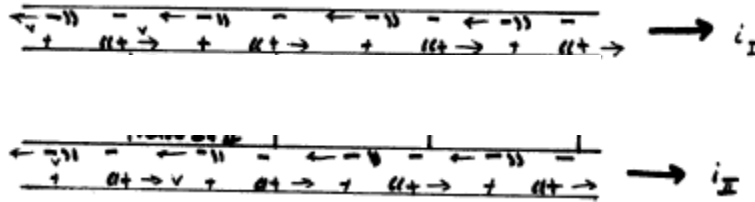
Let's consider the magnetic force between two currents in parallel wires.

- We'll assume in this discussion that electrical charge is invariant relativistically, i.e., a charge whose magnitude is Q in one frame is Q in all frames just like all observers measure c to be the same in all frames.
- We will begin by considering two idealized conductors containing an equal number of (+) and (-) charges at rest and equally spaced. Because the conductors are neutral, there is no net force between them.

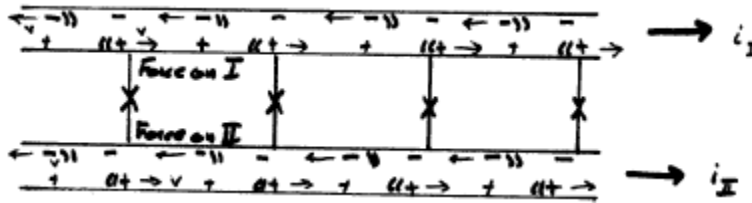


In the diagram below, currents flow through both wires in the same direction.

- Even though these currents are moving very slowly a lot of charges are involved so small relativistic effects may be discernable.
- Notice that the spacing of moving charges undergoes relativistic contraction, γ , as seen from a lab frame of reference, on *both* wires.

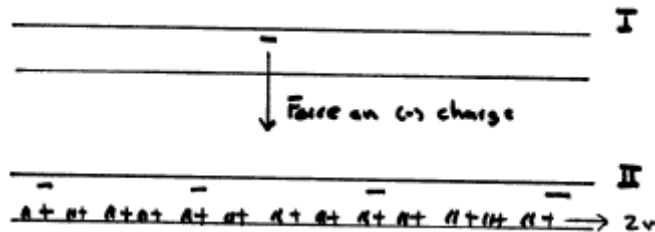


- Because the charge spacing in both wires is contracted by an identical factor, the dynamic situation here should be identical to the static situation previously considered.
- As we learned from electrodynamics an attractive force, in fact, exists between the wires.



Why?

Consider conductor II from the frame of reference of one of the negative (-) charges on conductor I.



- Because the negative charges in conductor II are moving with the same speed and in the same direction as the negative charge in conductor I, and are therefore at rest with respect to each other, so there is no length contraction of negative charge on conductor II with respect to conductor I.
- Now consider the relative motion between the (-) charge in I and the (+) charges in II, which are *not* at rest with respect to each other.
- The relative velocity of the (+) charges in II with respect to the (-) charge in I is $2v$ so their relative spacing is even closer than in the lab reference frame.
- Conductor II appears to have a net (+) charge in the frame of a (-) charge on I. Hence, the two are attracted.

Identical arguments show that the (+) charges on I look at the (-) charges on II in the same way, and that II behaves the same way with respect to I.

