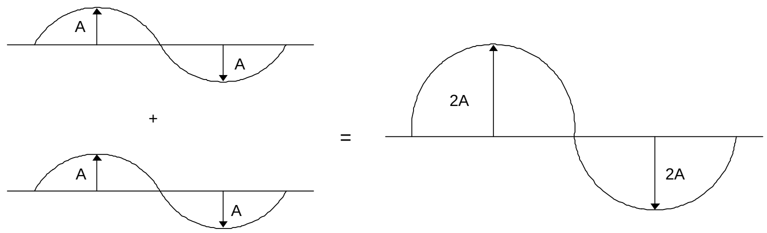
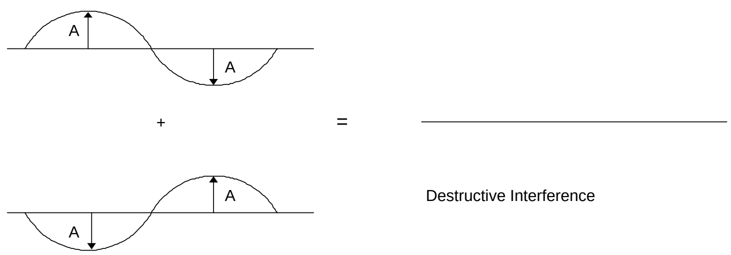
Interference of Light

Just as with sound waves, light waves, when combined, may interfere constructively, destructively or some combination of both.

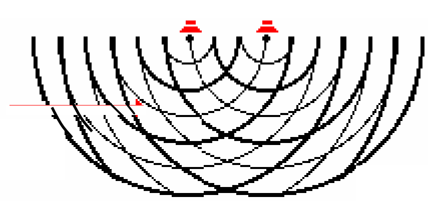


Constructive Interference



Destructive Interference

- In order for an interference pattern to be stable the waves must be emitted from *coherent* and *monochromatic* sources.
- Most natural light sources are both non-coherent and polychromatic so interference is not widely observed in nature.
- In order to create a stable interference pattern waves from different sources must maintain a constant phase relationship with each other.
- It is much easier to arrange coherent sound sources than light sources.
- Because the frequency of light waves (around 10^{15} Hz) is so high coherence between separate sources is difficult to arrange.
- Two loudspeakers driven by the same amp in mono are responding to the same inputs and will produce coherent waves.
- It is easy to introduce phase differences between the loudspeakers and constructive or destructive interference merely by either moving the loudspeakers around or moving around in the field produced by the loudspeakers.



Two Beam Interference

We'll add the electric fields of two light beams of identical frequency and wavelength that differ only by some initial phase difference:

$$E_1 = E_{01}\, sin(\omega t + \alpha_1)$$
$$E_2 = E_{02}\, sin(\omega t + \alpha_2)$$

where α is a constant that contains the phase difference between the waves.

$$E_R = E_1 + E_2 = E_{01}\, sin(\omega t + \alpha_1) + E_{02}\, sin(\omega t + \alpha_2)$$

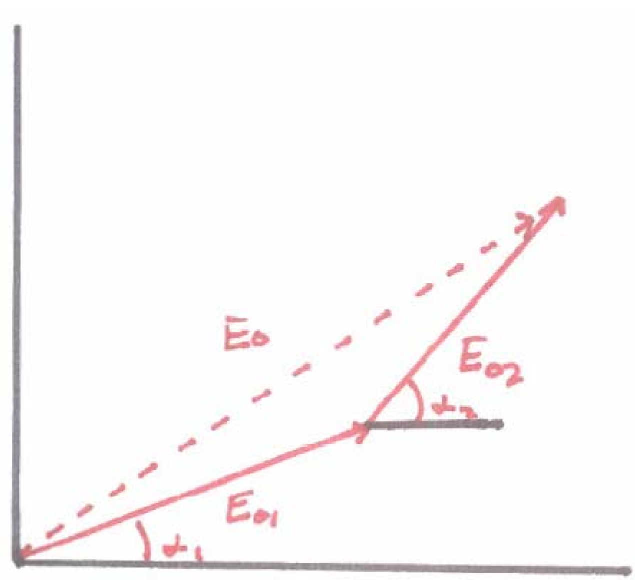
We'll invoke the trig identity:

$$\sin(\alpha + \beta) \equiv \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

so that:

$$E_R = (E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)\sin(\omega t) + (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)\cos(\omega t)$$

Notice that we can plot each of these component waves as *phasors* as shown at right by plotting the magnitude and phase angle of each.



$$E_R = E_{01} + E_{02} = E_0\cos\alpha + E_0\sin\alpha$$

$$E_0\cos\alpha = E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2$$

$$E_0\sin\alpha = E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2$$

$$E_R = E_0\cos\alpha\sin\omega t + E_0\sin\alpha\cos\omega t$$

and it is easily shown that this yields:

$$E_R = E_0\sin(\omega t + \alpha)$$

The resultant wave is another harmonic wave of the same frequency with an amplitude E_0 and a phase α that is related to the original waves as shown in the phasor diagram above.

