

PHYS 211 Examination 2

10-14-06

KEY

Directions: This exam is worth 58 points. Be sure to read each question carefully and choose the best answer. The exam will not be graded if all of the information above is not provided.

You must show all of your work, where appropriate, to receive credit for problems 11 - 17

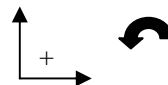
If you think that more than one answer is correct (multiple choice) choose the one you think is the best.

If you have any questions about the exam or the directions be sure to ask the exam proctor for clarification.

Materials you may use for this exam: TI30XA, CRC Math Tables Handbook. Please put away all cell phones, PDA's etc.

Words of wisdom: Good Luck!

For all problems unless otherwise indicated:



$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = v_0t + \frac{1}{2}at^2$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hyp}}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$F_f = \mu N$$

$$F = ma$$

$$\Gamma = I\alpha$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$W_{nc} = E_f - E_i$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$\text{P.E.}_{(\text{grav})} = mgh$$

$$W = F \times s = \Delta \text{KE} = \Delta \text{PE}$$

$$a_c = v^2/r$$

$$\Delta \text{KE} + \Delta \text{PE} = 0$$

$$\text{B.F.} = \rho_f g V$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \frac{v_t}{R} = 2\pi f$$

$$p = mv$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$Q = mc\Delta T$$

$$v = \lambda f$$

$$\Gamma = \text{force} \times \text{distance}$$

$$\text{deg} \times 2\pi/360^0 = \text{rad}$$

$$Q = mL$$

$$\text{K.E.} = \frac{1}{2}I\omega^2$$

$$\text{P.E.}_{(\text{elastic})} = \frac{1}{2}kx^2$$

$$P_{\text{gauge}} = \rho gh$$

$$P_{\text{absolute}} = P_{\text{atm}} + \rho gh$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$A_1 v_1 = A_2 v_2$$

$$\Delta U = Q - W$$

$$PV = nRT$$

$$B = (10) \log \frac{I}{I_0}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$$

$$a_t = R\alpha$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$I = I_{cm} + Mr^2$$

$$\rho = \frac{m}{v}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \Gamma = \vec{r} \times \vec{F}$$

$$dm = \rho dV$$

$$dm = \sigma dA$$

$$dm = \lambda d\ell$$

$$\vec{F} = -k\vec{x}$$

$$L = I\omega$$

$$\vec{F}\Delta t = \Delta\vec{p} = \vec{J}$$

$$W_s = \frac{1}{2}kx^2$$

$$a_r = \omega^2 r = \frac{v^2}{r}$$

$$s = \theta r$$

$$\frac{v_t}{r} = \omega = 2\pi f$$

$$a_t = R\alpha$$

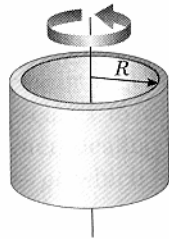
$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \times \vec{B} = AB \sin \theta$$

$$r_c = \frac{1}{M} \int r dm$$

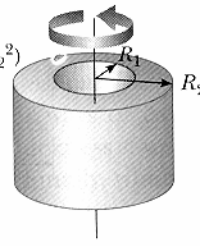
$$I = \int r^2 dm$$

Hoop or cylindrical shell
 $I_{CM} = MR^2$

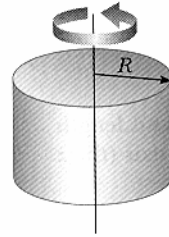


Hollow cylinder

$$I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$$

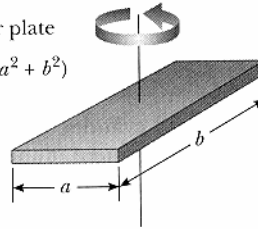


Solid cylinder or disk
 $I_{CM} = \frac{1}{2} MR^2$



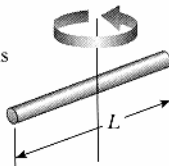
Rectangular plate

$$I_{CM} = \frac{1}{12} M(a^2 + b^2)$$



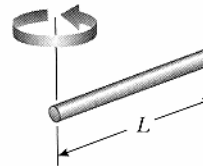
Long thin rod with rotation axis through center

