# Optics I – A General Description of Light and the Laws of Geometric Optics

So far we have considered the wave nature of light and its place in the electromagnetic spectrum. In this unit we will be less interested (though aware) of the wave nature of light.

In our study of light we will be interested in interactions of light beams with matter and with each other:

1. Reflection
2. Refraction
3. Polarization
4. Diffraction
5. Scattering
6. Interference

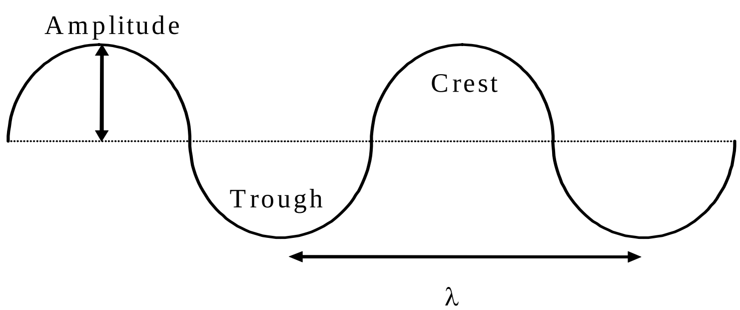
We will consider light to consist of rays or beams that are narrow, monochromatic (single wavelength) and move in straight lines through any homogenous media.

## Historical Figures and Their Contributions to Theories of Light

- Newton – corpuscular theory
- Huygens – wave theory
- Young – interference
- Maxwell – E/M equations
- Hertz – verified Maxwell's equations experimentally
- Michaelson & Morley – luminiferous ether
- Einstein – photons
- Bose – photons are indistinguishable particles

## Light – Physical Attributes

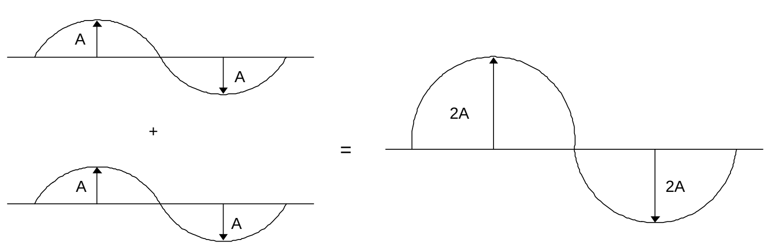
- Visible light exists between 400 – 700 nanometers ($10^{-9}$)
- White light consists of all frequencies in the visible range combined
- Wavelength – peak to peak distance
- Amplitude – ½ of the distance from peak to trough



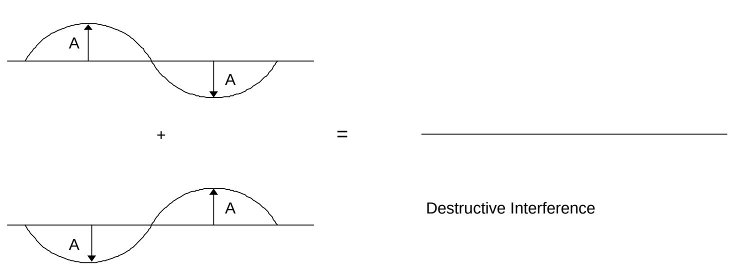

- Frequency – number of oscillations per second
- Photons are indistinguishable particles known as *bosons* (not subject to the Pauli Exclusion Principle). Electrons, protons and neutrons are *fermions* (subject to the P.E.P.)

## Interactions of Light Beams

- *Interference* - combining e/m waves together in such a manner as to cause either constructive (bright) or destructive (dark) patterns to result

