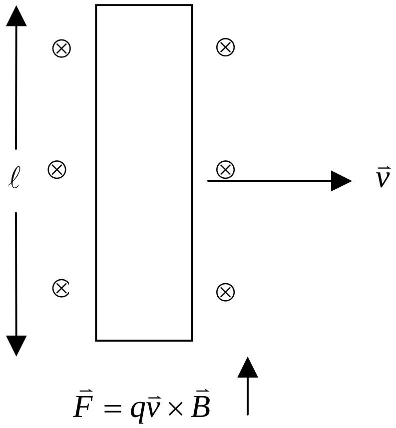
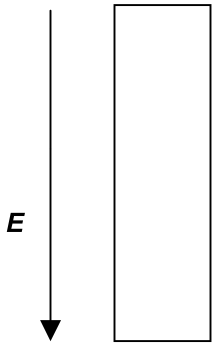
Electromagnetic Induction and Motional EMF

An EMF (source of electrical potential) created by a changing magnetic field is known as an *induced* EMF. Induced EMF's generate currents

Consider a conductor, length ℓ moving to the right in a magnetic field as shown below.

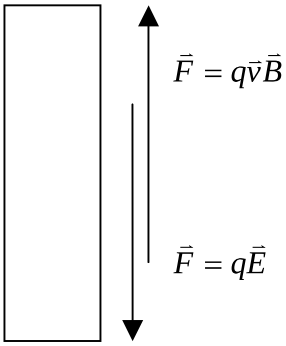


- A charge, $+q$ in the conductor experiences an electric force directed upward (be sure to verify this for yourself) of magnitude ***qvB***, due to the motion of the conductor through the field
- The magnetic force causes the + charges in the conductor to move upward and the negative charges to move downward



- The result of this process is a pileup of + charge at the upward end of the conductor and – charge at the downward end, creating a potential difference between the ends of the conductor and an ***E*** field pointing downward within the conductor.
- The ***E*** field within the conductor produces a force, ***qE***, also directed downward on positive charges. .

- The force *qvB* causes polarization of the conductor and establishes a potential difference between the top and bottom of the conductor.
- Due to this polarization an electric field is established within the conductor and any positive charge in this field experiences a force *q***E** directed *downward* - which grows as charges accumulate at the ends of the conductor (it also continues to experience a force of *qvB* directed upwards as long as the conductor continues to move to the right).



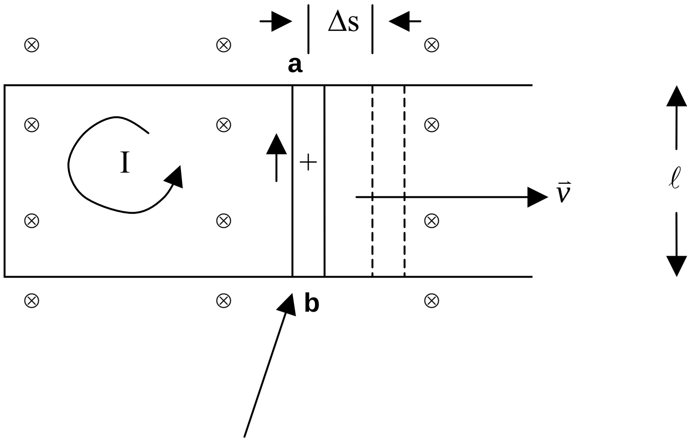
- The accumulation of charges at the ends of the conductor continues until $q\vec{E} = q\vec{v}\vec{B}$. When this occurs the forces within the conductor are in equilibrium and no more accumulation of charges at the ends of the bar occurs.
- The magnitude of the potential difference at equilibrium is:

$$\cancel{q}vB = \cancel{q}E \quad \because E = vB$$

Recalling that $V = Ed \rightarrow V = E\ell = vB\ell$

- The potential difference between the top and bottom of the conductor is ***vBℓ***

What if we connect this polarized conductor, a source of electrical potential, to an external circuit?



moving conductor and source of EMF

- A counterclockwise current is established due to the motion of the moving portion of the circuit.
- The moving element is a source of EMF.
- Charges move from lower to higher potential (as in other sources) in the moving segment, and from higher to lower throughout the rest of the circuit.
- Motional EMF:

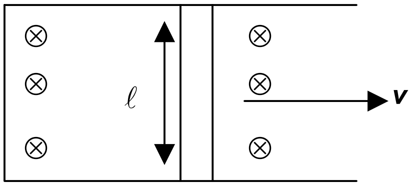
$$\xi = vB\ell.$$

- If resistance (R) is negligible in the sliding segment $\xi = V_{ab}$.
- In real conductors $\xi = V_{ab} - IR$ where the potential drop is due to the resistance in real conductors.
- The quantitative result here is applies generally to any source of electrical potential though the fundamental source of the EMF may be different.
- In this particular example we had the moving conductor moving with a velocity ⊥ to ***B***. In general the conductor could move in any plane and still produce a motional EMF as long as a component of that plane is perpendicular to ***B***. The general expression for motional EMF is:

$$\xi = vB\ell.\sin\theta \text{ or } \xi = \int_a^b \left(\vec{v} \times \vec{B}\right) \cdot d\ell$$

Example 1

Compute the motional EMF and the current in the circuit for the following



$$\ell = 0.1m$$
$$v = 0.1m \cdot s^{-1}$$
$$R = 0.01\Omega$$
$$B = 1.0T$$

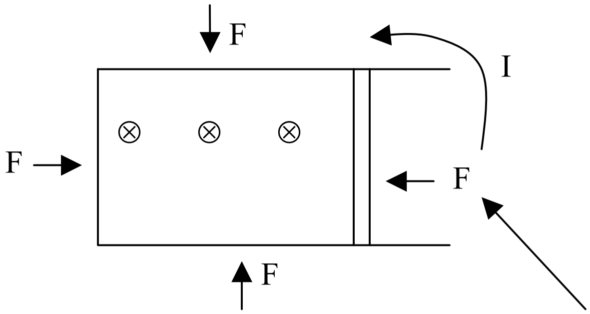
The motional EMF is: $\xi = vB\ell.\sin\theta$ = (0.1m/s)(1.0T)(0.1m) = 0.01 volts

The current in the loop is: $I = \xi/R$ = 1 Amp

This is essentially (schematically) how all electrical current is generated.

In the previous example the current flowing through the circuit moves in the presence of an external ***B*** field that generates a force on the conducting elements of the circuit as shown below.

$F = I\ell \times B = IB\ell\sin\theta$ = 0.1N (on the moving conductor)



This magnetic force opposes the motion of the conductor

In order to move the bar to the right at a constant velocity we must continue to apply a force to it that is equal in magnitude to **IℓB** force. In other words we must do work on the system.

Recalling that work per unit time is power:

$$P = Fv = (0.1N)\left(0.1m \cdot s^{-1}\right) = 0.01\ watts$$

or, equivalently:

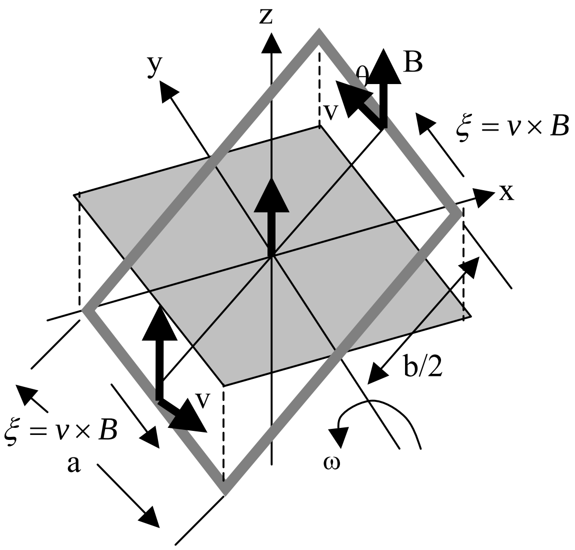
$$P = \xi I = \left(0.01V\right)\left(1A\right) = 0.01\ watts$$

The rate of energy conversion, ξI, equals the rate of mechanical energy input to the system.

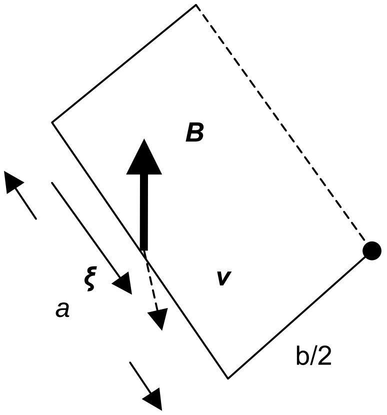
- We have conjured up a 0.01 watt electrical generator.
- We would have to supply 0.01 watts of mechanical energy to move the conductor to produce 0.01 watts of electrical energy assuming 100% conversion.
- Is this realistic? What is an obvious source of loss we have not computed?