$$I_{CM} = \frac{1}{12} ML^2$$

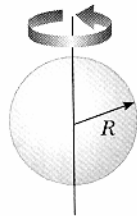


Long thin rod with rotation axis through end

$$I = \frac{1}{3} ML^2$$

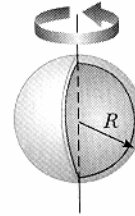


Solid sphere
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell

$$I_{CM} = \frac{2}{3} MR^2$$



$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t$$

$$\vec{F} = m\vec{a}$$

$$\vec{x} - \vec{x}_0 = \vec{v}_0t + \frac{1}{2}\vec{a}t^2$$

$$\theta - \theta_0 = \omega_0t + \frac{1}{2}\alpha t^2$$

$$\vec{F}\Delta t = \Delta\vec{p} = \vec{J}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$f_{friction} = \mu N$$

$$s = \theta r$$

$$\frac{v_t}{r} = \omega = 2\pi f$$

$$a_t = R\alpha$$

$$\vec{p} = m\vec{v}$$

$$Work = \vec{f} \cdot \vec{s} = \Delta E$$

$$a_r = \omega^2 r = \frac{v^2}{r}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hyp}}$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hyp}}$$

$$\vec{A} \times \vec{B} = AB \sin\theta$$

$$I = I_{cm} + MR^2$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$r_c = \frac{1}{M} \int r dm$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$KE_{lin} = \frac{1}{2}mv^2$$

$$PE_{grav} = mgh$$

$$PE_{spring} = \frac{1}{2}kx^2$$

$$KE_{rot} = \frac{1}{2}I\omega^2$$

$$\vec{F} = -k\vec{x}$$

$$\vec{F}_G = G \frac{m_1 m_2}{r^2} \hat{r}$$

$$I = \int r^2 dm$$

$$dm = \rho dV$$

$$dm = \sigma dA$$

$$dm = \lambda d\ell$$

$$P = P_a + \rho gh$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{\Gamma} = \vec{r} \times \vec{F}$$

$$B.F. = w_{fluid} = m_f g = \rho_f v g$$

$$PA = F$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\Gamma = I\alpha$$

$$U_g = G \frac{mM}{r}$$

$$n\lambda = d \sin\theta$$

$$B = (10) \log \frac{I}{I_0}$$

$$v = \lambda f$$

$$f_n = n \frac{v}{2L}, n \frac{v}{4L}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\psi = \psi_0 \sin(kx \pm \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T}$$

$$f = \frac{1}{T}$$

$$\Delta p_m = v\rho 2\pi f A$$

$$I = \frac{P^2}{\rho v}$$

$$\text{difference} = 20 \log \frac{d_1}{d_2}$$

$$Q = mc\Delta T$$

$$PV = nRT$$

$$V = IR$$

$$P = I^2 R$$

$$1/R_{\text{eq}} = 1/R_1 + 1/R_2$$

$$R_{\text{eq}} = R_1 + R_2$$

$$E = Pt$$

$$c/\lambda = f$$

$$E = hf$$

$$E_n = -\frac{13.6}{n^2} eV$$

$$s = \frac{L\lambda}{b} m \quad m = 0, \pm 1, \pm 2, \dots$$

$$s = \frac{L\lambda}{d}$$

$$r_n = \frac{L\lambda}{d} K_n$$

$$\tau = L/R$$

$$I(t) = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$V(t) = V_0 [1 - e^{-\frac{t}{RC}}]$$

$$I(t) = \frac{V}{R} [1 - e^{-\frac{t}{\tau}}]$$

$$V(t) = V_0 [1 - e^{-\frac{t}{\tau}}]$$

$$\tau = RC$$

$$1/d_o + 1/d_i = 1/f$$

$$m = -d_i/d_o$$

$$m = h_i/h_o$$

$$CV = Q$$

$$X_C = 1/2\pi f C$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$h = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$F = k \frac{q_1 q_2}{r^2}$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