The law of cosines may be applied to yield:

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$

and the phase angle may determined by:

$$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2}$$

Now recall that irradiance (the time average value of the Poynting vector) of a light beam is:

$$E_e = \varepsilon_0 c\langle E_0\rangle^2$$

So the irradiance at a point *P* due to two beam interference is:

$$E_e = \varepsilon_0 c\langle E_p\rangle^2 = \varepsilon_0 c\langle E_p \cdot E_p\rangle = \varepsilon_0 c\langle (E_1 + E_2)\cdot(E_1 + E_2)\rangle$$

which may be recast in the form:

$$E_e = \varepsilon_0 c\langle E_1^2 + E_2^2 + 2E_1 \cdot E_2\rangle$$

It is apparent that the first two quantities in the bracketed term are the individual contributions to the sum at *P* while the third term (the *interference* term) is the simultaneous contribution from both beams due to their interaction with each other.

- The interference term is indicative of the wave nature of the beams of light.
- If light behaved like classical particles there would be no interference and the irradiance would be: $E_e = \varepsilon_0 c\langle E_1^2 + E_2^2\rangle$, i.e. solely dependant on the separate contributions of the individual waves.
- The interference term varies between zero and $2E_1E_2$ depending on the orthogonality of the two beams.

It may be shown that:

$$E_e = E_{e1} + E_{e2} + 2\sqrt{E_{e1}E_{e2}}\cos\delta$$

where δ is the phase difference between the two waves due to either a path length difference or an actual phase difference (for our purposes here they are functionally the same.)

When $\cos\delta = +1$ constructive interference yields the maximum irradiance:

$$E_e = E_{e1} + E_{e2} + 2\sqrt{E_{e1}E_{e2}} \quad \text{(maximum irradiance)}$$

a condition that occurs whenever the phase difference is $\delta = 2m\pi$, where $m = 0,\pm1,2,3,...$

When $\cos\delta = -1$ destructive interference yields the minimum irradiance:

$$E_e = E_{e1} + E_{e2} - 2\sqrt{E_{e1}E_{e2}} \quad \text{(minimum irradiance)}$$

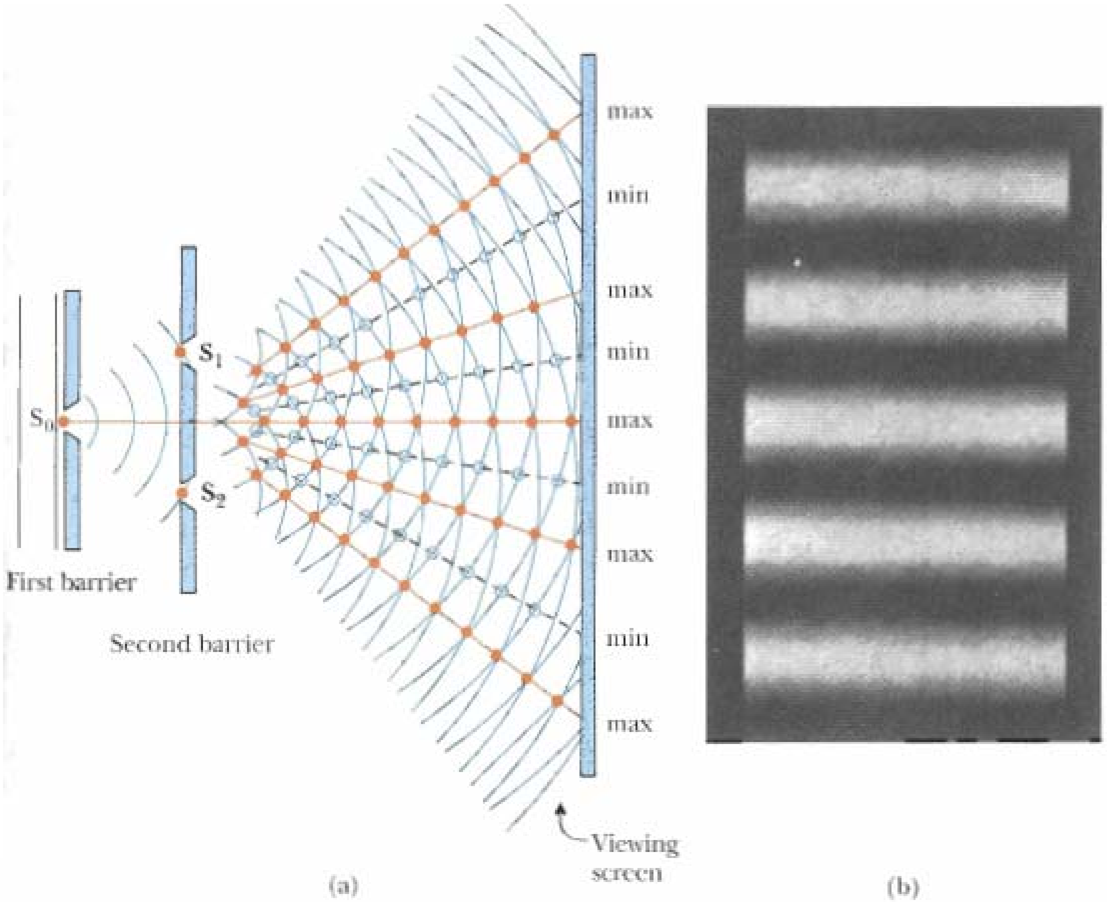
a condition that occurs whenever the phase difference is $\delta = (2m+1)\pi$ where $m = 0,\pm1,2,3,...$

It is also apparent that complete destructive interference can occur when $E_{e1} = E_{e2} = E_{e0}$ and that the irradiance, in this case, varies between:

$$E_{e\max} = 4E_{e0} \text{ and } E_{e\min} = 0$$

Alternatively, it may be shown that the irradiance between two equal interfering beams may be written:

$$E_e = 4E_0 \cos^2\left(\frac{\delta}{2}\right)$$



Courtesy of *Physics for Scientists and Engineers*, Serway and Beichner, 5th Ed.