Example 2

Consider the induced EMF from a conducting loop rotating as shown below with an angular velocity ω.



First let's examine the motional EMF produced by each side "*a* "of the loop.



$v = r\omega = \left(\frac{b}{2}\right)\omega$ The velocity of side *a.*

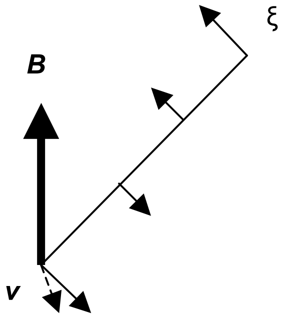
$$\xi = vB\ell.\sin\theta = vBa\sin\theta = \frac{b}{2}\omega Ba\sin\theta$$

Since there are two *a* sides for this loop these two EMF's add and the total EMF is:

$$\xi = \omega abB\sin\theta$$

Notice that this EMF is directed in such a manner that it will cause current to flow through the *a* sides of the loop in the direction shown.

On the "*b*" sides of the loop (top and bottom):



The magnetic forces on *b* sides are transverse to the conductor and do not contribute an EMF in a direction that produces current flow down the length of either side *b*

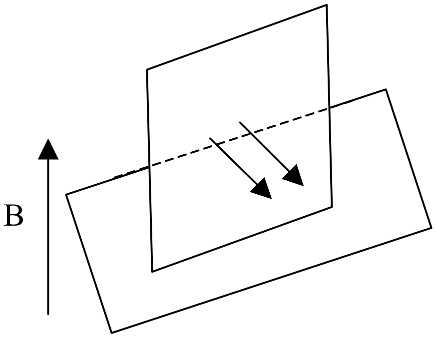
The total EMF of the loop is therefore produced solely from the sides *a*.

$$\xi_{total} = \omega abB\sin\theta$$

Noting that *ab* = the area of the loop (*A*) and that $\theta = wt$:

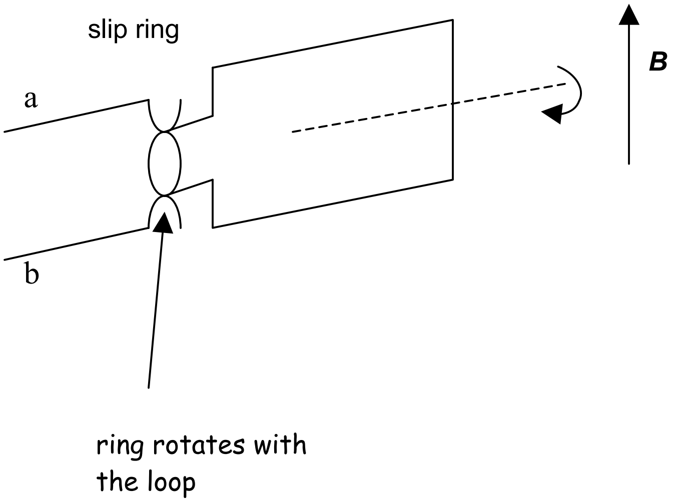
$$\xi_{total} = \omega AB\sin\omega t$$

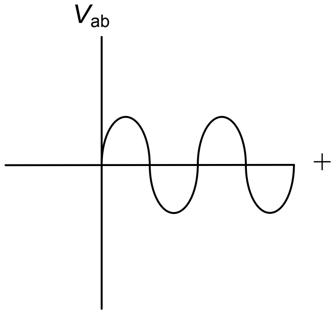
Note:



- ξ varies sinusoidally with time.
- ξ = ξ_{max} when *sin* ωt = 1 this occurs when ***B*** is parallel to the plane of the loop and is perpendicular to μ and ***A***
- $\xi_{max} = \omega AB$
- ξ = $\xi_{max}\sin\omega t$
- Basis for the construction of an electrical generator or alternator

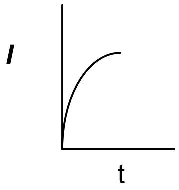
The Alternator



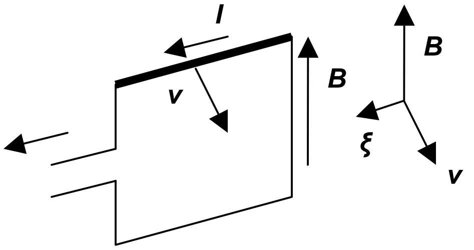


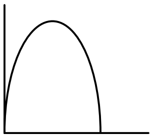
Notice current reverses direction in the loop as it turns. To see this look only at one segment of the loop as it rotates through 360^0. The angles given are between the ***B*** and μ vectors.

