Relativity

Einstein's Postulates:

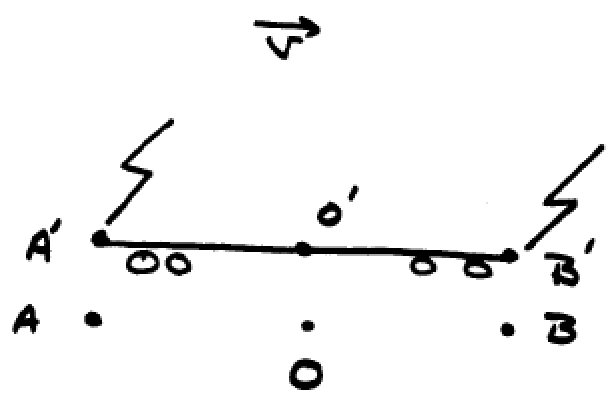
- The laws of physics are the same in all inertial frames of reference. There is no absolute or preferred frame of reference. There is no way to detect absolute motion.
- Observers in all inertial systems measure the same value for the speed of light in a vacuum.

An *inertial frame* is one in which a free body experiences no acceleration. As a consequence:

- An experiment done on a platform moving at constant velocity will yield the same results as one performed in a laboratory.
- All motion is relative

Simultaneity

Consider a railroad flat car traveling along a set of tracks when it is struck at both ends at the same time by identical lightning bolts. Exactly how this might occur isn't important but the effect of such an occurrence is.



The bolts will leave scorch marks on both ends of the rail car at points A' and B' and on the ground at points A and B beneath the ends of the rail car.

Notice that as a consequence of the motion of the rail car an observer on the ground at point O (midway between the lightning bolts) sees a different sequence of events than an observer in the middle of the rail car at O′.

- The observer at O sees both events occur simultaneously.
- The observer at O′ sees light from the front lightning bolt before the back and concludes front of the car is struck first.

So the question is who is right? The answer is *both*.

Two events that are simultaneous in one reference frame are not necessarily simultaneous in another reference frame moving with respect to the first.

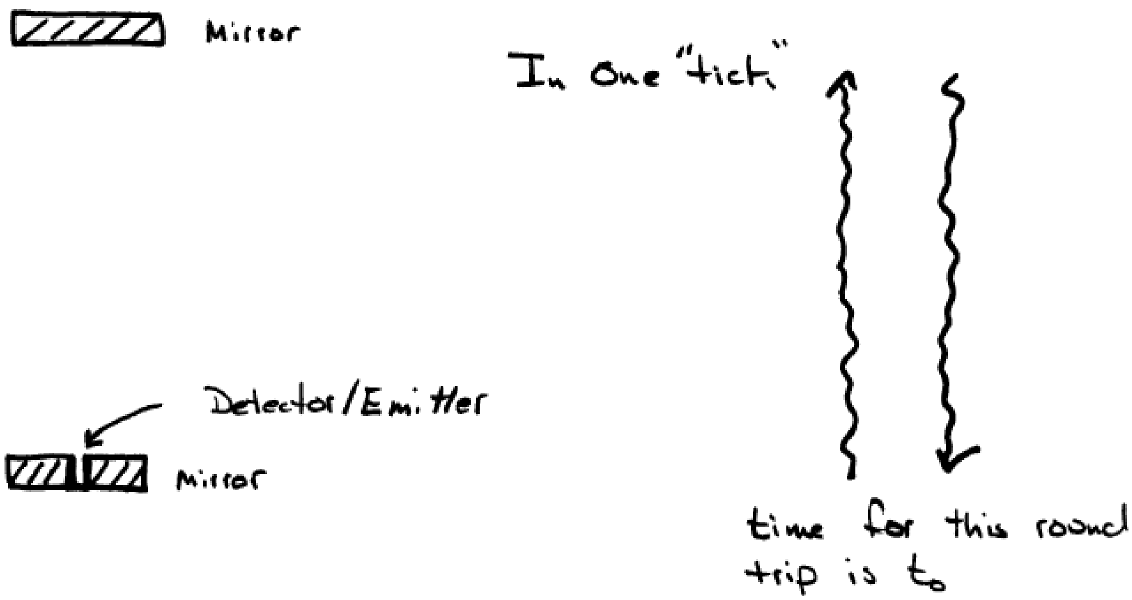
Notice that there is no inference here that the events can change order (travel backwards in time) only that they can occur simultaneously or one always precedes the other.