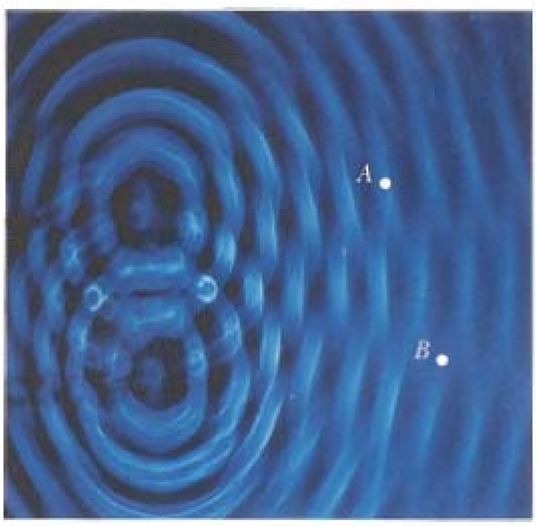
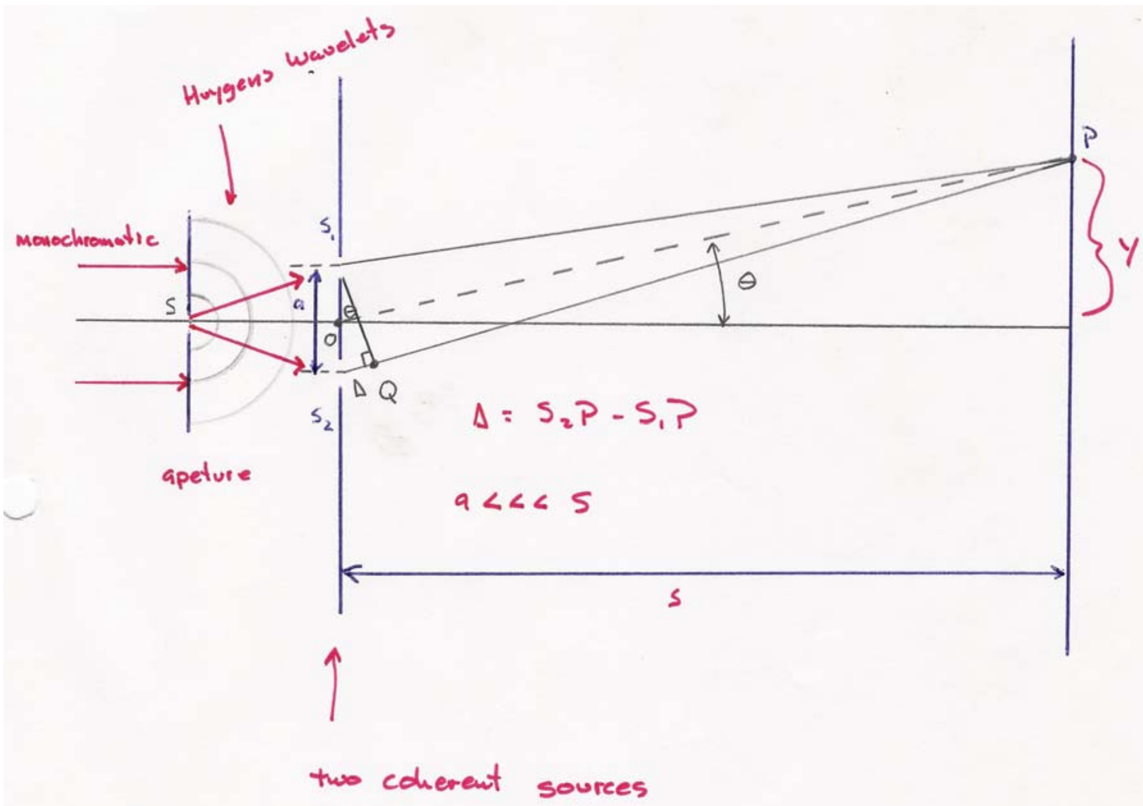


Figure 37.2 An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive (*A*) and destructive (*B*) interference. *(Richard Megna/Fundamental Photographs)*

Young's Double Slit

Consider a beam of light incident upon two slits, S_1 and S_2, as shown below. This arrangement insures that the beams that leave the slits are coherent.

We'll choose an arbitrary point *P* on a distant screen and look at the pattern of interference which results from combining the two beams at this point. The path length difference between the two beams is $S_2P - S_1P = \Delta$.