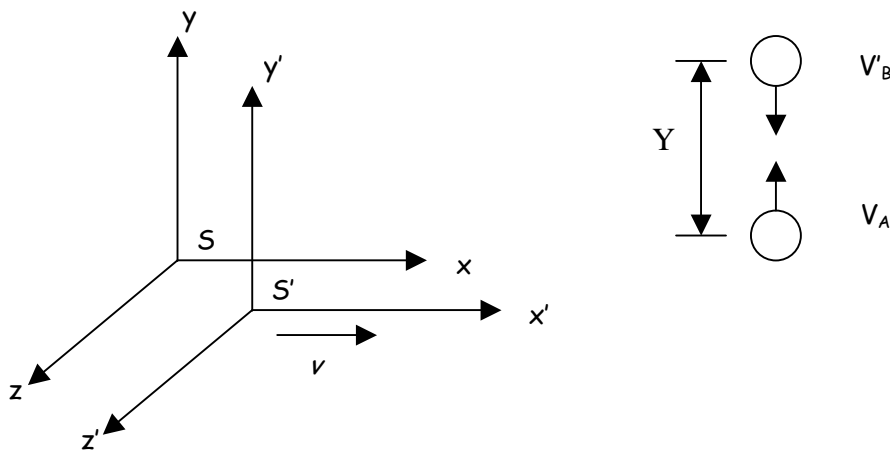
- Note that from a laboratory frame this attractive force does not exist electrically. The attraction should not occur because the conductors are both electrically neutral and no Coulomb force should exist between them.
- The force that does exist, $\vec{F}_B = q(\vec{v} \times \vec{B})$, that is attributed to "magnetism" in the lab frame is seen to be a Coulomb force $\vec{F}_z = \frac{kq^2}{r^2}$ in the frame of reference of either conductor.
- Electric and magnetic forces are not different forces. Electric and magnetic forces are manifestations of the Coulomb interaction between the charged particles.
- The only remaining issue with this is the difficulty in reconciling the concept of a current carrying element that is neutral in one frame and charged in another with invariance of charge.
- But current occurs in loops. For every element that an observer finds to have one charge there must exist another element with current flowing in the opposite direction that the same observer finds to have the opposite charge!
- Invariance, therefore, is observed. The "magnetic" forces always act between different parts of the same circuit while the circuit as a whole appears to be electrically neutral to all observers. All magnetic phenomena can be interpreted on the basis of Coulomb's Law.

Relativistic Mass

Classically, when a force is applied to a mass it accelerates and increases the objects kinetic energy. But relativistically this acceleration cannot continue without limit since the velocity of the mass could exceed the speed of light in free space.

- Energy is still conserved even when relativity is taken into account so the mass of an object must increase as its speed increases so that the work done in accelerating this mass is converted to kinetic energy in such a manner that v does not exceed c .

To illustrate this let's consider a situation in which it is easy to show that energy is conserved, e.g., an elastic collision.



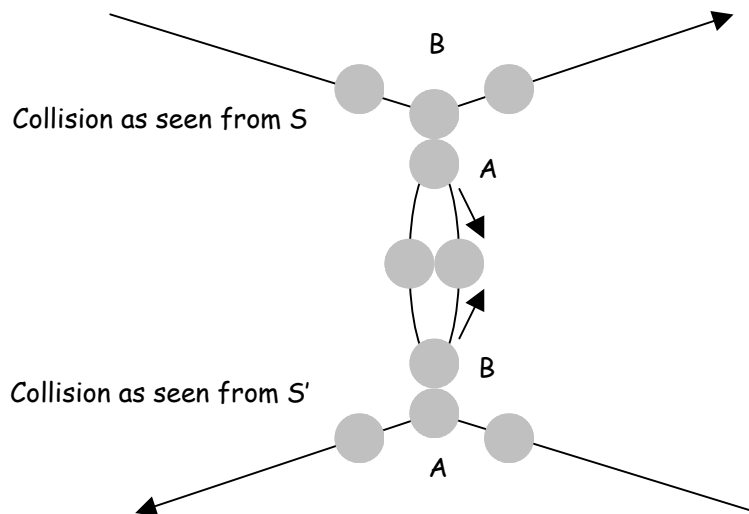
- We'll evaluate an elastic collision between two particles, A and B , as seen from observers in inertial frames S and S' that are in uniform motion with respect to each other.
- Particles A and B are identical to each other in every way when considered in the frame of reference in which they are at rest.
- The frames of reference are oriented as shown, S' moves to the right with a velocity of v along the x -axis with respect to S .
- Before the collision process begins A is at rest in S and B is at rest in S' .