$$f_o = (2\pi\sqrt{LC})^{-1}$$

$$X_L = 2\pi f L$$

$$I_m Z = V_m$$

$$I_m X_C = V_C$$

$$I_m X_L = V_L$$

$$P = I^2 R$$

$$E = Pt$$

$$F = ma$$

$$1/R_{eq} = 1/R_1 + 1/R_2$$

$$R_{eq} = R_1 + R_2$$

$$v = \sqrt{\frac{F}{m/L}}$$

$$c/\lambda = f$$

$$E_n = -\frac{13.6Z^2}{n^2} eV$$

$$n\lambda = d\sin\theta$$

$$\beta = (10) \log \frac{I}{I_0}$$

$$\tau = L/R$$

$$I = \frac{P}{A}$$

$$I(t) = \frac{V}{R} e^{-\frac{t}{RC}}$$

$$V(t) = V_0 [1 - e^{-\frac{t}{RC}}]$$

$$\tau = RC$$

$$I(t) = \frac{V}{R} [1 - e^{-\frac{t}{\tau}}]$$

$$V(t) = V_0 [1 - e^{-\frac{t}{\tau}}]$$

$$\sin\theta = 1.22 \frac{\lambda}{D}$$

$$1/d_o + 1/d_i = 1/f$$

$$m = -d_i/d_o$$

$$m = h_i/h_o$$

$$f_n = n \frac{v}{2l}$$

$$f_n = n \frac{v}{4l} \quad n = 1, 2, 3, \dots$$

$$X_L = 2\pi fL$$

$$V = IR$$

$$I_m Z = V_m \quad I_m X_C = V_C$$

$$I_m X_L = V_L$$

$$CV = Q$$

$$X_C = 1/2\pi fC$$

$$E = hf$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$v = \lambda f$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$h = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

$$F = k \frac{q_1 q_2}{r^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$f_o = (2\pi\sqrt{LC})^{-1}$$

$$EA = q/\epsilon_0$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = N \frac{\mu_0 I}{2R}$$

$$B = n\mu_0 I$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$k = 9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

$$S = c\epsilon_0 E^2$$

$$S = \frac{c}{\mu_0} B^2$$

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$

$$F = l \frac{\mu_0 I I'}{2\pi r}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$F = IB \sin \theta$$

$$\gamma = (1 - v^2/c^2)^{1/2}$$

$$V = k \frac{q}{r}$$

$$CV = Q$$

$$\epsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$

$$\epsilon = vBl \sin \theta$$

$$\ln e^x = x$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta = m \frac{\lambda}{d}$$

$$P = IV = I^2 R$$

$$v_R = V_R \sin \omega t$$

$$2t = m\lambda_{\text{film}}$$

$$C = \epsilon_0 \frac{A}{d}$$

$$\Delta t = \Delta t_0 / \gamma$$

$$\Delta L = \Delta L_0 \gamma$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \epsilon_0 E^2 Ad$$

$$\sum B_{\text{parallel}} \Delta \ell = \mu_0 I_{\text{enc}} = \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

$$v = v_0 + at$$

$$\omega = \omega_0 + \alpha t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$PV = nRT$$

$$x - x_0 = v_0 t + \frac{1}{2} at^2$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$B = 10 \log \frac{I}{I_0}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hyp}}$$

$$\omega = 2\pi f = \frac{v}{R}$$

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hyp}}$$

$$v = \omega A$$

$$E = mc^2$$

$$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$F_f = \mu N$$

$$\Gamma = I\alpha$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$W_{nc} = E_f - E_i$$

$$\text{K.E.} = \frac{1}{2}mv^2 = q\Delta V$$

$$\text{P.E.}_{(\text{grav})} = mgh$$

$$W = F \times s = \Delta \text{KE} = \Delta \text{PE}$$

$$a_c = v^2/r$$

$$\Delta \text{KE} + \Delta \text{PE} = 0$$

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{L} = I\boldsymbol{\omega}$$

$$a_t = \Delta v / \Delta t = r\alpha$$

$$\text{deg} \times 2\pi/360^0 = \text{rad}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

$$\Gamma = \text{force} \times \text{distance}$$

$$H = A \frac{(T_2 - T_1)}{\sum_i R_i}$$

$$Q = mL$$

$$\text{K.E.} = \frac{1}{2}I\omega^2$$

$$\text{P.E.}_{(\text{elastic})} = \frac{1}{2}kx^2$$

$$P_{\text{gauge}} = \rho gh$$

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$A_1 v_1 = A_2 v_2$$

$$\text{B.F.} = \rho_f g V$$

$$\rho = \frac{m}{v}$$

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\lambda = h/p$$

$$1/s + 1/s' = 1/f$$

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$

$$P = \sigma A e T^4$$

$$\Delta U = Q - W$$

$$dW = PdV$$

$$Q = mc\Delta t$$

$$W = \int_{v_i}^{v_f} PdV$$

$$P_{\text{absolute}} = P_{\text{atm}} + \rho gh$$

$$e = 1 - \frac{T_c}{T_h}$$

$$\text{COP}_{HP} = \frac{Q_h}{W}$$

$$\Delta S = \int_i^f \frac{dQ_r}{T}$$

Problems 1 - 10 are worth 1 point each

1. The definition of a totally elastic collision is:

- a) one in which the objects collide and bounce off each other
- b) one in which momentum is conserved but kinetic energy is not
- c) a collision between two Gumbies
- d) all of the above
- e) none of the above