- When $S_2P - S_1P = m\lambda$, constructive interference occurs
- When $S_2P - S_1P = \left(m + \frac{1}{2}\right)\lambda$, destructive interference occurs

  $m = 0, \pm 1, \pm 2, ...$

The conditions for interference are:

- $S_2P - S_1P = \Delta = m\lambda \approx a\sin\theta$ (constructive)
- $S_2P - S_1P = \Delta = \left(m + \frac{1}{2}\right)\lambda \approx a\sin\theta$ (destructive)

  $m = 0, \pm 1, \pm 2, ...$

Notice that a phase difference between the two beams, δ, is equivalent to a path length difference between the two beams, Δ so that:

$$\delta = \frac{2\pi}{\lambda}\Delta$$

It then follows that the irradiance at the point of interference varies as a cosine function that also depends on λ and Δ:

$$E_e = 4E_0 \cos^2\left(\frac{\delta}{2}\right) \rightarrow E_e = 4E_0 \cos^2\left(\frac{\pi}{\lambda}\Delta\right) = 4E_0 \cos^2\left(\frac{\pi}{\lambda} a \sin\theta\right)$$

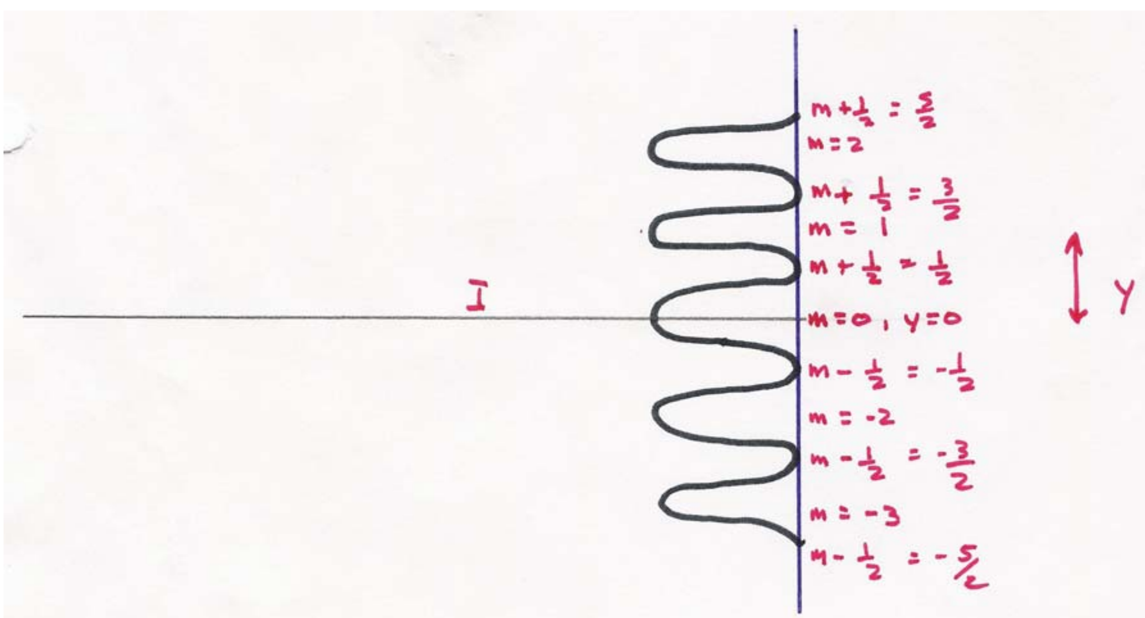
For points near the optical axis where y << s:

$$\sin\theta \approx \tan\theta \approx \frac{y}{s}$$

$$E_e = 4E_0 \cos^2\left(\frac{\pi}{\lambda} a \frac{y}{s}\right)$$

As the cosine term varies from ±1 and 0 (usually as a function only of y), the intensity of the pattern on the screen varies from $4E_0$ to zero, i.e. constructive and destructive interference occur.

The pattern that results is an alternating series of bright and dark fringes.



The location of the bright fringes (intensity maxima): $y_m = m\frac{\lambda s}{a}, \quad m = 0, \pm 1, \pm 2...$

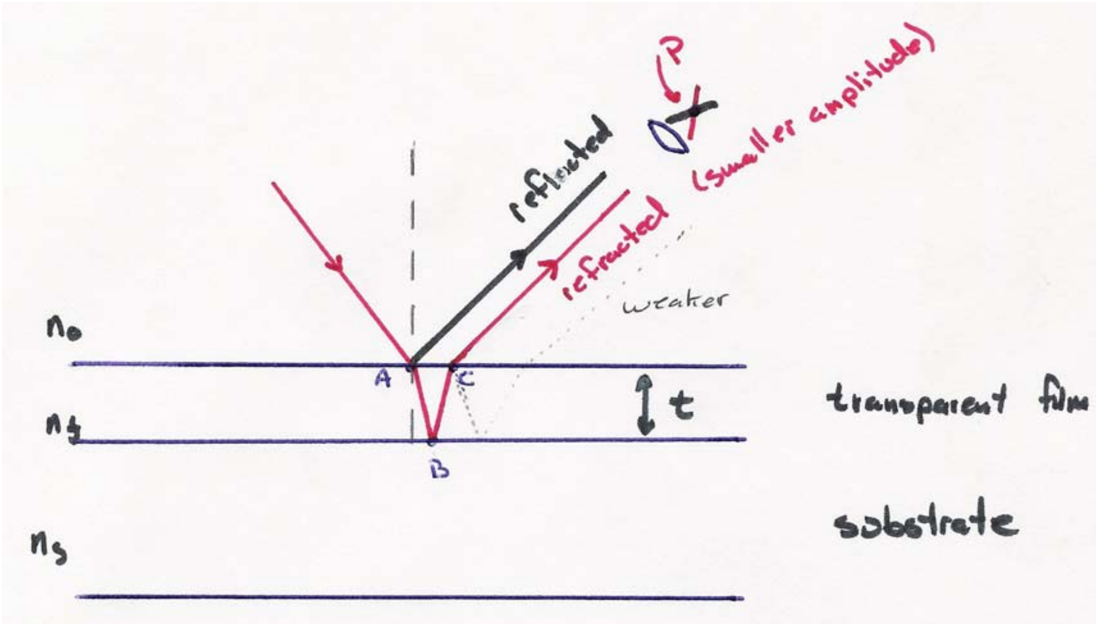
and the separation of the intensity maxima is: $\Delta y = \frac{\lambda s}{a}$

The Young's double slit experiment is very important historically because it is used to demonstrate the wave nature of light.