- Now let's assume that A is propelled upward in the +y direction with a speed V_A at the same instant that B is propelled downward in the -y' direction with an identical speed of V'_B from a distance of Y apart (the same for both observers since the relative motion is in x only).
- In this case the behavior of A as seen from S is the same as the behavior of B as seen from S'.
- After the two particles collide A rebounds in the -y direction with a speed of V_A and B rebounds in the +y direction with a speed of V'_B .
- An observer in S concludes that the collision occurs at the position $y = \frac{1}{2}Y$ while an observer in S' concludes that the collision occurs at the position $y' = \frac{1}{2}Y$.
- For the observer in S the roundtrip time, T_0 , for particle A is:

$$T_0 = \frac{Y}{V_A}$$

and it is the same as for B in S':

$$T_0 = \frac{Y}{V_B}$$



- Since linear momentum is conserved in elastic collisions its application in the S frame yields:

$$m_A V_A = m_B V_B$$

where all quantities are as measured in the S frame.

- If T is the time required for B to make its round trip as measured in S then:

$$V_B = \frac{Y}{T} \quad (1)$$

- But in S' , B's round trip is T_0 where:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2)$$

So even though both observers see the same collision they disagree on the amount of time it takes for the particle from the other frame to make the collision and return.

Now combining equations (1) and (2):

$$V_B = \frac{Y}{\frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}} = \frac{Y \sqrt{1 - \frac{v^2}{c^2}}}{T_0}$$

and from a previous result:

$$V_A = \frac{Y}{T_0}$$

Now we use these velocities in our conservation of momentum equation:

$$m_A V_A = m_B V_B \rightarrow m_A \frac{Y}{T_0} = m_B \frac{Y \sqrt{1 - \frac{v^2}{c^2}}}{T_0}$$

which is true if:

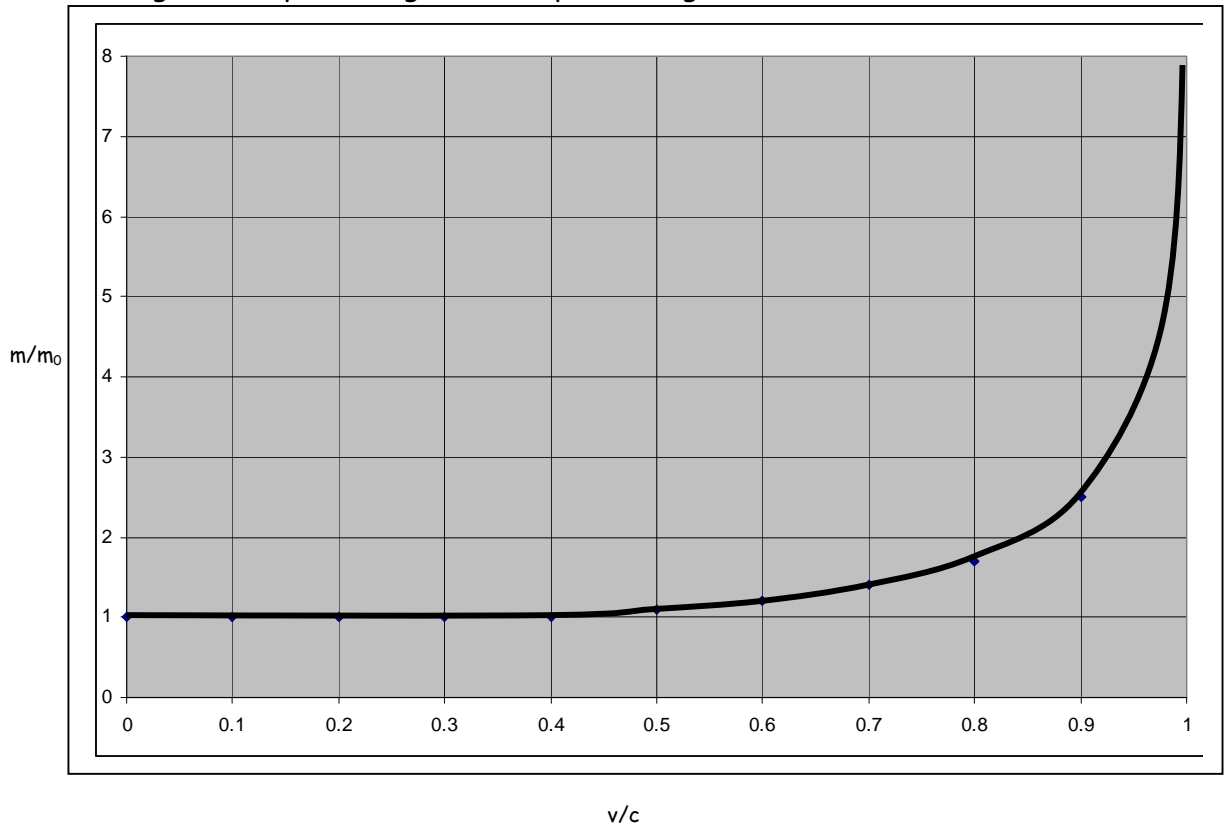
$$m_A = m_B \sqrt{1 - \frac{v^2}{c^2}}$$