- $\theta = 0^0 \rightarrow 90°$ ***I*** starts at zero and is increases to $\boldsymbol{I}_{max}$ + (ccw)

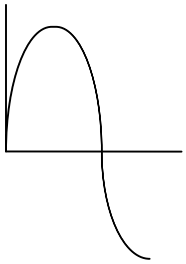


- $\theta = 90^0 \rightarrow 180^0$ ***I*** begins at its maximum value and decreases to zero, still + (ccw)

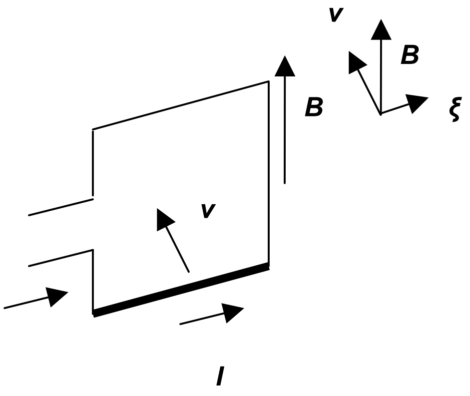


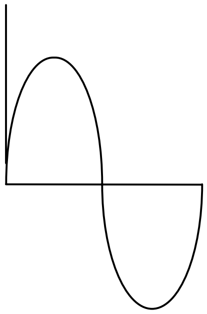


- $\theta = 180^0 \rightarrow 270^0$ $\boldsymbol{I}$ starts at zero and is increasing – (cw)



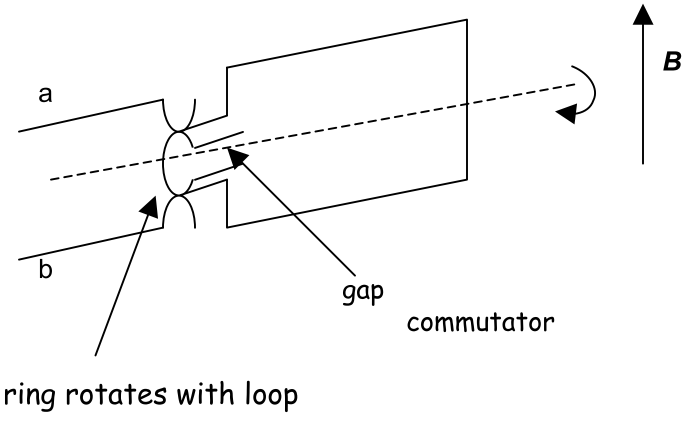
- $\theta = 270^0 \rightarrow 360^0$ $\boldsymbol{I}$ starts – $\boldsymbol{I}_{max}$ and decreases to zero – (cw)

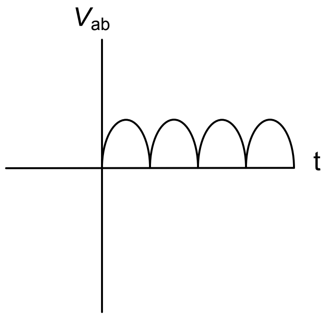




So the direction of the current in side *a* reverses each half cycle as it does in the entire loop.

Let's modify this device as follows:

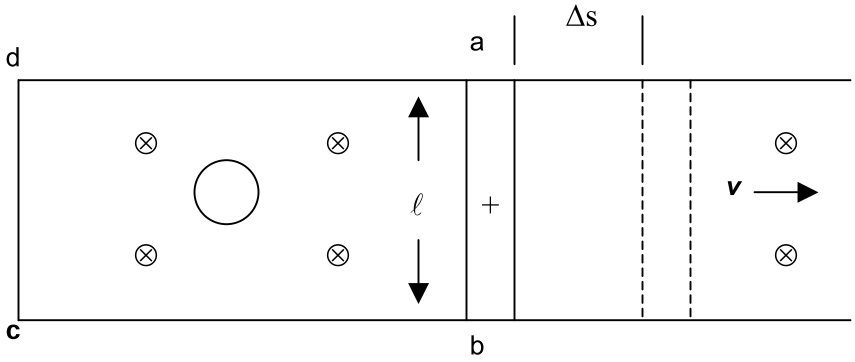




At the angular positions where the current reverses itself, the connections to the external circuit are reversed. The EMF is always is the same direction but varies from 0 to some maximum value. This "half wave" may be easily *rectified* or converted from alternating to direct current.

