

# PHYS 212 Examination 2

Name (print): Key

Signature \_\_\_\_\_

min

12

max

95

$\bar{x}$

63.6%

STD

25.6

Problem 1 \_\_\_\_\_

Problem 2 \_\_\_\_\_

Problem 3 \_\_\_\_\_

Problem 4 \_\_\_\_\_

Problem 5 \_\_\_\_\_

**Total** \_\_\_\_\_

**Directions:** This exam contains six problems worth 20 points each for a possible 120/100 points. Your solutions should be written as neatly as possible and arranged in a logical manner. Credit will be awarded on the basis of thought, compactness, and neatness of the written solution. Remember to use basic physical principles in solving the problems. Show all of your work. I will not award full points for a problem with a solution that I am unable to decipher even if the answer is correct.

An equation sheet has been provided. CRC handbooks are allowed. Calculator rule is in effect. Good Luck!

**Problem 2.** Consider the charged disk of the Problem 1. If  $x$  is sufficiently large with respect to  $R$ , the potential at  $P$  is:

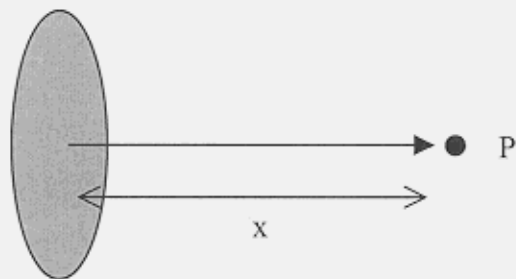
$$V = \frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x)$$

Under these conditions, what is the electric field at  $P$ ?

$$\begin{aligned} \vec{E} &= -\vec{\nabla} V \quad +10 \\ &= -\frac{d}{dx} \left( \frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x) \right) \\ &= -\frac{\sigma}{2\epsilon_0} \left( \frac{1}{2} (x^2 + R^2)^{-1/2} (2x) - 1 \right) \\ &= \frac{\sigma}{2\epsilon_0} \left( 1 - \frac{x}{\sqrt{x^2 + R^2}} \right) \quad +10 \end{aligned}$$

**Problem 1.** The disk shown below has radius  $R$  and uniform charge density  $+Q$ . If  $R$  is very large compared to  $x$ , compute:

- the electric field at  $P$
- the potential at  $P$
- the force on a charge  $q$  at  $P$



$$\bullet \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \quad q_{enc} = QA$$

$$\vec{E}(QA) = \frac{QA}{\epsilon_0}$$

$$\vec{E} = \frac{Q}{2\epsilon_0} \hat{x} + C$$

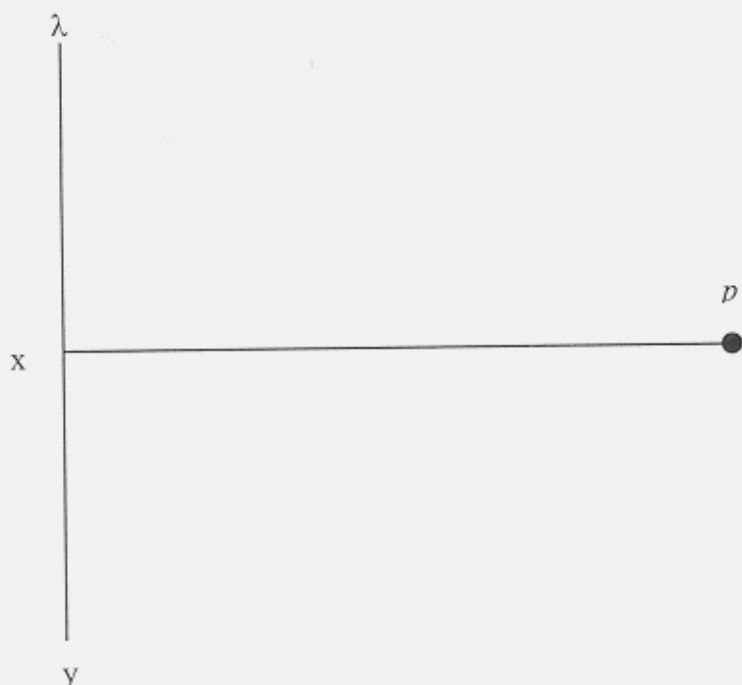
$$\bullet V_P = -\int E dx$$

$$= -Ex = -\frac{Qx}{2\epsilon_0} + C$$

$$\bullet \vec{F} = q\vec{E}$$

$$= \frac{Qq}{2\epsilon_0} \hat{x} + C$$

**Problem 3.** Consider a uniform charge distribution  $\lambda$ , distributed along a long line, as shown below, and the force exerted by it on a charge,  $q$ , at point  $p$  some distance  $x$  from the line. Compute the force on the charge  $q$ , the electric field at  $p$ , and the potential at point  $p$ .