It follows that the relativistic mass of an object in motion, m , as compared to the same object at rest in the frame of reference of an observer depends on velocity and is given by:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is always larger than m_0 .

- Relativistic mass increases are significant only when v is a significant percentage of c .
- In practice it is possible to accelerate only very small particles to any significant percentage of the speed of light.



Example 4 Find the mass of a proton ($m_0 = 1.6726 \times 10^{-27} \text{ kg}$) traveling with a velocity of $0.90c$ with respect to an observer in a lab.

The ratio of $v/c = 0.90$ so $v^2/c^2 = 0.81$ and:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow m_p = \frac{1.6726 \times 10^{-27} \text{ kg}}{\sqrt{1 - 0.81}} = 3.84 \times 10^{-27} \text{ kg}$$

This is about 2.3 times the protons' rest mass.

Relativistic Momentum

Recall that linear momentum is defined as

$$\vec{p} = m\vec{v}$$

so relativistic momentum may be defined

$$\vec{p} = m\vec{v} = \frac{m_0\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Second Law

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{d}{dt}m\vec{v}$$

$$= \frac{d}{dt} \left(\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

Relativistic Mass and Energy

Recalling that $W = KE = \int_0^s \vec{F} \cdot d\vec{s}$ and using the relativistic second law:

$$F = \frac{d}{dt}mv$$

$$KE = \int_0^s \frac{d(mv)}{dt} ds = \int_0^{mv} v d(mv) = \int_0^v v d \left(\frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

Integration by parts:

$$\left(\int xdy = xy - \int ydx\right)$$

$$KE = \frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 \int_0^v \frac{v dv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$KE = \frac{m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \left| m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} \right|_0^v$$

$$KE = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$$

$$KE = mc^2 - m_0 c^2$$

This is more commonly written:

$$mc^2 = m_0 c^2 + KE$$

or

$$E = E_0 + KE \text{ (total energy of an object)}$$

When an object is at rest $KE = 0$ but it still has a *rest energy* of $m_0 c^2$.

$$E_0 = m_0 c^2$$

When an object is moving its energy is:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass and energy are not conserved independently.

Example 5 The sun radiates about 4×10^{26} Watts of power from the conversion of matter to energy. How much does the mass of the sun decrease per second due to this conversion?

$$m_0 = \frac{E_0}{c^2} = \frac{4 \times 10^{26} J}{9 \times 10^{16} \frac{m^2}{s^2}} = 4.4 \times 10^9 \frac{kg}{s}$$

This is a lot of mass but the sun's total mass is about 2.0×10^{30} kg so it is miniscule by comparison.