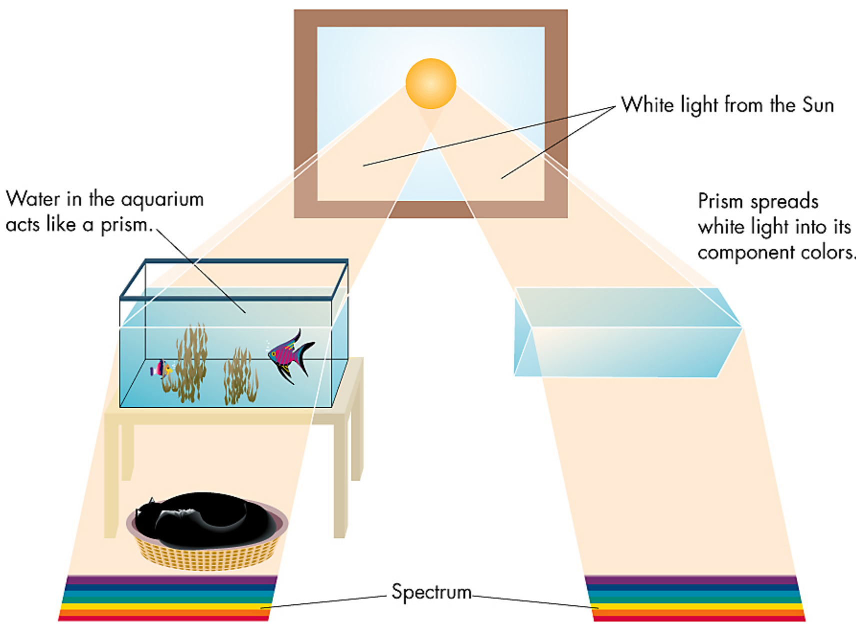



Constructive Interference

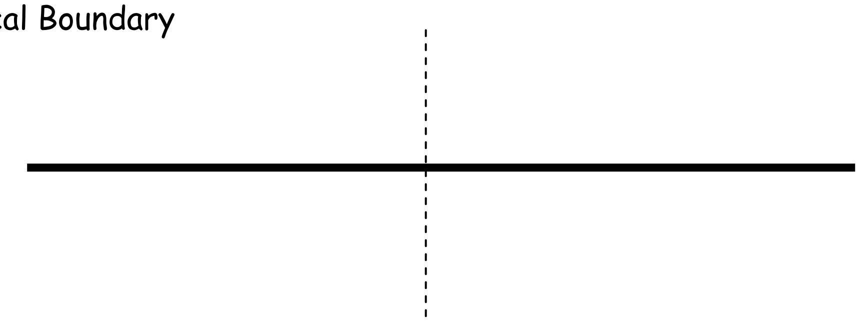



Destructive Interference

- *Diffraction* – bending of light or any e/m waves as they pass through narrow openings or around sharp corners. Explained by *Huygen's Theory* (below).
- *Reflection* - a form of scattering that may be described with a simple geometric relationship, i.e. angle of incidence equals angle of reflection (The Law of Reflection). There are two types of reflection: specular and diffuse.
- *Scattering* – what happens, in general, when light or any e/m waves interact with matter.
- *Polarization* – orientation of e/m fields in space
- *Refraction* – the change of speed and direction that occurs when light goes from one medium to another. Refraction of white light, which contains all wavelengths from 400 to 700 nanometers results in *dispersion*.



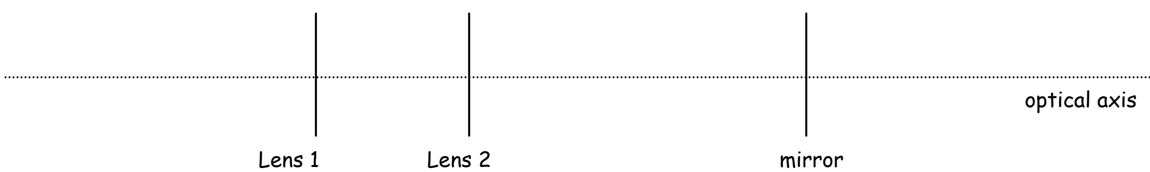
## An Optical Boundary



- An optical boundary is an interface between any two dissimilar media.
- An optical boundary is shown as above with a perpendicular line or *normal* drawn crossing the boundary between the two media (the heavy line above). This template will be important for ray tracing during our study of reflection and refraction.
- When a beam of light encounters an optical boundary three things may happen: reflection, transmission and refraction or some combination of both.

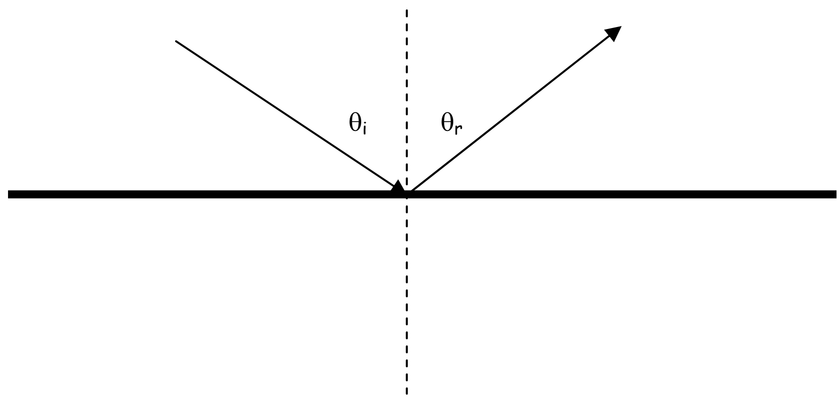
## An Optical System

An optical system consists of one or more refracting and or reflecting devices and is designed to produce an image by modifying beams of light that pass through them.