Faraday's Law

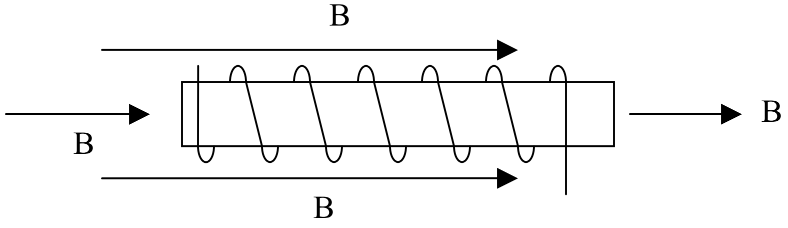
Faraday's Law relates changes in magnetic flux to EMF



- We have already looked at this circuit using the concept of magnetic to produce a motional EMF.
- We can also examine the change in magnetic flux through this circuit. A change in magnetic flux produces EMF.
- Changes in magnetic flux may be due to changes in the magnetic field strength or direction, or a change in the area being penetrated by the magnetic field lines (which applies here?).
- When the conductor moves to the right at distance *Δs*, the area enclosed by *abcd* increases by: $\Delta A = \ell \Delta s$
- The change in magnetic flux through *abcd* is: $\Delta\Phi_m = \vec{B}\cdot\Delta\vec{A} = B\ell\Delta s\cos\theta$. Since ***B*** and ***A*** are parallel $\Delta\Phi_m = B\ell\Delta s$
- The time rate of change of the magnetic flux is $\frac{\Delta\Phi_m}{\Delta t} = B\ell\frac{\Delta s}{\Delta t} = vB\ell$
- Faraday's Law: $\xi = -\frac{\Delta\Phi_m}{\Delta t} = -\frac{\Delta(BA)}{\Delta t}$
- Faraday's Law of induction is valid for any circuit for which there is time varying magnetic flux, even circuits in which there is no evident motion.
- The change in flux may be caused by a change in area with respect to time, a change in magnetic field with respect to time, or both
- The negative sign is a matter of convention.
- For multiple loops $\xi = -n\frac{\Delta\Phi_m}{\Delta t}$

Example 3

Consider a solenoid; *n* = 500 loops, *r* = 4 cm as shown below. The magnetic field is changing at a rate of 0.2T per second. Find the induced EMF in this solenoid



B is increasing at $0.2T \cdot s^{-1} = \frac{\Delta B}{\Delta t}$ while the area remains unchanged. The change in magnetic flux here is due to a changing magnetic field rather than a change in area penetrated by the magnetic field..

$$\Phi_m = BA$$

$$\frac{\Delta \Phi_m}{\Delta t} = A\frac{\Delta B}{\Delta t}$$

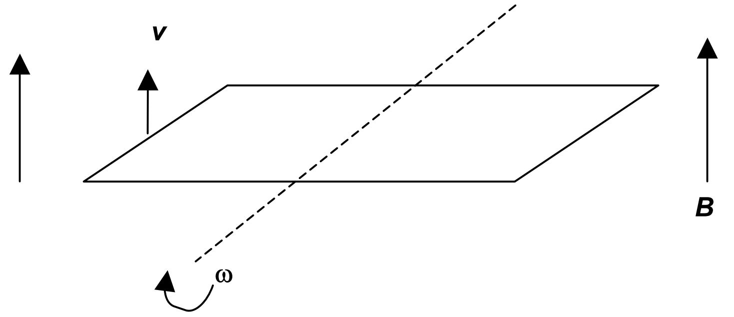
A = π(.04m)² = 0.00503m²

$$\because \frac{\Delta \Phi_m}{\Delta t} = A\frac{\Delta B}{\Delta t} = (.00503m^2)(0.2T \cdot s^{-1}) = .00101\,T \cdot m^2 \cdot s^{-1} = .00101 wb \cdot s^{-1}$$

$$\xi = -n\frac{\Delta \Phi_m}{\Delta t} = (500)(.00101T \cdot m^2 \cdot s^{-1}) = 0.505 \text{ Volts}$$

Which direction does the induced current flow in the coils?