The energy equivalent of 1 kg of matter (about the mass of your text) is:

$$m_0 c^2 = (1kg)(3 \times 10^8 m \cdot s^{-1}) = 9 \times 10^{16} J$$

Kinetic Energy at Low Speeds

When relative velocity, v , is small, relativistic KE should reduce to Classical KE.

$$KE = mc^2 - m_0c^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0c^2$$

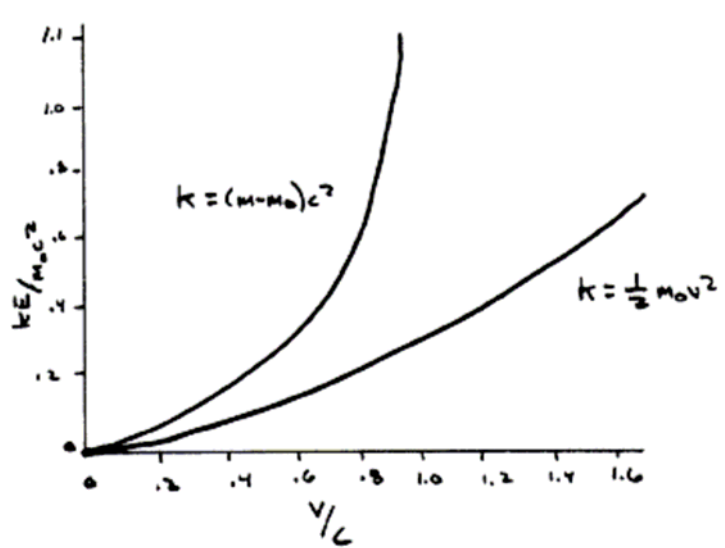
When $\frac{v^2}{c^2} \ll 1$, we can use the binomial approximation $(1+x)^n \approx 1+nx$, valid for $|x| \ll 1$. This yields:

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{1}{2} \frac{v^2}{c^2} \quad v \ll c$$

$$\Rightarrow KE \approx \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) m_0c^2 - m_0c^2$$

$$KE \approx m_0c^2 + \frac{m_0c^2}{2} \frac{v^2}{c^2} - m_0c^2$$

$$KE \approx \frac{1}{2} m_0v^2 \quad v \ll c$$



The classical formula works well for velocities less than $0.1c$ (3×10^7 m/s). At $0.5c$, the classical formula understates KE by 19%.

Momentum - Energy Relationship

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{total energy}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{relativistic momentum}$$

Notice that when $m_0 = 0$ and $v < c$:

$$E = p = 0$$

A massless particle ($m_0 = 0$) with a velocity of less than c can't have any momentum or energy.

But when $m_0 = 0$ and $v = c$:

$$\left. \begin{array}{l} E = \frac{0}{0} \\ p = \frac{0}{0} \end{array} \right\} \begin{array}{l} \text{These are indeterminate forms.} \\ E \text{ and } p \text{ can have any values.} \end{array}$$

So massless particles may exist, possessing both momentum and energy, as long as they travel at the speed of light.

Do such particles exist? Such a particle would have momentum and energy, but no rest energy. Consider:

$$E^2 = \frac{m_0^2 c^4}{1 - \frac{v^2}{c^2}}$$

$$p^2 = \frac{m_0^2 v^2}{1 - \frac{v^2}{c^2}}$$

$$p^2 c^2 = \frac{m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}}$$

$$E^2 - p^2 c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}}$$

$$E^2 - p^2 c^2 = \frac{m_0^2 c^4 \left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$E^2 - p^2 c^2 = m_0^2 c^4$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} = \sqrt{E_0^2 + p^2 c^2}$$

According to this, if $m_0 = 0$:

$$E = pc \quad \text{massless particles}$$

None of this proves the existence of massless particles, only that relativistic mechanics does not exclude this possibility that massless particles may exist provided the $v = c$ and $E = pc$.

Massless particles do, in fact, exist. The photon and the neutrino are well documented examples of massless particles.

General Relativity

- Special relativity is concerned only with objects and motion in inertial frames.
- General relativity is concerned with accelerations and non-inertial frames of reference.
- The force of gravity arises from the warping of spacetime around a massive object.
- Objects moving through spacetime contours warped by the gravity of a massive object follow curved paths.
- Light is affected by the warping of spacetime and therefore by gravity

The Principle of Equivalence

An observer in a closed laboratory cannot distinguish between the effects produced by a gravitational field and those produced by an acceleration of the laboratory.