Young's double slit still has a lot of utility as an easy method of experimentally determining the wavelength of a beam of monochromatic light since:

$$\frac{\Delta y a}{s} = \lambda$$

Two Beam Interference in Thin Dielectric Films



We will begin by considering only the first two rays to emerge from a film of uniform thickness *t* and index of refraction *n*, as shown above (in reality internal reflection from the upper boundary produces many more refracted beams). We'll also assume near-normal incidence so that the path length of the refracted beam may be expressed in terms of the thickness of the film.

- The wavelength, λ_n, in a medium whose refractive index is *n* is given by: $\lambda_n = \frac{\lambda_o}{n}$ where λ_0 is the wavelength in free space.

The path length difference between the reflected and refracted beams is:

$$\Delta = \left(AB + BC\right) = 2t$$

or, in terms of the index of refraction in the film,

$$\Delta = n_f\left(AB + BC\right) = n_f\left(2t\right)$$

It is apparent that when $2n_f t = \frac{\lambda_0}{2}$, or in terms of the wavelength in the film, $2t = \frac{\lambda_n}{2}$, the two beams interfere destructively based on the path length difference, Δ, since the path length difference is the equivalent of a phase shift of half a wavelength.

When $2n_f t = \lambda_0$, or $2t = \lambda_n$, the two beams interfere constructively since the path length difference is the equivalent of a phase shift of a full wavelength.

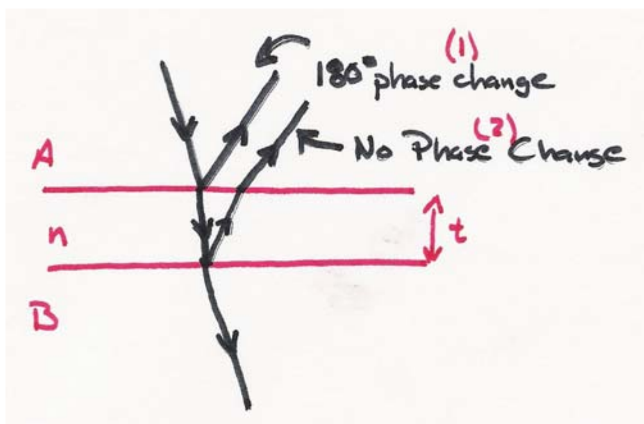
There exists, however, a complication. Recall that waves often undergo a phase shift upon reflection. In beams of light:

external reflections $(n_f > n_o) \rightarrow 180^0$ (λ phase shift)

internal reflections $(n_o > n_f) \rightarrow 0^0$ (no phase shift)

- If *both* reflections are internal or external – no *relative* phase shift between the beams occurs.
- The reflected phase shifting causes beams that would interfere constructively due to path length difference alone to interfere destructively.

Consider the thin film surrounded by air shown at right.



- Beam 1 is externally reflected with a phase change of 180° with respect to the incident wave.
- Beam 2 is internally reflected and undergoes no phase change upon reflection with respect to the incident wave. It is, however, 180 degrees out of phase with the Beam 1.

If only the reflections are considered the beams would interfere destructively but we must also consider the actual difference in path length $2t$. Hence if:

$$2t = \frac{\lambda_n}{2} \text{ or } t = \frac{\lambda}{4}$$

the waves recombine in phase which leads to the requirement for *constructive interference* for mixed reflections:

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \qquad m = 0, 1, 2, 3, \ldots$$

$$2tn_f = \left(m + \frac{1}{2}\right)\lambda_0$$

If $2t$ is a multiple of λ_n the two waves combine out of phase and *destructive interference* for mixed reflections results:

$$2t = \lambda_n$$

$$t = \frac{\lambda_n}{2}$$

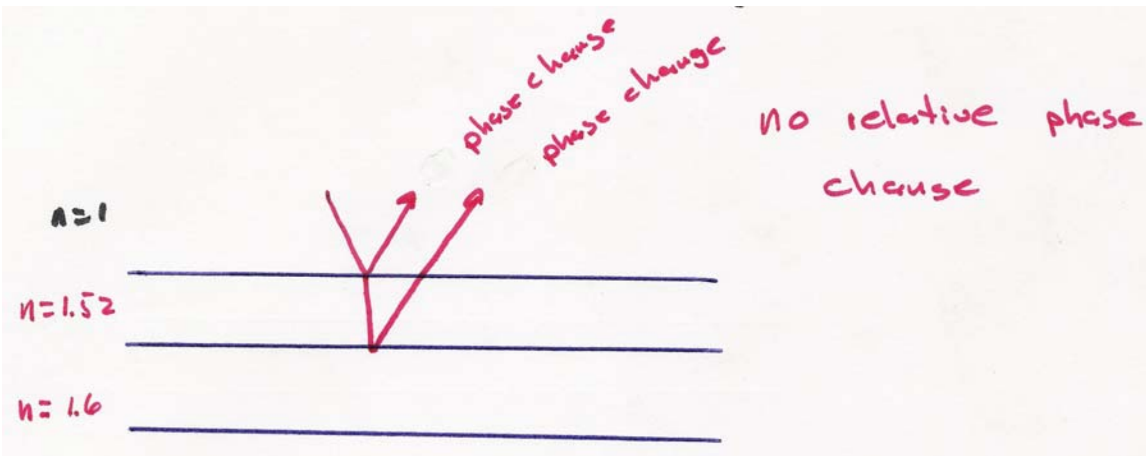
$$2t = m\lambda_n \text{ or } 2tn_f = m\lambda_0 \qquad m = 0, 1, 2, 3, \ldots$$