Faraday's Law and Rotating Loops



- In this position the loop produces the minimum amount of induced EMF.
- The magnetic flux through the loop has its maximum value
- The flux through the loop changes as the loop rotates through the field. At a position 90^0 from this orientation the flux through the loop will have it's minimum value of zero.

$$\xi = -\frac{d\Phi_m}{dt} = -\frac{d(BA)}{dt}$$

In this case the area being penetrated by the magnetic field is changing

$$\vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t = \Phi_m \rightarrow \Phi_m = B\frac{dA}{dt} = BA\cos\omega t$$

It may be shown that $\frac{d}{dt}A\cos\omega t = -\omega A\sin\omega t$

Hence: $\xi = -\frac{d\Phi_m}{dt} = \omega AB\sin\omega t$ which is in agreement with the result previously derived

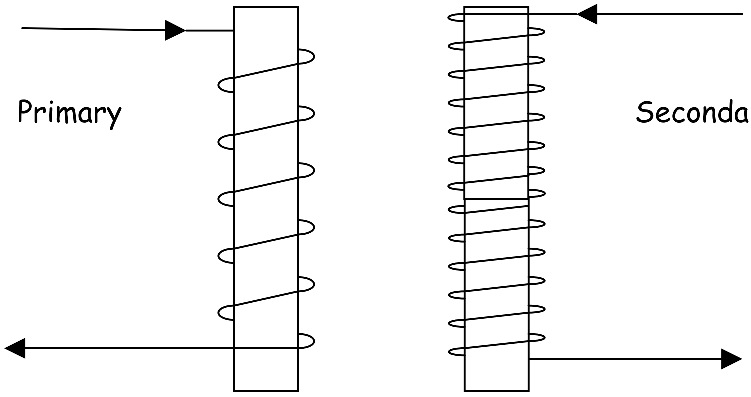
Transformers

Transformers are devices which exploit the constantly changing magnetic flux of alternating currents to induce currents in devices that do not physically touch each other. Transformers are widely used for both electronic isolation (for low electronic noise and for safety) and to step up or step down AC voltages

Some advantages of AC over DC:

- Easier to step up and down
- Easier to transmit
- Can use high voltage & low current to reduce power losses in transmission lines

Most transmission lines contain about 500kV which must be stepped down (converted to) lower voltages for household or office operation.



$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow$$ the transformer equation

$N_S > N_P \Rightarrow$ Step up transformer

$N_S < N_P \Rightarrow$ Step down transformer

Power In = Power Out. Transformers trade voltage for current

$$V_P I_P = V_S I_S \rightarrow \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