## Reflection and Refraction

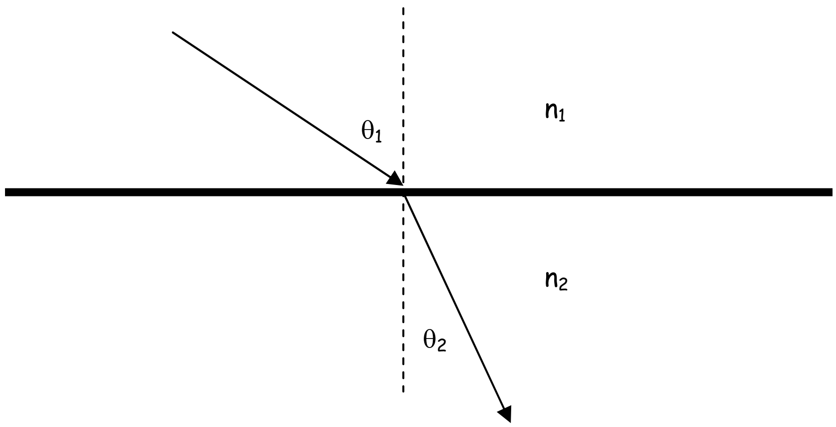
The Law of Reflection $\theta_i = \theta_r, coplanar$




The geometry and type of reflection depends on:

- The frequency/wavelength of the incoming wave
- The incident angle of the incoming wave
- The composition of the material off which the beam is being reflected. In general very good conductors are also very good reflectors
- The smoothness of the reflective surface
- Reflection may be thought of as a specific form of scattering that follows a specific geometric relationship.
- Some reflection occurs even at optical boundaries between transparent media.
- External reflections occur when the medium that contains both the incident and reflected rays has a lower index of refraction than the reflective medium.

**The Law of Refraction** $n_1 \sin\theta_1 = n_2 \sin\theta_2$



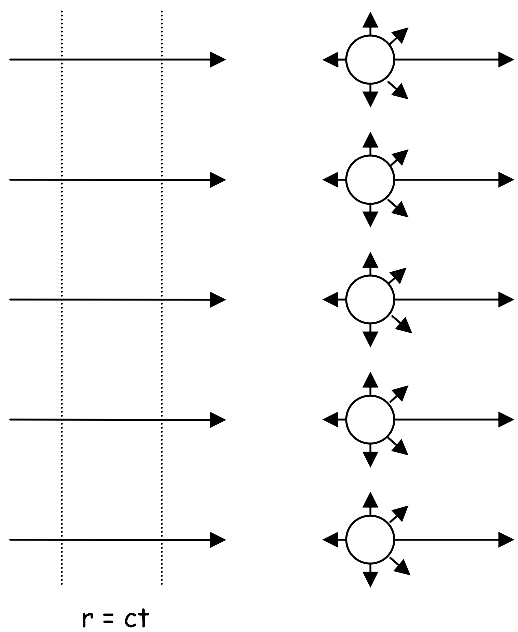

The geometry and type of refraction depends on:

- The frequency/wavelength of the incoming wave
- The incident angle of the incoming wave
- The opacity or transparency of the media
- The material on each side of the optical boundary, specifically a property known as the index of refraction of the material which is a measure of the speed with which light travels in the material as compared to the speed of light in free space
$$n = \frac{c}{v}$$
- Refraction or bending of a light beam as it crosses an optical boundary occurs due to the beam slowing down as to goes from a less to more dense medium or speeding up as it goes from a more dense to a less dense medium. *Frequency remains unchanged.*
- Since $v = \lambda f$, a monochromatic beam of light has a shorter wavelength in a dense medium than it does in free space: $\lambda_n = \frac{\lambda_0}{n}$
- The amount of refraction depends both on the frequency/wavelength of the light beam and the material in which it is being refracted.
- Light beams bend towards the normal when $n_1 < n_2$. Light beams bend away from the normal when $n_1 > n_2$.
- Broad spectrum, *polychromatic* light (such as white light) has components that bend at different angles in a given media

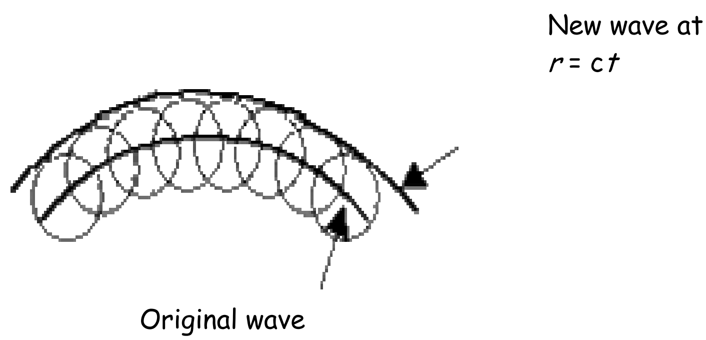
- Red light bends the least and blue the most as white light enters a dense transparent medium.
- The differential bending of each element of polychromatic or white light is known as dispersion.
- Dispersion is common in raindrops, lenses and prisms.
- Dispersion separates any beam of polychromatic light into its component colors.
- In the case of white light all colors of the visible spectrum result from dispersion.

