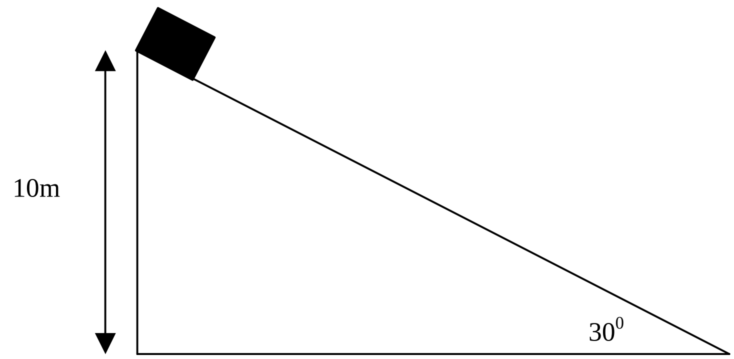
Kinematics Revisited

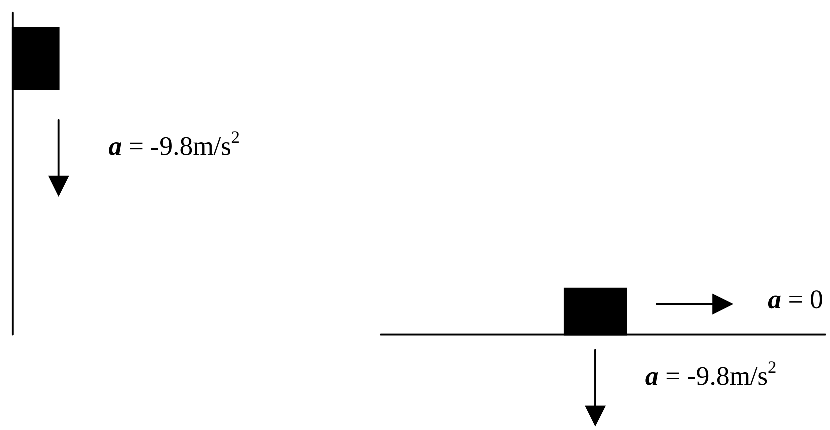
Let's reconsider motion in a plane, this time neither entirely along either the *x* or the *y* coordinate:

Consider the following block at rest on a frictionless inclined plane:



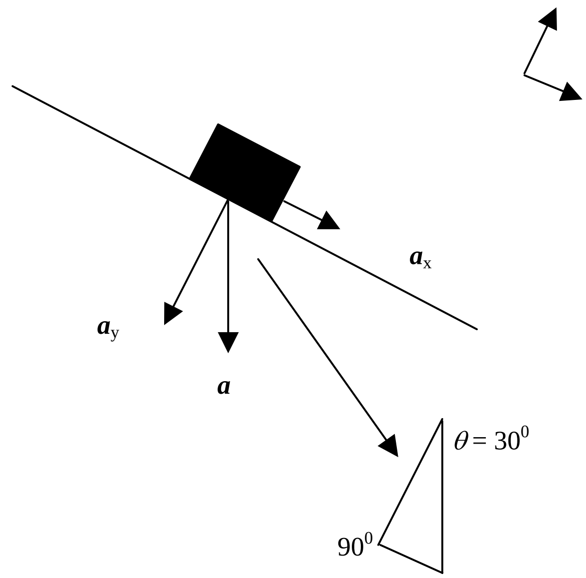
When released, how long does it take for the block to reach the bottom of the incline?

- We must be able to determine the acceleration in the plane of motion, i.e., along the surface of the incline.



Consider the two example planes above, one vertical, and one horizontal.

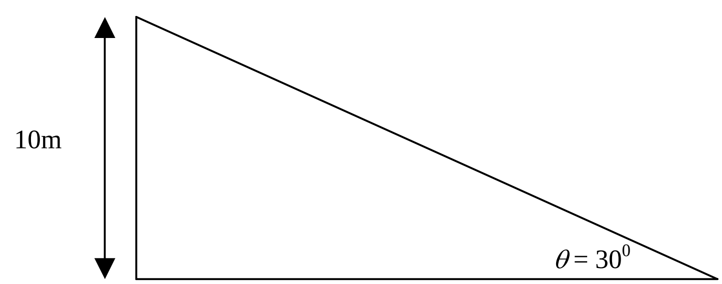
- If we ignore the effects of friction the block on the vertical plane slides with acceleration due to gravity (just as if it were in free fall).
- The block on the right, however, is constrained to move in the horizontal plane. Since the force of gravity acts downward, and no forces act on the block in the horizontal plane, the block on the right does not accelerate.
- The situation in the problem under consideration falls somewhere between these two extremes.



- We are interested in the component of acceleration in the +*x* direction as indicated by the arrows above the diagram since this is the only plane in which the object can slide.
- Notice that the angle θ in the vector triangle has the same value as the angle of the incline.
- Do you know why that must be the case?

$$a_x = a \sin 30^0 = (9.8 m/s^2)(0.5) = 4.9 m/s^2$$

Next we must find the length of the incline:



$$y = h \sin\theta \therefore \frac{y}{\sin\theta} = h = x = \frac{10m}{0.5} = 20m$$

So the incline is 20 meters in length.

Now let's find the time it takes the block to slide +20 meters under the given acceleration:

$$\vec{x}-\vec{x}_0=\vec{v}_0 t+\frac{1}{2}\vec{a}t^2 \Rightarrow t=\sqrt{\frac{\vec{x}-\vec{x}_0}{\frac{1}{2}\vec{a}}}=\left(\frac{+20m}{2.45m/s^2}\right)^{\frac{1}{2}}=2.86s$$

And the velocity at the bottom of the incline is:

$$\vec{v}=\vec{v}_0+\vec{a}t=\left(4.9m/s^2\right)\left(2.86s\right)=+14m/s$$

Now let's compare these values to those that we would obtain from an object in free fall dropped from a height of 10 meters (the height of the inclined plane).

$$\vec{y}-\vec{y}_0=\vec{v}_0 t-\frac{1}{2}\vec{g}t^2 \Rightarrow t=\sqrt{\frac{\vec{y}-\vec{y}_0}{\frac{1}{2}\vec{g}}}=\left(\frac{-10m}{-4.9m/s^2}\right)^{\frac{1}{2}}=1.43s$$

$$\vec{v}=\vec{v}_0-\vec{g}t=-\left(9.8m/s^2\right)\left(1.43s\right)=-14m/s$$

The sign of the velocity vector is different only because of the way we defined our coordinates. Does this result surprise you?