Relativity – Time Dilation and Length Contraction

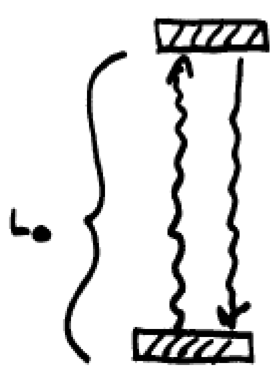
Time Dilation – A clock in motion with respect to an observer appears to tick more slowly than one at rest.

Convention: t_0 is the frame of the observer.
t - time in the moving frame.

Now consider a clock with keeps track of time with "ticks" that involve the transit of light between two mirrors.



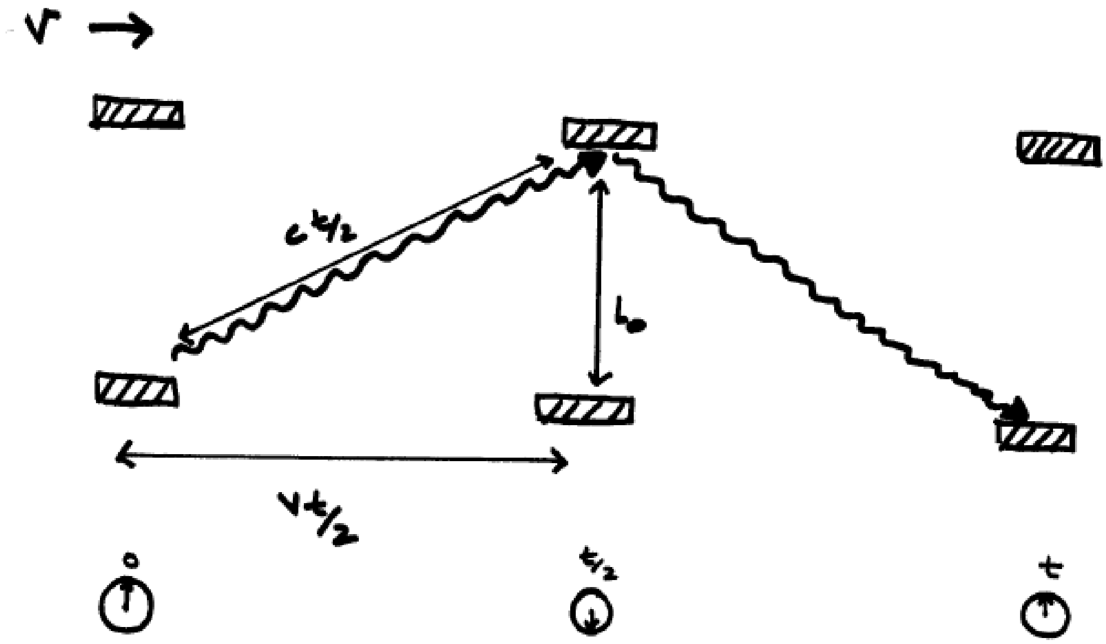
In a lab with such a clock at rest with respect to an observer:



- The time required to reach upper mirror is $\frac{t_0}{2}$
- The time for a round trip between the mirrors is t_o

Since $d = vt, \quad L_o = c\frac{t_0}{2} \rightarrow t_0 = \frac{2L_0}{c}$

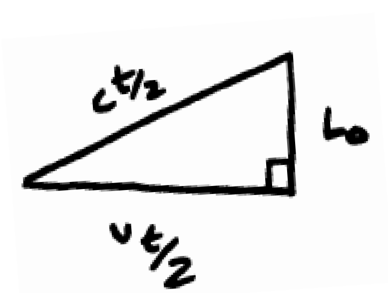
Now consider an identical clock in motion with respect to the observer in the lab:



The light "ticks" now appear to follow a diagonal path:

- In time $\frac{t}{2}$ the horizontal distances the clock travels is $v\frac{t}{2}$
- In time $\frac{t}{2}$ the distance the beam of light travels is $c\frac{t}{2}$

Let's examine the geometry of this situation and see if we can use it to relate *v, c,* and *t.*



$$a^2 + b^2 = c^2$$

$$\left(\frac{ct}{2}\right)^2 = L_o{}^2 + \left(\frac{vt}{2}\right)^2$$

$$\frac{c^2t^2}{4} = L_o{}^2 + \frac{v^2t^2}{4}$$

$$\frac{t^2}{4}\left(c^2 - v^2\right) = L_o{}^2$$

$$t^2 = \frac{4L_o^{\ 2}}{c^2 - v^2}$$

$$t^2 = \frac{(2L_o)^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{\frac{2L_o}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Recall that for a stationary clock we obtained a relationship between t_0 and L_0 that applies here as well: $t_o = \frac{2L_o}{c}$, and substituting this into the numerator of the equation above we obtain:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- t – time on moving clock
- t_o – time on clock at rest
- v – relative speed
- c – speed of light

Note that according to this relationship, t is always > t_o. This relationship is known as relativistic *time dilation*.

In relativistic mechanics

- the sequence of events is unchanged
- one cannot peer into the future
- time does not run backwards

Example 1 What is the speed of a spacecraft whose clock runs 2 second slower per hour than the clock of an observer on earth?

t = time observer on clock in spacecraft
t_o = time observed on clock on earth

We solve for *v*:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{t_o}{t}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$1 - \frac{{t_o}^2}{t^2} = \frac{v^2}{c^2}$$

$$v = c\sqrt{1 - \frac{t_0^2}{t^2}}$$

$$\approx 3.33\%c$$

Evidence for special relativity - muon (μ±) production

- Muons are created by collisions with cosmic rays (γ rays) and atomic nuclei high in the atmosphere.
- A muon has a mass of 207 times that of an electron, has the charge of ±e, and decays into an electron or a positron after a mean lifetime of 2μs before decaying into other particles.
- Muons have a speed of .998c (2.994×10^8 m/s) and are found in profusion at the earth's surface.

Classically there should not be any muons detected at the earth's surface since

muon lifetime (2ms) x muon speed (.998c) = 600 m

which means that muons decay after traveling about 600 meters from their point of origin. Since muons are created at heights of 8000m and above none should reach the earth's surface.

Now consider relativity. Since muons travel at relativistic speeds we must use special relativity to compute the lifetime of a muon in the frame of reference of an observer on earth. Given $t_o = 2 \times 10^{-6} s$, $v = .998c$:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2 \times 10^{-6} s}{\sqrt{1 - \frac{(0.998c)^2}{c^2}}}$$

$$t = 31.6 \mu s$$

In this amount of time, $d = vt = (2.994 \times 10^8 m \cdot s^{-1})(31.6 \times 10^{-6} s) = 9500m$

So a ground-based, stationary observer records the lifetime of a muon as 31.6μs and in this amount of time a muon travels 9500 meters which is why they are found in abundance at Earth's surface.

The corollary to time dilation is *length contraction*. To an observer traveling with the muon the path traveled is shortened by virtue of motion. This length contraction occurs only along the direction of motion. The contraction is by the factor $\sqrt{1-\frac{v^2}{c^2}}$.

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 9500m\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 600m$$

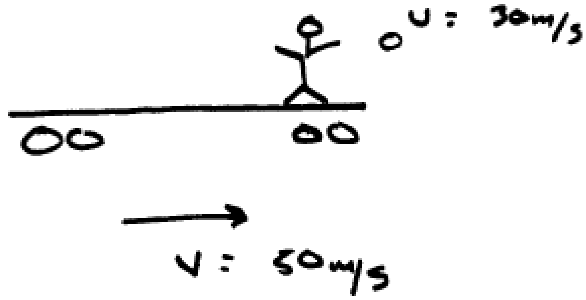
So an observer moving with the muon measures a path of 600 meters before decay.