## Huygens Model of Light Wave Propagation

**A plane wave**

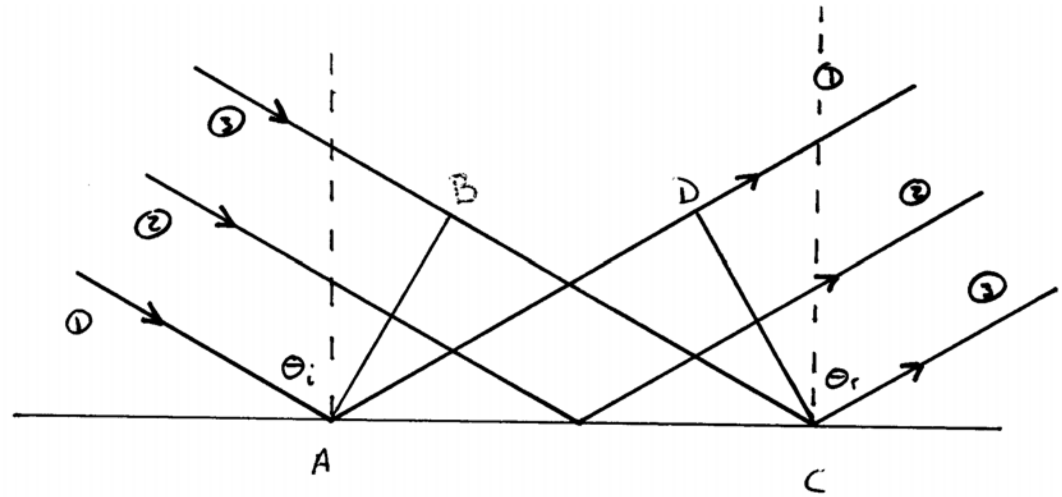



**A spherical wave**

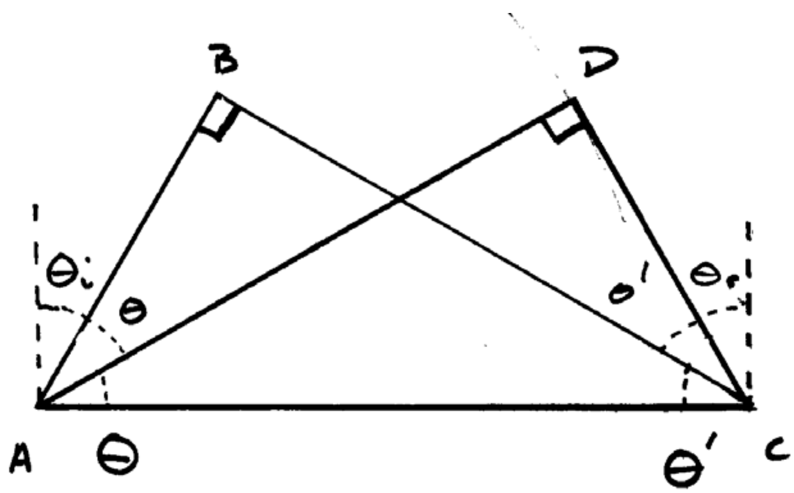



- Christian Huygens was the first to propose that light was a wave.
- Light waves traveling together through space or any dense medium form *wavefronts*.
- Any point on a wavefront is capable of acting as a new source of the wave.
- In general waves self-propagate through space by this method.
- This is known as ***Huygens Theory*** and may be used to explain *reflection, refraction, diffraction* and *interference*.

Huygens Principle Applied to Reflection




$\overline{AB}$ is the last plane wavefront that is totally unreflected
$\overline{DC}$ is the first plane wavefront that is totally reflected




As ray **3** travels from **B** → **C**, ray **1** is reflected from **A** and produces a wavelet of radius $\overline{AD}\ (= ct)$

Since the wavelets having radii **BC** and **AD** are in the same medium, they must have the same velocity, *v*, this leads to the conclusion:

$$\overline{BC} = \overline{AD} = ct \qquad \text{(in air)}$$
$$= vt \qquad \text{(any other medium)}$$

Notice that this is not a self-fulfilling prophecy. There is no requirement here that these segments have the same length other than the fact that Huygens' theory specifies that any point along a wavefront may act as the source of a new wave that propagates forward with a velocity of c.