2. Momentum is conserved:

- a) in the absence of external forces
- b) in the absence of external torques
- c) when all torques and forces are balanced
- d) all of the above
- e) none of the above

3. Moment of Inertia is:

- a) equivalent to work
- b) equivalent to heat
- c) the rotational analog of mass
- c) an epiphany
- d) all of the above

4. Torque is:

- a) a force that causes a rotational acceleration
- b) force \times distance
- c) in units of $\text{N} \cdot \text{m}$
- d) in units of $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
- e) all of the above

5. You are stranded on the side of the road with a flat tire and are attempting to remove the lug nuts from your wheels. If your mass is 80kg and you are able to push down on the nut wrench with 50% of your weight. Compute the torque you supply to loosen a stubborn nut with a 1.0 meter bar (assuming you push on the very end) and determine which of the following are true.

- a) you supply 172 N · m of torque
- b) you supply 392 N of force
- c) you supply 784 N of force
- d) you supply 118 kg of torque
- e) none of the above

6. A 10 kg solid wheel of radius 0.1 meters spinning at a rate of 10 rad/s comes to a complete stop in 20 seconds. Which of the following are true:

- a) $\alpha = 0.5 \text{ rad/s}^2$
- b) the displacement is 100 radians (about 16 revolutions)
- c) the moment of inertia with respect to the axle is $0.05 \text{ kg} \cdot \text{m}^2$
- d) the torque required to change the momentum of the wheel is $0.025 \text{ N} \cdot \text{m}$
- e) all of the above

7. A centripetal force:

- a) $m \frac{v^2}{r}$
- b) $G \frac{m_1 m_2}{r^2}$
- c) $\mu_k mg$
- d) all of the above are true
- e) none of the above are true

8. Linear momentum:

- a) is conserved in the absence of external forces
- b) $p=mv$
- c) has units of $\text{kg} \cdot \text{m/s}$
- d) time rate of change is equal to force
- e) all of the above

9. *Angular momentum:*

- a) is conserved in the absence of external torques
- b) $L = I\omega$
- c) requires torque to change
- d) keeps bicycles upright
- e) all of the above

10. A 1500 kg automobile collides with an immovable wall. If the auto was initially traveling 15.0 m/s:

- a) its momentum was initially 22,500 kg · m/s
- b) momentum is not conserved in this process
- c) the change in momentum was 22,500 kg · m/s
- d) the final momentum is zero
- e) all of the above.

Problem 11. (6 points) Consider a pulley with two masses m_1 & m_2 attached by a light cord as shown below. The pulley is a solid disk with a mass of 5 kg and a radius of 0.5 meters. If $m_1 = 6$ kg and $m_2 = 3$ kg, develop an equation to determine the speed of the system when the 6 kg mass has fallen, starting at rest, a distance of 3.0 meters.

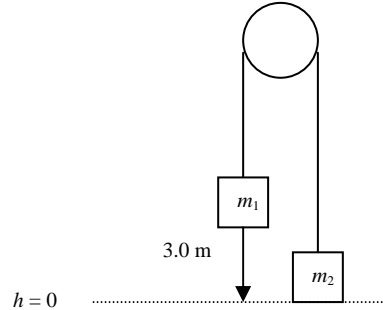
$$\bullet F = ma$$

$$\bullet \Gamma = I\alpha$$

$$\underline{m_1} :$$

$$\sum F_x = 0$$

$$\sum F_y = T_1 = m_1g - m_1a$$



$$\underline{m_2} :$$

$$\sum F_x = 0$$

$$\sum F_y = T_2 = m_2g + m_2a$$

$$T_1 - T_2 = (m_1g - m_1a) - (m_2g + m_2a)$$

$$\underline{m_p} :$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \Gamma = T_1r - T_2r = I\alpha$$

$$T_1 - T_2 = \frac{1}{2}m_p a$$

$$\therefore \frac{1}{2}m_p a = (m_1g - m_1a) - (m_2g + m_2a)$$

$$\Rightarrow a = \frac{g(m_1 - m_2)}{\frac{1}{2}m_p + m_1 + m_2} = 2.6 \frac{m}{s^2}$$

$$v_f^2 = v_i^2 + 2a(\Delta y)$$

$$v_f = 3.9 \frac{m}{s}$$

Or use conservation of energy for the same result (see your notes).