These conditions apply only when the film is surrounded by a common medium (a glass slide in air, water between two glass slides, etc.) or when the index of refraction on both sides of the film is either lower or higher than that in the film resulting in a mixture of external and internal reflections.

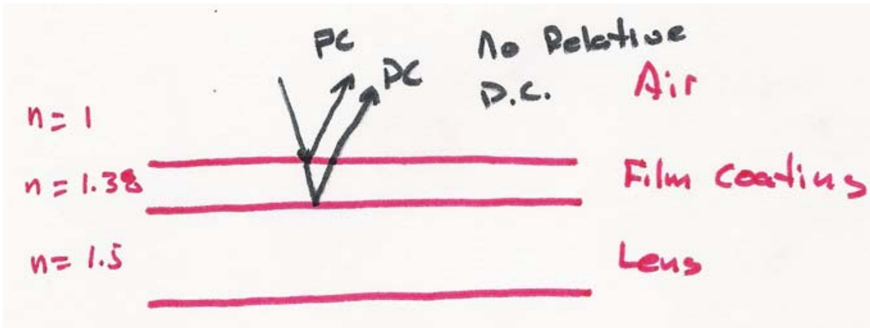
If the film is located between two different media, one of lower index of refraction and one of higher index of refraction, the conditions for constructive and destructive interference are reversed by virtue of the fact that both reflections will be of the same type, either external or internal. In this case:

$2t = m\lambda_n$ or $2tn_f = m\lambda_0$ m = 0, 1, 2, 3, ...   (constructive interference)

$2t = \left(m + \frac{1}{2}\right)\lambda_n$ or $2tn_f = \left(m + \frac{1}{2}\right)\lambda_0$ m = 0, 1, 2, 3, ...   (destructive interference)



Example 1 Consider a non-reflecting film on a camera lens. Antireflection coatings on lenses are designed to exploit destructive interference to quench reflection from the lens in visible light. A commonly used coating is M_gF_2, n = 1.38.



Since the film is located between media of two different refractive indices, one lower and one higher than that of the lens, the reflections are matched (both external) and the path length difference is the sole determinant in the type of interference that occurs. In this case the condition for destructive interference is:

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \quad \text{or} \quad 2t_n = \left(m + \frac{1}{2}\right)\lambda$$

If $\lambda_f = \dfrac{\lambda_0}{n_f}$ in the film and $t = \dfrac{\lambda_f}{4}$, the condition for destructive interference is:

$$2tn_f = \left(m + \frac{1}{2}\right)\lambda_0$$

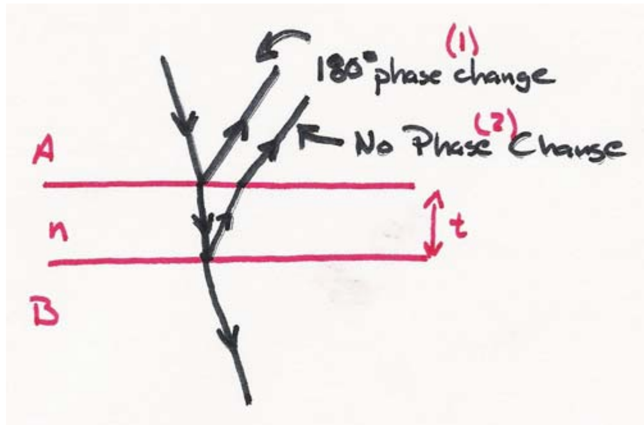
For complete extinction to occur, the reflected and refracted waves should have the same amplitude. In practice the concept is to reduce reflection from 4 – 5% to less than 1%.

If we assume that $\lambda_0 = 550nm$, the minimum thickness (m = 0) of the coating is:

$$t = \frac{\lambda_0}{4n_f} = \frac{550 \times 10^{-9} m}{(4)(1.38)} = 99.6nm$$

Notice that if the coating has an index of refraction greater than the glass of the lens then reflectivity is increased under the same conditions. This is how one way windows and reflecting sunglasses are made.

Example 2 Consider a layer of methylene iodide (n = 1.756), between two layers of glass (n = 1.5). What must the minimum thickness of the film be if light of 600 nm is to be strongly reflected?



The condition for maximum constructive interference in a film surrounded by a common medium.

$$2t = \left(m + \frac{1}{2}\right)\lambda_n$$

$$2tn_f = \left(m + \frac{1}{2}\right)\lambda_0$$

$$t = \frac{\lambda_0}{4n_f}$$

$$t = \frac{600 \times 10^{-9}\,m}{(4)(1.756)} = 85.4 \times 10^{-9}\,m$$

So the minimum thickness is 85.4 nanometers.