$$\lambda l = q$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E \cdot d\vec{A} = \frac{\lambda l}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$\vec{E} = \frac{\lambda \hat{r}}{2\pi r \epsilon_0} = \frac{2k\lambda}{r} \hat{x} \quad +7$$

$$\vec{F} = \vec{E}q$$

$$= \left( \frac{2k\lambda}{r} \hat{x} \right) (q) = \frac{2k\lambda q}{r} \hat{x} \quad +10$$

$$V = - \int E \cdot dx$$

$$= - \frac{2k\lambda}{r}(x) = \frac{2k\lambda x}{r} \quad +7$$

**Problem 4.** A solid insulating sphere of radius  $a$  with a uniform charge density  $\rho$  is surrounded by a concentric conducting sphere of inner radius  $b$ , outer radius  $c$ . What is the electric field (a) when  $r = 0$  (b)  $a < r < b$  (c)  $b < r < c$  (d)  $c < r$ ? It is OK to express your answer in terms of  $r$ .

a.  $\vec{E} = \frac{kq}{r^2} = 0$  +5

b.  $\vec{E} = \frac{kq}{r^2} = \frac{k\rho}{r^2}$  +5

c.  $\vec{E} = 0$  +5

d.  $\vec{E} = \frac{kq}{r^2}$  +5

Problem 5. Using

$$V = k \int \frac{dQ}{r}$$

to compute the potential at the *center* of a uniformly charged sphere of radius  $R$ , I obtain the expression:

$$V_o = \frac{3kQ}{2R}$$

while using

$$V_B - V_A = - \int \vec{E} \cdot d\vec{s} \quad (\vec{E} = \frac{kQ}{r^2} \hat{r})$$

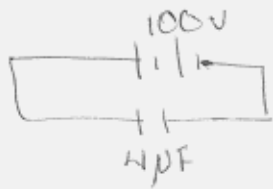
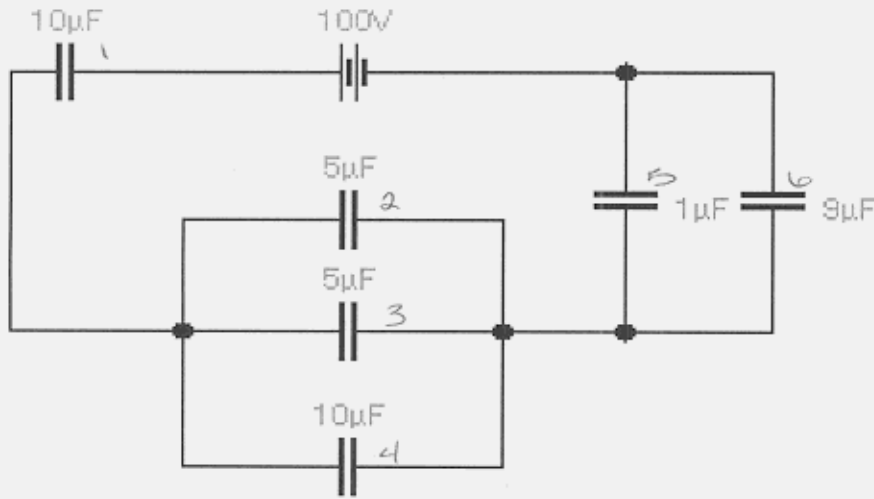
I obtain the expression:

$$V_o = \frac{kQ}{R}$$

Which expression is correct? What is wrong with the other computation?

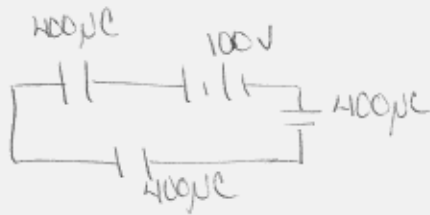
First <sup>+10</sup> expression is correct. The second ignores the discontinuity at  $r=R$  <sup>+10</sup>

**Problem 6.** Consider the circuit below. What is the charge on each capacitor and what is the potential across each capacitor.



$$QV = Q$$

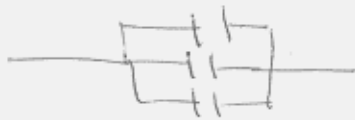
$$(4\mu F)(100V) = 400\mu C$$



$$QV = Q$$

$$V = \frac{Q}{C}$$

$$= \frac{400\mu C}{10\mu F}$$



$$V = \frac{400\mu C}{20\mu F} = 20V$$

Cap	Charge	Pot
1	400µC	40V
2	100µC	20V
3	100µC	20V
4	200µC	20V
5	40µC	40V
6	360µC	40V