Kinematic Equations from Newton's Second Law

Begin with Newton II

$$\vec{F} = m\vec{a}$$

Rearranging

$$\frac{F}{m} = a = \frac{dv}{dt}$$

Rewriting

$$dv = adt$$

Integrating

$$\int dv = \int adt = a\int dt$$ **acceleration is a constant here**

$$v = at + C$$ **we combine the two constants of integration from each indefinite integral into one which we must evaluate using initial conditions: at t = 0, v = v_0**

$$v_0 = a(o) + C$$

$$\therefore C = v_0$$

$$\rightarrow v = v_0 + at$$ **Equation 1**

In the same manner

$$v = \frac{dx}{dt}$$

$$vdt = dx$$

$$\int vdt = \int dx$$

$$\int (v_0 + at)dt = \int dx$$

$$\int v_0 dt + \int atdt = \int dx$$

$$v_0 \int dt + a \int tdt = \int dx$$

$$v_0 t + \frac{1}{2}at^2 + C = x$$

again we combine the constants of integration from each indefinite integral into one which we evaluate using initial conditions: at *t* = 0, *x* = x_0

$$v_0(0) + \frac{1}{2}a(0)^2 + C = x_0$$

$$\therefore x_0 = C$$

$$\rightarrow x - x_0 = v_0 t + \frac{1}{2}at^2$$ **Equation 2**

Now rearrange (1) to solve for *t* and substitute into (2)

$$\frac{v - v_0}{a} = t \qquad \textbf{(1)}$$

$$x - x_0 = v_0 \frac{v - v_0}{a} + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2 \qquad \textbf{(2)}$$

$$v^2 = {v_0}^2 + 2a(x - x_0)$$