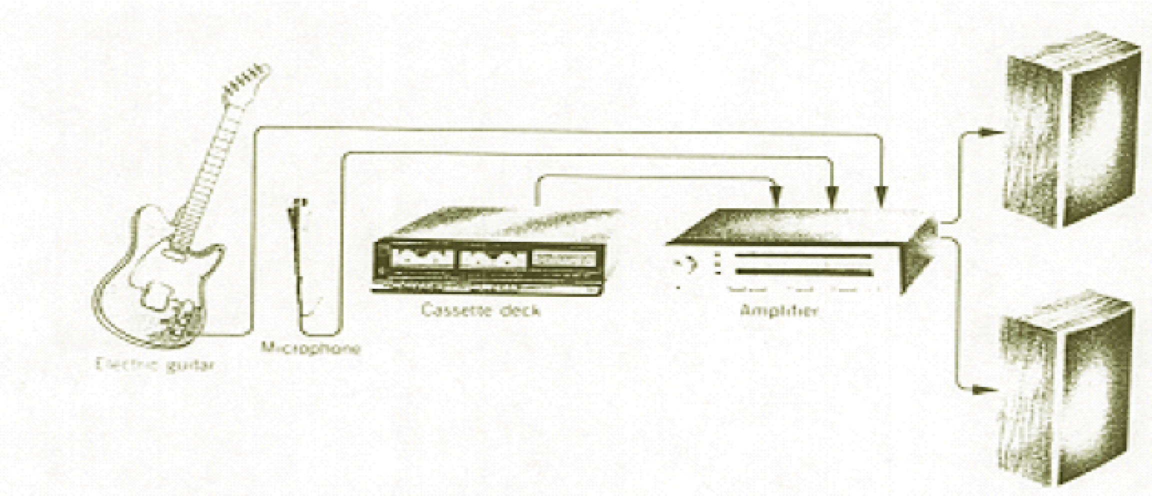
or

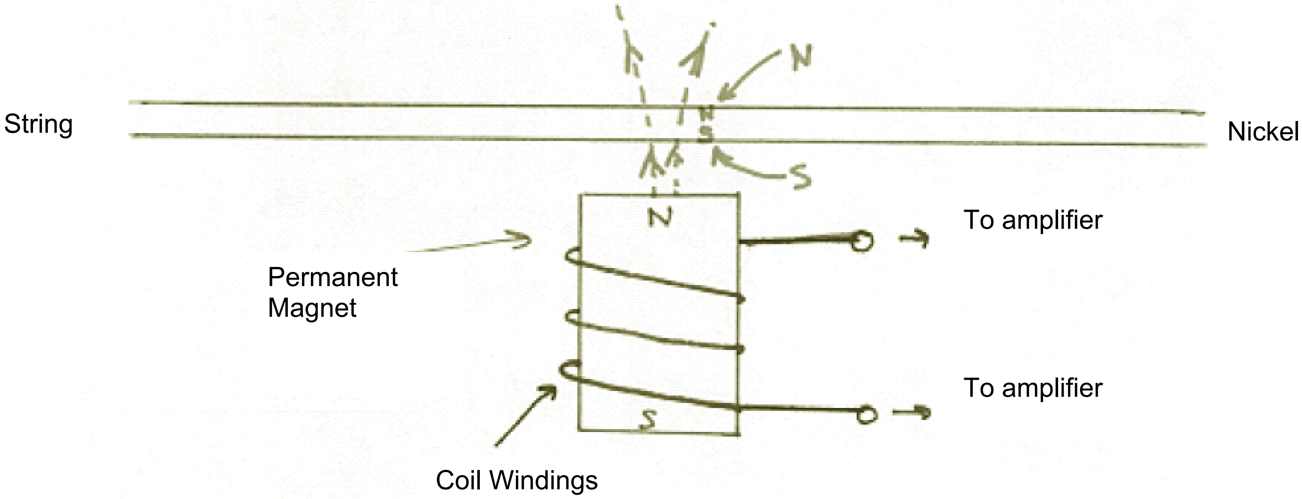
$$\frac{N_P}{N_S} = \frac{I_S}{I_P}$$

Transducers

A transducer is a device that converts mechanical motion to an electrical signal or vice versa. Transducers generally work by exploiting the behavior of conductors in magnetic flux

The Electric Guitar





- Virtually all electric guitars use electromagnetic pickups
- Most guitars have at least two pickups for each string and some have three.
- These pickups are positioned at different locations under the string, so that each is sensitive to different harmonics produced by the vibrating string.
- Electric guitar strings are generally made of nickel – a material that is easy to magnetize

- The pickup consists of a coil of wire with a permanent magnet located inside the coil.
- The magnet in the pickup induces a magnetic field in the string as shown above
- The magnetic field of the string vibrates with the string as it is plucked
- The change in magnetic flux due to the vibration of the field lines across the coil windings produces an oscillating change if EMF in the coil
- An oscillating current is produced at the same frequency as that of the vibrating string
- The EMF produced is very weak (a few microvolts) and must be amplified to a level of a few millivolts (*line level*) in order to drive any subsequent electronic devices. The device that does this is referred to as a *preamp.* Preamps are also responsible for modifying the tonal characteristics of the signal as well
- In order to drive a loudspeaker the signal must be amplified again by a *power amplifier* to a level of 10 – 70 volts.
- A loudspeaker is another transducer that converts electricity to mechanical motion which in turn produces an acoustic wave

Loudspeakers and microphones

- We have previously shown how a vibrating object such as a guitar string produces an acoustic wave. We have also seen how acoustic waves propagate through the air and how the energy in acoustic waves can be transferred to an object some distance from the source of the wave causing it to vibrate at the same frequency as the source. This is how acoustic waves transfer energy from one point to another.

- As noted previously, sound waves consist of very small displacement amplitudes and minute fluctuations of pressure. As sound waves travel through the air they are attenuated because small amounts of energy are lost in collisions as air molecules move back and forth. When sound waves impinge upon a surface they impart very small energy pluses to the incident material (because their displacement amplitudes are small).