The factor $\sqrt{1-\frac{v^2}{c^2}}$, which appears often in relativistic equations is often referred to as γ.

Velocity Addition

When two objects are in motion with respect to a stationary observer, under relativistic conditions, adding their velocities is not trivial.

Consider a platform moving to the right as shown below. Since the velocities involved are *non*-relativistic,



the total velocity of the ball with respect to an observer standing on the ground is 50 m/s + 30 m/s = 80 m/s

But if one substitutes a flashlight beam for the ball and uses the same classical velocity addition $v_{total} = v + u$ which is greater than *c*. This violates the second postulate of special relativity.

Relativistic velocity addition is accomplished with the Lorentz velocity transformations:

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

Primed variables are in the *moving* frame with respect to an observer.

Example 2 Assume in the example given above that the rail car is moving with a velocity of 0.8c and the ball is released with a velocity relative to the person on the *flatcar* of 0.6c. What velocity does a stationary observer on the ground observe?

We want the velocity in the stationary frame u_x.

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

- $u_x' \equiv$ the velocity of the second object in motion with respect to the first object in motion.
- $v \equiv$ the velocity of the first object in motion with respect to the observer at rest.
- The signs of these velocities matter only to the extent that objects moving in opposite directions should have an opposite sign.

$$u_x = \frac{0.6c + 0.8c}{1 + \frac{(0.6c)(0.8c)}{c^2}} = .946c$$

Try this with $u_x' = c$ and notice that the result is $u_x = c$.

Example 3 Two spacecraft, A and B, move toward each other with velocities measured on earth of A = 0.75c and B = 0.85c. Find the velocity of B with respect to A.

Assume that A is in motion with respect to earth and B is in motion with respect to A.

$$u_x' = \frac{u_X - v}{1 - \frac{u_x v}{c^2}}$$

We want to find the velocity of B with respect to A so we employ the primed transformation where:

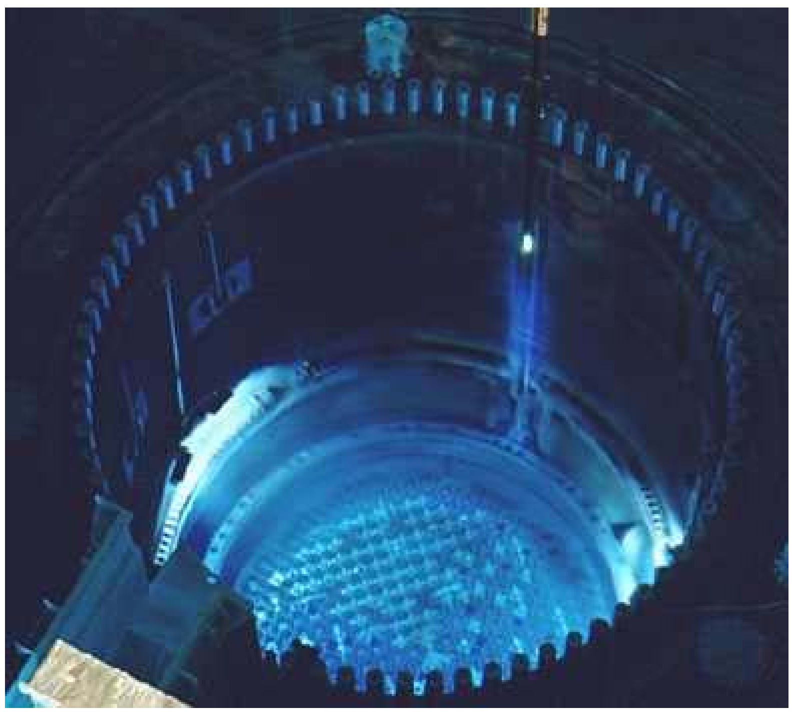
$u_x \equiv$ motion of B with respect to earth
$v \equiv$ motion of A with respect to earth

$$u_x' = \frac{-0.85c - 0.75c}{1 - \frac{(-0.85c)(0.75c)}{c^2}}$$

$$= -0.98c$$

Cerenkov Radiation

- The speed of light in free space, c, is inviolate.
- No massive particle can travel at the speed of light
- Nothing can travel faster than the speed of light
- In dense material light slows down, i.e., $v < c$, and some particles are able to move faster than light in such media.
- When charged particles move faster than light in a dense media light waves similar to the bow waves created by a canoe moving down a stream faster than the velocity of the stream itself are created.
- These light waves are known as Cerenkov radiation and are useful for measuring the speeds of such particles.