- The incident and reflected wavefronts and the segment **AC** form two congruent right triangles, **ACB** and **CAD.**
- Both of these triangles share hypotenuse $\overline{AC}$.
- Right triangle relationships yield:

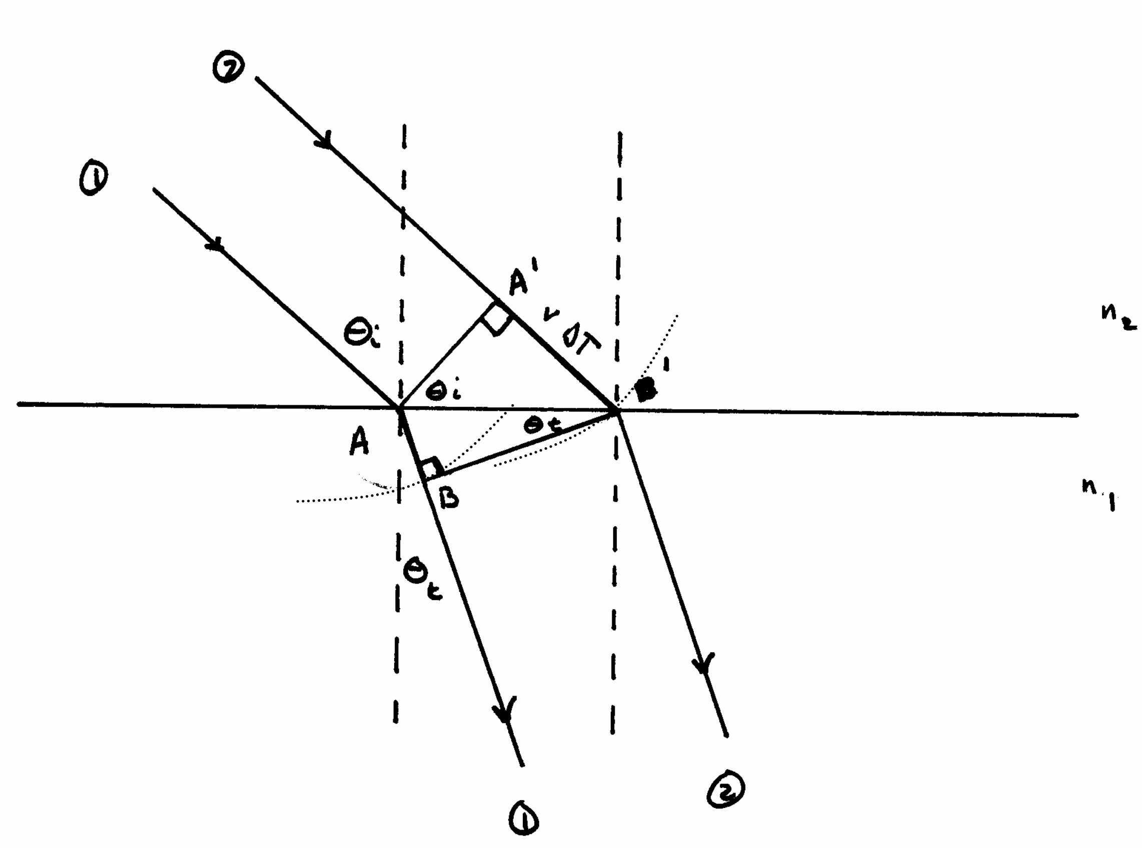
$$\sin\theta = \frac{\overline{DC}}{\overline{AC}} \frac{(opp)}{(hyp)}$$

$$\sin\theta' = \frac{\overline{BA}}{\overline{AC}} \quad \frac{(opp)}{(hyp)}$$

$$\Rightarrow \quad \frac{\overline{DC}}{\overline{AC}} = \frac{\overline{BA}}{\overline{AC}} \quad \therefore \quad \sin\theta = \sin\theta'$$

$$\Rightarrow \quad \sin\theta_i = \sin\theta_r \quad \therefore \quad \theta_i = \theta_r$$

Huygens Principle Applied to Refraction




In time interval Δt ray **1** moves from **A** → **B** and ray **2** moves from A′ → **B**′.

The radius of an incoming wavelet centered at **A'** is r = **A'B'** = $v_2$ ΔT.
The radius of an outgoing wavelet centered at **A** is r = **AB** = $v_1$ ΔT.

Notice again that we have two right triangles **AA'B'** and **ABB'** that share a common hypotenuse **AB'**

$$\sin\theta_i = \frac{\overline{A'B'}}{\overline{AB'}} \quad \therefore \quad \overline{A'B'} = \overline{AB'}\sin\theta_i = v_2\Delta T$$

$$\sin\theta_t = \frac{\overline{AB}}{\overline{AB'}} \quad \therefore \quad \overline{AB} = \overline{AB'}\sin\theta_t = v_1\Delta T$$

Now if we divide the first equation by the second:

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_2\Delta T}{v_1\Delta T}$$

The indices of refraction are related to the speed of the waves in the media $n_1$ and $n_2$: $v_2 = \frac{c}{n_2}$ and $v_1 = \frac{c}{n_1}$, this leads to the conclusion:

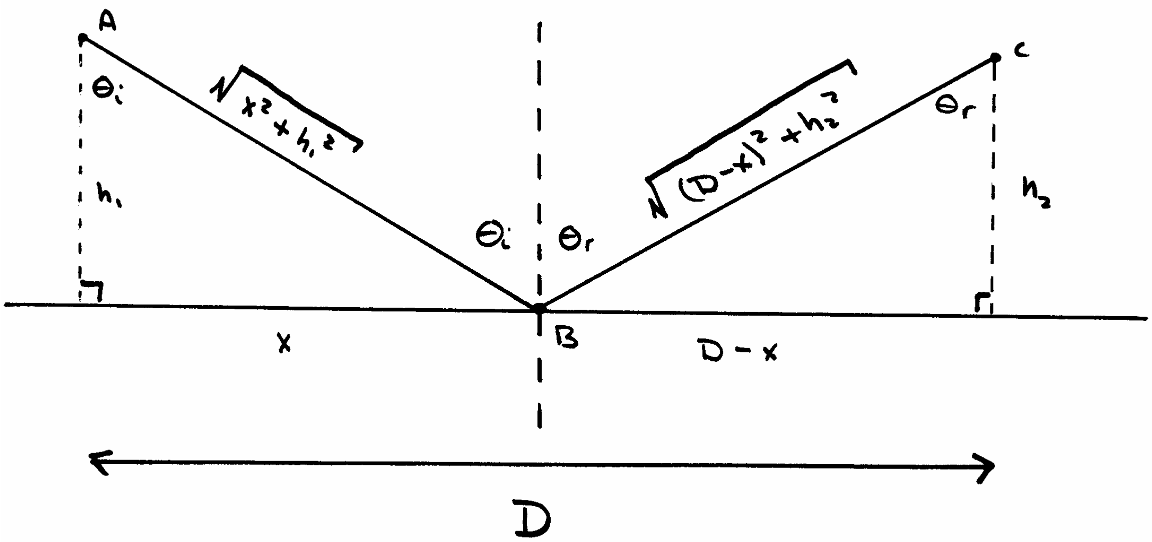
$$\sin\theta_i \frac{c}{n_1} = \sin\theta_t \frac{c}{n_2}$$

$$\therefore n_1 \sin\theta_t = n_2 \sin\theta_i$$

Fermat's Principle Applied to Reflection

Fermat's Principle determines the path of a ray by minimizing its time of transit function.

Consider the progress of a ray from A to C, in a common medium, as shown below.




Notice that we have chosen to express all distances in terms of the variable *x*. This, again, to avoid any circular reasoning. The time of transit is:

$$t = \frac{\overline{AB}}{v} + \frac{\overline{BC}}{v} = \frac{\sqrt{x^2 + h_1^{\ 2}}}{v} + \frac{\sqrt{(D-x)^2 + h_2^{\ 2}}}{v}$$

To minimize time with respect to distance we take the derivative of f(t) with respect to distance and set it to zero, $\frac{dt}{dx} = 0$:

$$\frac{dt}{dx} = \frac{\frac{1}{2}\left(x^2 + h_1^{\ 2}\right)^{-\frac{1}{2}}(2x)}{v} + \frac{\frac{1}{2}\left[(D-x)^2 + h_2^{\ 2}\right] \cdot 2(D-x) \cdot (-1)}{v} = 0$$

$$\frac{\left(x^2 + h_1^{\ 2}\right)^{-\frac{1}{2}}(x)}{v} - \frac{\left[(D-x)^2 + h_2^{\ 2}\right]^{-\frac{1}{2}}}{v} \cdot (D-x) = 0$$

$$\frac{x}{\sqrt{x^2 + h_1{}^2}} = \frac{D - x}{\sqrt{(D - x)^2 + h_2{}^2}}$$

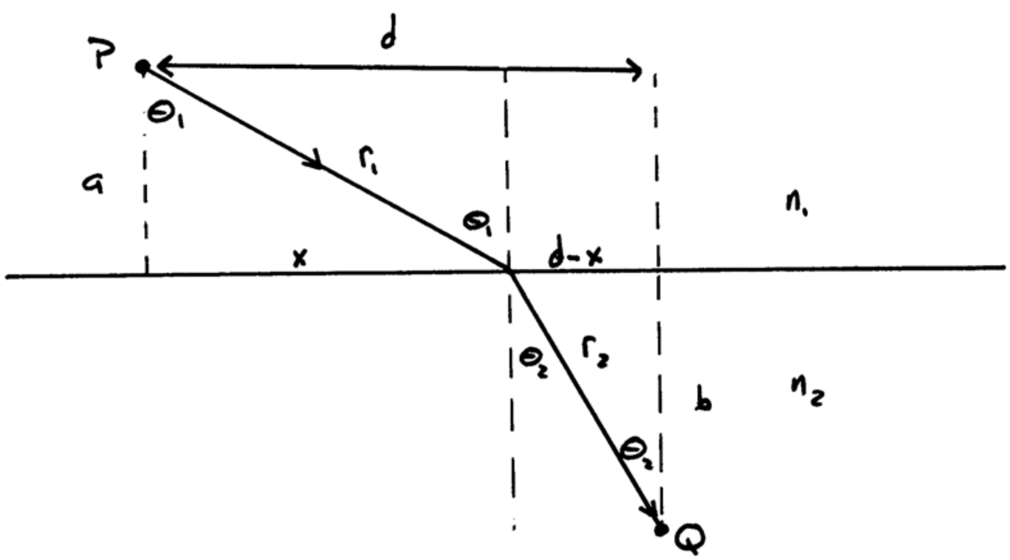
Referring to the triangles in the diagram above:

$$\frac{x}{\overline{AB}} = \frac{D - x}{\overline{BC}}$$

$$\sin\theta_i = \sin\theta_r$$

$$\theta_i = \theta_r$$

## Fermat's Principle applied to refraction




Consider a ray moving from point **P** to point **Q** and crossing an optical boundary between two different media in the process. **P** is in medium 1, **Q** is in medium 2

The velocity of the beams : v in medium 1 is $\frac{c}{n_1}$, v in medium 2 is $\frac{c}{n_2}$

The transit time from **P** → **Q** is:

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{\frac{c}{n_1}} + \frac{\sqrt{b^2 + (d - x)^2}}{\frac{c}{n_2}}$$

Again, to minimize time with respect to distance we take the derivative of f(t) with respect to distance and set it to zero, $\frac{dt}{dx} = 0$:

$$\frac{dt}{dx} = \frac{\frac{1}{2} n_1 \left(a^2 + x^2\right)^{-1/2} 2x}{c} - \frac{\frac{1}{2} n_2 \left(b^2 + \left(d - x\right)^2\right) \cdot 2\left(d - x\right)}{c} = 0$$

$$\frac{n_1 x}{c\left(a^2 + x^2\right)^{1/2}} - \frac{n_2 (d - x)}{c\left[\left(b^2 + (d - x)^2\right)\right]^{1/2}} = 0$$

Referring to the diagram above: $\frac{x}{\left(a^2 + x^2\right)^{1/2}} = \sin\theta_1$ and $\frac{d - x}{\left(b^2 + \left(d - x\right)^2\right)^{1/2}} = \sin\theta_2$

$$\frac{n_1}{c} \sin\theta_1 = \frac{n_2}{c} \sin\theta_2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

**Example 1** A beam of monochromatic light (632.8 nm) from a He-Ne laser impinges on the smooth surface of a transparent medium at an angle of $40^0$ to the normal. The refracted beam is measured at $26^0$ to the normal. What is the index of refraction of the transparent media?

From Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1)(\sin 40^0)}{\sin 26^0} = 1.47$$

This corresponds closely to the index of refraction of fused quartz.

What is the speed of light in this material?

$$v = \frac{c}{n} = \frac{3\times10^8 m\cdot s^{-1}}{1.47} = 2.04\times10^8 m\cdot s^{-1}$$

What is the wavelength of light in this material?

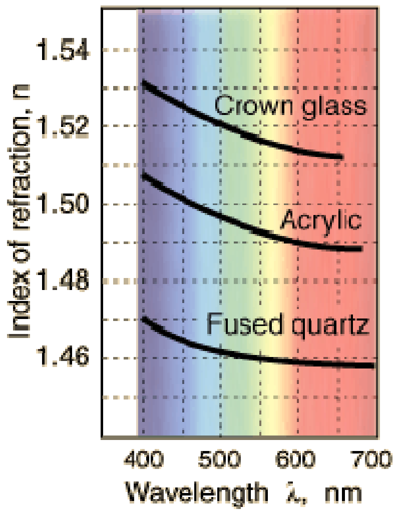
$$\lambda_n = \frac{\lambda_0}{n} = \frac{632.8nm}{1.47} = 430nm$$

What is the frequency of this light beam?

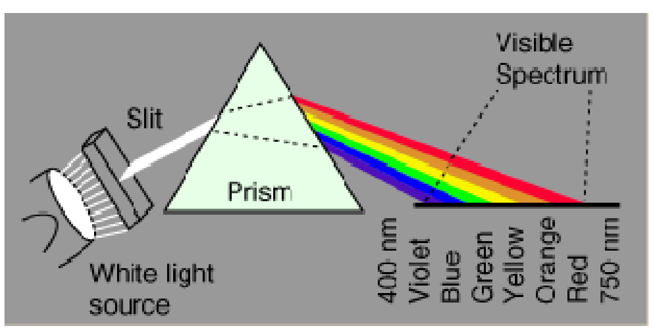
$4.7 \times 10^{14}$ Hz using the values from either side of the optical boundary.

Dispersive Media

A dispersive media is one in which *n*, the index of refraction, varies with λ. Although nearly all common transparent media are dispersive the rate at which *n* varies with λ varies from material to material.