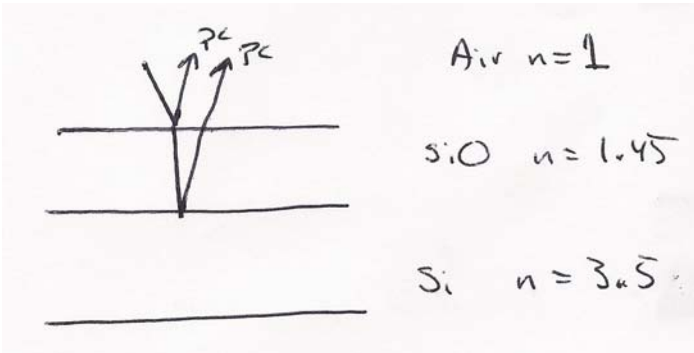
What are the next three thicknesses of coating that will work?

$$t = \frac{3\lambda_0}{4n_f} = 256.2nm$$

$$t = \frac{5\lambda_0}{4n_f} = 427.1nm$$

Notice that Δt is about 171 nanometers. This represents a tight tolerance.

Example 3 Compute the minimum thickness for a nonreflecting coating for a solar cell.



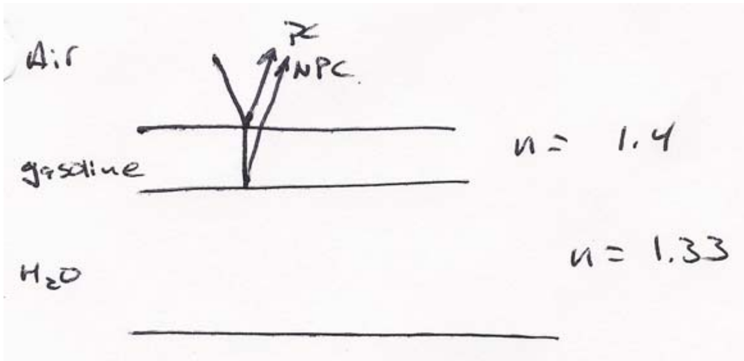
Assume: $\lambda_0 = 550nm$

Destructive interference with no relative phase change:

$$2tn_f = \left(m + \frac{1}{2}\right)\lambda_0$$

$$t = \frac{\lambda_0}{4n_f} = 94.8nm$$

Example 4 In a thin film of gasoline on water the film appears yellow instead of white because blue light has been removed. What is the minimum thickness of the film?



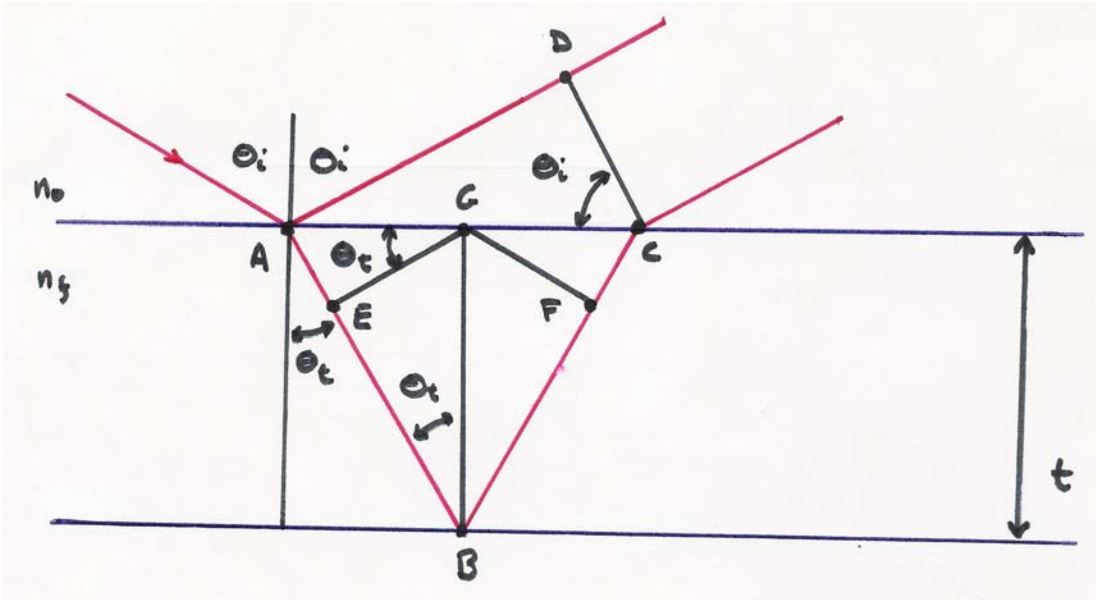
$$\lambda_{blue} = 469nm$$

Notice that destructive interference occurs due to mixed reflections when *t* = 0 (*m* = 0) (what does this imply abut the edge of a film of gasoline on a wet surface?). The next minimum thickness that will work is:

$$2tn_f = m\lambda_0$$

$$t = \frac{\lambda_0}{2n_f} = 167.5nm$$

Non–normal rays (arbitrary angle of incidence)



The phase difference along the plane defined by points C and D is due to path difference (Δ) between A → D and A → B → C.