Cerenkov radiation in the core of the nuclear reactor Photo by Charles Bell
Courtesy of http://www.physlink.com/Education/AskExperts/ae219.cfm

The Twin Paradox

Consider two twins, J and R, whose lifespans constitute biological clocks. J enters the space program and is sent on a mission to explore a nearby star system while R stays behind observing the voyage from mission control. .

J and R are both 20 when J blasts off into space. If we assume that J's spaceship is capable of making the voyage at a speed of about 0.8c (ignoring the acceleration at liftoff) then everything that J does appears to take longer in R's frame of reference by a factor of:

$$\sqrt{1-\frac{v^2}{c^2}} \rightarrow \sqrt{1-\frac{(0.8c)^2}{c^2}} = 0.6 = 60\%$$

This means that everything J does, breathe, walk and age seems to take longer by a factor of 3/5. For every three beats of J's heart, for instance, R's heart beats five times.

Finally J returns to earth 50 years later according to R and mission control. In J's frame of reference the same journey has taken only 30 years so upon their reunion J is 50 years old and R is 70 years old.

As much as this runs against intuition this is not the paradox. The difference in ages is simply a consequence of special relativity and is a function of the way in which our Universe operates. Although relativistic effects have been measured with great accuracy they are not apparent without velocities approaching the speed of light. Our everyday experiences have not equipped us to notice relativistic effects.

The "paradox" occurs because neither frame of reference in this problem can be considered to be the preferred frame. For this reason J should be able to apply the same relativistic calculations to R assuming that the earth is moving away at a speed of 0.8c. Given this apparent symmetry why should either twin be older than the other?

The solution to the paradox lies in the fact that the two situations are not symmetric. When J arrives at the distant star system then turns around to come back he switches inertial frames while R remains in the same inertial frame throughout. R is entitled to apply time dilation while J is not.

J also moves along a length contracted path along his voyage:

$$L = L_o\sqrt{1-\frac{v^2}{c^2}} \rightarrow 20ly\sqrt{1-\frac{(0.8c)^2}{c^2}} = 12ly$$

In J's inertial frame time goes by at the usual rate but his voyage to the star takes

$$\frac{L}{v} = \frac{12ly}{0.8c} = 15years$$

while the return voyage takes another 15 years for a total of 30 years. So although J's lifetime seems longer to R it does not seem so to J.

Relativistic Mass and Energy

Classically, when a force is applied to a mass it accelerates and increases the objects kinetic energy. But relativistically this acceleration cannot continue without limit since the velocity of the mass could exceed the speed of light in free space.