Images above and below Courtesy of Rod Nave and Hyperphysics



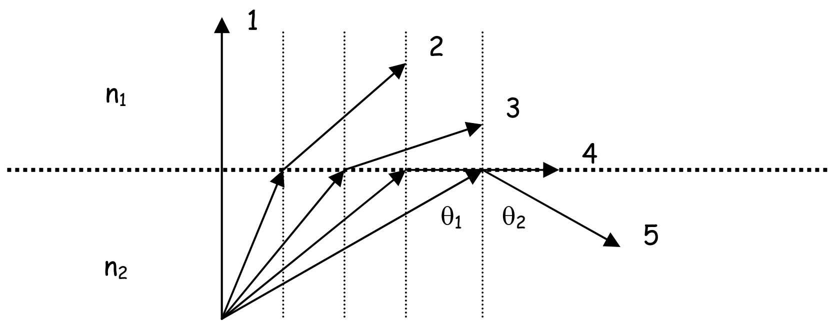

A prism is a device often used to exploit dispersion to separate polychromatic light into its component colors.

Total Internal Reflection

*Internal* reflections occur when light originates in a medium of greater density (higher index of refraction) and travels toward a medium of lesser density (lower index of refraction). A flashlight beam originating underwater aimed up toward the air/water boundary is such an example.

For large angles of incidence no light tends to be transmitted across the optical boundary and total internal reflection results.

Consider the 5 beams below. Each originates from an object in medium 2 where $n_2 > n_1$.

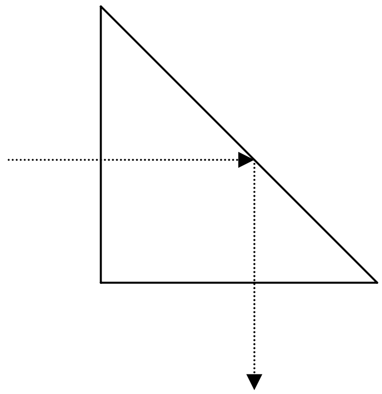



- In each case the beams are bent away from the normal because $n_2 > n_1$.
- Beams 1 – 3 are refracted at the boundary with a small amount of reflection directing part of the beams back into the originating medium.
- Beam 4 is refracted but does not cross the optical boundary
- Beam 5 is *reflected* and remains in the originating medium.
- Beam 4 approaches the optical boundary at the critical boundary. At angles of incidence greater than the critical boundary no refraction occurs. This leads to a condition known as *total internal reflection*.
- Snell's Law may be used to determine the critical angle for any two media:

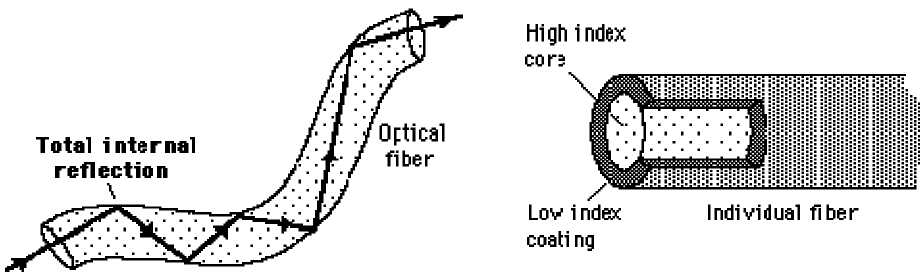
$$n_1 \sin\theta_c = n_2 \sin 90^0 = n_2 \rightarrow \sin\theta_c = \frac{n_2}{n_1} \quad \text{(for } n_2 > n_1\text{)}$$

- Notice that at angles greater than the critical angle Snell's Law yields invalid results.
- The critical angle for a water-air boundary is about $49^0$, for a diamond-air boundary $24^0$, and for glass-air $41^0$.
- Total internal reflection is 100% efficient at angles greater than the critical angle

- Prisms are often used to redirect light beams via total internal reflection

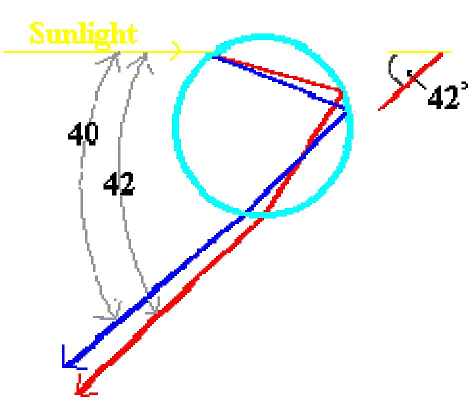


- Light travels down *fiber-optic* cables via a series of total internal reflections.




Courtesy of Rod Nave and Hyperphysics

- Fiber-optic cables have very low signal transmission losses. Most of the signal loss in a typical fiber optic cable occurs at the ends of the cable.
- Fiber-optic cables are capable of bandwidths of 50 Thz (tera = $10^{12}$ Hz). Analog video requires about 5 MHz of bandwidth, digital video less depending on frame rate and compression. High quality digital Audio requires less than 10 kHz. A phone conversation requires just a few kilohertz.
- Rainbows are formed, in part, by total internal reflection.

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