Problem 12 (6 points) The angular position of a reference line on a spinning wheel is given by $\theta = 3t^2 - 7t + 4$, where t is in seconds and θ in radians. Find the instantaneous

values for angular velocity and acceleration. Is the velocity constant? Is the acceleration constant? What is the velocity and what is the acceleration at $t = 5s$?

$$\theta = (3t^2 - 7t + 4) \text{rad}$$

$$\omega = \frac{d}{dt} \theta = (6t - 7) \frac{\text{rad}}{s} \quad \bullet \text{ Velocity is not constant.}$$

$$\alpha = \frac{d}{dt} \omega = 6 \frac{\text{rad}}{s^2} \quad \bullet \text{ Acceleration is constant.}$$

$$\omega(t = 5s) = 23 \frac{\text{rad}}{s}$$

$$\alpha(t = 5s) = 6 \frac{\text{rad}}{s^2}$$

Problem 13 (6 points) In the "roto ride" problem we looked at when we were considering centripetal forces, the force in the +y direction is the force of friction which depends on the normal force which, in turn, depends on the speed of rotation. If the speed of rotation is greater than that needed to keep the person suspended the normal force increases. This means that the force of friction must also increase. Since this force now exceeds mg (gravity) why doesn't the person begin to accelerate upward?

As the ride's rate of rotation increases, the force of friction in the positive direction also increases. At some point the force upward will become larger than the downward gravitational force. As this happens a downward force of friction occurs to maintain equilibrium of forces. This is what keeps the rider in position as the ride rotates faster.

Problem 14. (8 points) A solid sphere (mass 10 kg, radius 0.1 meters) rolls from rest down a rough inclined plane a distance of 14.1 meters from a height of 10 meters. Determine:

- the initial energy of this system
- final energy of this system
- the final distribution of the energy of the system
- the translational velocity of this system at the bottom of the incline
- the angular velocity of this system at the bottom of the incline
- the linear acceleration of the system
- the angular acceleration of the system
- What causes the sphere to roll without slipping?

$$E_i = E_f = mgh = 980J$$

$$\theta = \sin^{-1}\left(\frac{10m}{14.1m}\right) = 45 \text{ deg}$$

$$\sum F_x = mg \sin \theta - f_s = ma$$

$$f_s = ma - mg \sin \theta$$

$$\sum F_y = 0$$

$$\sum \Gamma = Rf_s = I\alpha$$

$$f_s = \frac{I\alpha}{R}$$

$$mg \sin \theta - ma = \frac{I\alpha}{R}$$

$$mg \sin \theta - ma = \frac{2}{5}ma$$

$$a = \frac{5}{7}g \sin \theta = 4.95 \frac{m}{s^2}$$

$$\alpha = \frac{a}{R} = 49.5 \frac{rad}{s^2}$$

$$v_f^2 = v_i^2 + 2a(\Delta x)$$

$$v_f = 11.82 \frac{m}{s}$$

$$\omega = \frac{v}{R} = 118.2 \frac{rad}{s}$$

Or use conservation of energy (see your notes) for the same result

$$K_r = \frac{1}{2}I\omega^2 = 280J$$

$$K_t = \frac{1}{2}mv^2 = 700J$$

Friciton causes the sphere to roll without slipping.

Problem 15 (6 points) Compute the moment of inertia of a thin disk of uniform density and radius r .

$$I = \int r^2 dm \quad dm = \sigma dA \quad A = \pi r^2 \quad dA = 2\pi r dr$$

$$I = \int r^2 \sigma 2\pi r dr$$

$$I = 2\pi\sigma \int r^3 dr \quad I = 2\pi \frac{m}{\pi r^2} \left(\frac{1}{4} r^4\right) = \frac{1}{2} mr^2$$

Problem 16. (4 points) An egg (mass = 1 kg) is dropped to the ground from a height of 3 meters. If the egg can survive an average force of 100 Newtons upon hitting the ground, what must be the thickness of the cushioning in order to protect the egg?