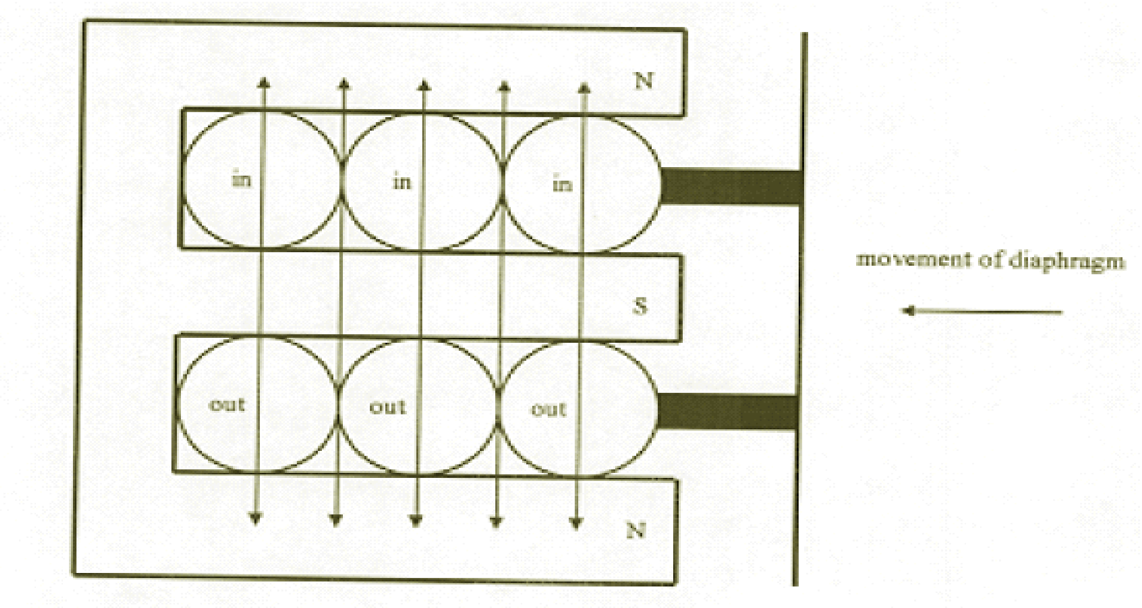
- If the material upon which the sound wave falls is a rigid solid with low density and high incompressibility, the wave may be transmitted for large distances within the material without much attenuation. In general, however, they impinge upon a solid object. As sound waves move away from their source they are also subject to inverse square losses.

- Because sound waves suffer losses from several sources, they must be amplified to travel long distances and still arrive at the source with sufficient volume for clarity. This is the primary purpose of a sound system.

- In order to amplify an acoustic wave we must find a way of converting it to an electrical signal. Devices that convert acoustic waves to electrical signals are, of course, transducers.

- There are many types of transducers. We will examine a very common type of transducer known as a *linear electromagnetic motor*. A guitar pickup is one example of a linear electromagnetic motor

- Linear electromagnetic motor are based on electromagnetic induction. Electromagnetic induction occurs whenever a change in magnetic flux occurs in the presence of a conductor. The diagram below contains a cross-sectional view of a simplified linear electromagnetic motor.



- This particular LEM consists of an iron magnet and a coil of copper wire attached to a diaphragm. As a sound wave impinges upon the diaphragm it vibrates at the same frequency as the wave. The greater the SPL (amplitude) of the wave, the greater the amplitude of vibration imparted to the diaphragm and attached wire coil.

- As the coil moves back and forth within the magnetic field of the fixed magnet, an alternating current is induced that changes direction each time the coil changes direction. The larger the amplitude of oscillations, the stronger the induced voltage.

- Notice that as long as the movement of the diaphragm does not exceed the elastic limits of the mounting system, this system behaves exactly like a simple harmonic oscillator. The waveform generated is therefore sinusoidal.

- This is generally how a *dynamic microphone* operates. In order to contain the infrastructure needed to create this transformation in a compact package it is necessary to limit the size of the magnet and wire coil.

- Large diaphragms and massive wire coils, due to their large inertia, will not vibrate as easily as smaller diaphragms and light coils unless the SPL of the incoming wave is extremely high.

- Because of these limitations, most dynamic microphones produce extremely low-voltage signals typically a few microvolts that must be amplified by other electronic devices in order to gain enough strength to drive a loudspeaker.

- There are other ways of converting an acoustic wave to an ac signal: capacitors, piezoelectrics, ribbons, among others..

- Loudspeakers are transducers that are essentially microphones in reverse, i.e., they convert electrical signals to acoustic waves. Most loudspeakers use linear electromagnetic motors to do this. Alternating current flowing into a wire coil in the presence of a fixed magnetic field causes the coil to oscillate at the frequency of the source signal. If the coil is attached to a diaphragm, acoustic waves will be produced