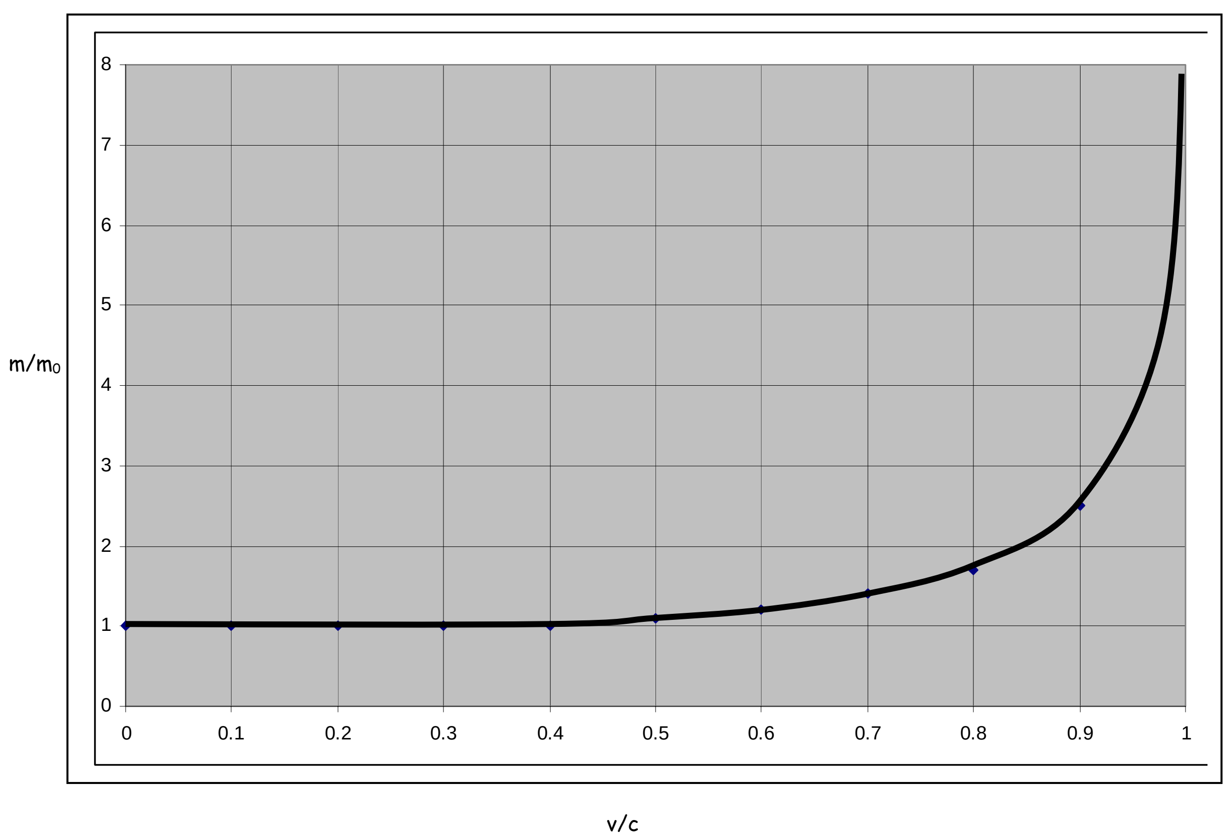
Energy is still conserved even when relativity is taken into account so the mass of an object must increase as its speed increases so that the work done in accelerating this mass is converted to kinetic energy in such a manner that v does not exceed c.

It may be shown that the relativistic mass of an object in motion, *m*, as compared to the same object at rest in the frame of reference of an observer depends on velocity and is given by:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where *m* is always large that m_0.

Relativistic mass increases are significant only when *v* is a significant percentage of *c*. In practice it is possible to accelerate only very small particles to any significant percentage of the speed of light.



It may also be shown that an object at rest has the energy:

$$E_0 = m_0 c^2$$

When an object is moving its energy is:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass and energy are not conserved independently.

Example 4 Find the mass of a proton (m_0 = 1.6726 × 10^{-27}kg) traveling with a velocity of 0.90c with respect to an observer in a lab.

The ratio of v/c = 0.90 so v^2/c^2 = 0.81 and:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow m_p = \frac{1.6726 \times 10^{-27}\, kg}{\sqrt{1 - 0.81}} = 3.84 \times 10^{-27}\, kg$$

This is about 2.3 times the protons' rest mass.

Example 5 The sun radiates about 4 × 10^{26} Watts of power from the conversion of matter to energy. How much does the mass of the sun decrease per second due to this conversion?

$$m_0 = \frac{E_0}{c^2} = \frac{4 \times 10^{26}\, J}{9 \times 10^{16}\, \frac{m^2}{s^2}} = 4.4 \times 10^9\, \frac{kg}{s}$$

This is a lot of mass but the sun's total mass is about 2.0 × 10^{30}kg so it is miniscule by comparison.

The energy equivalent of 1 kg of matter (about the mass of your text) is:

$$m_o c^2 = (1kg)\left(3 \times 10^8\, m \cdot s^{-1}\right) = 9 \times 10^{16}\, J$$