$$v = \sqrt{2gh} = 7.6 \frac{m}{s} \downarrow \text{ (negative direction)}$$

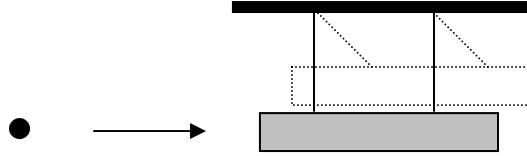
$$\Delta p = p_f - p_i = 7.6 N \cdot s$$

$$\bar{F}\Delta t = \Delta p = 7.6 N \cdot s \quad \Delta t = .0767 s$$

$$\bar{v} = \frac{v_f - v_i}{2} = 3.8 \frac{m}{s}$$

$$d = \bar{v}\Delta t = \left(3.8 \frac{m}{s}\right)(.0767 s) = .29 m$$

Problem 17. (6 points) A 0.0025 kg bullet imbeds itself in the wooden block of a ballistic pendulum. If the wooden block has a mass of 0.500 kg, and the bullet-block combination rises to a height of 2.0 meters after the collision, what is the initial velocity of the bullet?



Conservation of energy:

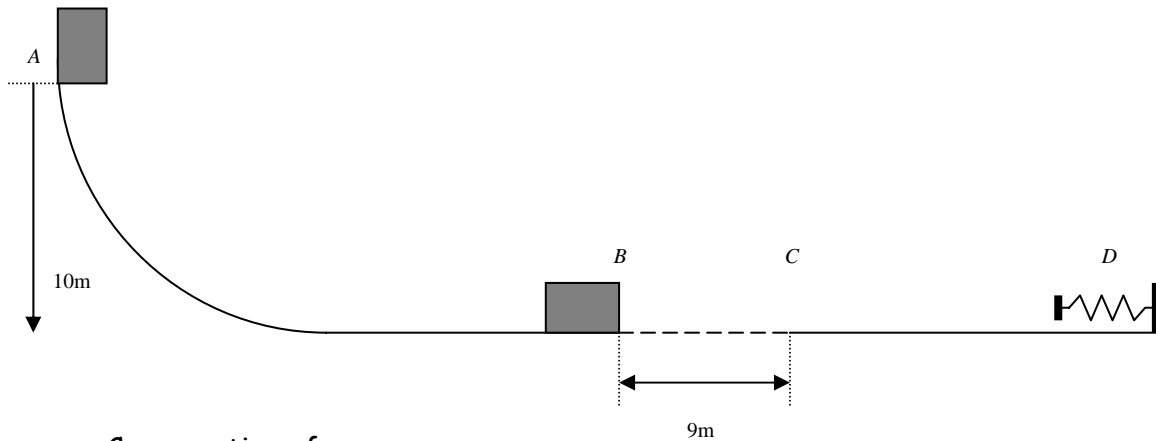
$$E_f = (m + M)gh = 9.85J$$

$$9.85J = \frac{1}{2}(m + M)v^2 \quad v = 6.3 \frac{m}{s}$$

Conservation of momentum:

$$(m + M)v = mv_{bullet} \quad v_{bullet} = 1266 \frac{m}{s}$$

Problem 18. (6 points) A 5 kg block is released from rest at point *A* on track *ABCD* as shown below. The block collides in a perfectly inelastic manner with another block, initially at rest, of mass 5 kg located just before section *BC*. The track is smooth except for section *BC*, of length 9 meters, $\mu_k = 0.531$. The blocks hit the spring ($k = 4000 \text{ N/m}$) and compress it from its equilibrium position. Determine how far the spring is compressed.



Conservation of energy:

$$E_i = m_1gh = 490J$$

$$m_1gh = \frac{1}{2}m_1v^2 \quad v = 14 \frac{m}{s}$$

Conservation of momentum:

$$m_1v_i = (m_1 + m_2)v_f \quad v_f = 7 \frac{m}{s}$$

Kinetic energy of blocks:

$$\frac{1}{2}(m_1 + m_2)v_f^2 \quad K.E. = 245J$$

Energy lost to friction:

$$W = F \times d \quad F = \mu_k N = \mu_k(m_1 + m_2)g$$

$$W = (\mu_k(m_1 + m_2)g) \times d = 468J$$

This result implies the blocks do not have enough energy to move across the rough area. The answer is the blocks do not reach the spring thus it is not compressed.