Unlike previous examples, the path in the film A → B → C is much greater than 2*t* for angles of incidence much greater than near-normal.

$$\Delta = n_f(AB + BC) - n_o(AD)$$

$$\Delta = \lfloor n_f(AE + FC) - n_o AD \rfloor + n_f(EB + BF)$$

$$n_o \sin\theta_i = n_f \sin\theta_t$$

$$AE = AG\sin\theta_t = \left(\frac{AC}{2}\right)\sin\theta_t$$

$$AD = AC\sin\theta_i$$

$$2AE = AC\sin\theta_t = AD\left(\frac{\sin\theta_t}{\sin\theta_i}\right) = AD\left(\frac{n_o}{n_f}\right)$$

$$n_o AD = 2n_f AE = n_f(AE + FC)$$

$$\Delta = n_f\left(EB + BF\right) = 2n_f EB$$

If $EB = t\cos\theta_t \rightarrow \Delta = 2n_f t\cos\theta_t$

Note that the path difference is expressed in terms of angle of refraction and that the angle of incidence may be determined by Snell's Law if needed.

- For near-normal incidence $\Delta = 2n_f t$ as before.
- The phase difference between the waves is: $\delta = k\Delta = \frac{2\pi}{\lambda_o}\Delta$.
- The net phase change must also take into account any phase changes occurring upon reflection (as before).

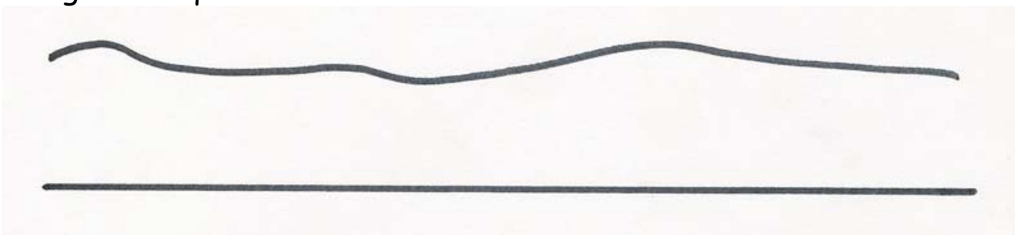
If $\Delta_p = 2n_f\cos\theta_t$ is the path length difference and Δ_r is the phase difference due to reflection then:

$$\Delta p + \Delta r = m\lambda \qquad \text{constructive interference}$$

$$\Delta p + \Delta r = \left(m + \frac{1}{2}\right)\lambda \qquad \text{destructive interference}$$

where *m* = 0, 1, 2, …

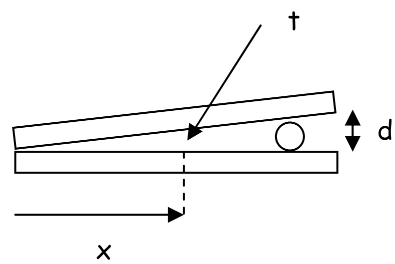
Fringes of Equal Thickness



If the thickness, *t*, of a film varies then the path length $\Delta = 2n_f t\cos\theta_t$ differs with out any variation of angle of incidence.

- A bright or dark fringe for a given angle of incidence will be associated with a particular thickness of the film as the Δ dependant conditions of constructive and destructive interference change with the change in thickness.
- Such fringe patterns are called fringes of equal thickness

Consider the arrangement of glass slides as shown at the right. The gap between the slides serves as a film of unequal thickness.



If a beam of monochromatic light strikes this arrangement at near-normal incidence:

$$\Delta = 2n_f t\cos\theta_t = 2n_f t$$

and the conditions for constructive and destructive interference, respectively, are:

$$\Delta_p + \Delta_r = 2n_f t + \Delta_r = m\lambda$$

$$\Delta_p + \Delta_r = 2n_f t + \Delta_r = \left(m + \frac{1}{2}\right)\lambda$$

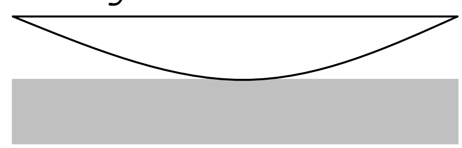
where Δ_r is either $\lambda/2$ or 0 depending on the whether or not a reflective phase shift exists. Note that as *x* decreases, *t* decreases and $\Delta_p \to 0$.

The pattern of equally spaced light and dark fringes seen in the film are known as Fizeau fringes. These are *virtual fringes* and cannot be projected onto a screen (i.e., require a lens or the eye to resolve).

If the incident light is sunlight the film will have fringes of different color. An example of this is oily pavement after a rainstorm.

Newton's Rings

- Since Fizeau fringes represent areas of equal thickness in a thin film their contours reveal any non-uniformities in the thickness the film.
- This effect is commonly exploited to measure the smoothness of a surface by creating an air film between it and some very flat reflective surface, illuminating the film with monochromatic light, and looking for the presence of Fizeau fringes.
- A common application of this technique is measuring the smoothness of lenses or mirrors.
- In this case the optical surface is not flat but has some radius of curvature and the air film beneath it will be in the form of a circular "wedge" producing Fizeau fringes in the form a series of concentric rings around the point of contact with the flat surface known as *Newton's rings*.
- At the point of contact the thickness of the film, t, is zero and phase difference between the reflected rays is π ($\lambda/2$) due to the external and internal reflections.



- The center of the fringe pattern is dark and is surrounded by a series of bright and dark concentric fringes.
- If the lens surface is smooth the fringes are smooth.
- This technique may also be used to measure the radius of curvature of the lens surface
- If the flat surface is transparent a complementary fringe pattern is formed by the light transmitted through it.