PHYS 211 Examination 4

Name (print): ______________________________

Signature _________________________________

Problem 1 _______________

Problem 2 _______________

Problem 3 _______________

Problem 4 _______________

Problem 5 _______________

Problem 6 _______________

Total _______________

Directions: Time allowed: 2.5 hours. There are six problems worth varying points each. You may leave the room at any time provided you are not gone for more than a couple of minutes, and do not take anything with you. An equation sheet has been provided. No other ancillary materials are allowed on this exam except your *CRC Math Tables Handbook*. The calculator rule is in effect. Good Luck!

$$\vec{v}=\vec{v}_0+\vec{a}t \qquad \vec{w}=\vec{w}_0+\vec{a}t \qquad \vec{F}=m\vec{a}$$

$$\vec{x}-\vec{x}_0=\vec{v}_0t+\frac{1}{2}\vec{a}t^2 \qquad q-q_0=w_0t+\frac{1}{2}at^2 \qquad \overline{F}\Delta t=\Delta\vec{p}=\vec{J}$$

$$v^2=v_0{}^2+2a\left(x-x_0\right) \qquad w^2=w_0{}^2+2a\left(q-q_0\right) \qquad f_{friction}=mN$$

$$s=qr \qquad \frac{v_t}{r}=w=2pf \qquad a_t=Ra$$

$$\vec{p}=m\vec{v} \qquad Work=\vec{f}\cdot\vec{s}=\Delta E \qquad a_r=w^2r=\frac{v^2}{r}$$

$$\cos q=\frac{adjacent}{hyp} \qquad \vec{A}\cdot\vec{B}=AB\cos q \qquad \cos q=\frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$$\sin q=\frac{opposite}{hyp} \qquad \vec{A}\times\vec{B}=AB\sin q \qquad I=I_{cm}+MR^2$$

$$\tan q=\frac{opposite}{adjacent} \qquad f=\frac{1}{T}=\frac{1}{2p}\sqrt{\frac{k}{m}} \qquad R=\frac{v_0{}^2\sin 2q}{g}$$

$$r_c=\frac{1}{M}\int r\,dm \qquad v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i} \qquad v_{2f}=\frac{2m_1}{m_1+m_2}v_{1i}$$

$$KE_{lin}=\frac{1}{2}mv^2 \qquad PE_{grav}=mgh \qquad PE_{spring}=\frac{1}{2}kx^2$$

$$KE_{rot}=\frac{1}{2}Iw^2 \qquad \vec{F}=-k\vec{x} \qquad W_s=\frac{1}{2}kx^2$$

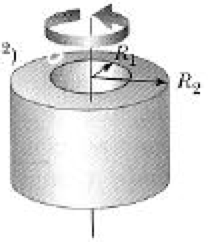
$$I=\int r^2dm \qquad \begin{aligned} dm&=r\,dV\\ dm&=s\,dA\\ dm&=l\,d\ell \end{aligned}$$

$$\vec{L}=\vec{r}\times\vec{p} \qquad \frac{d\vec{L}}{dt}=\Gamma=\vec{r}\times\vec{F}$$

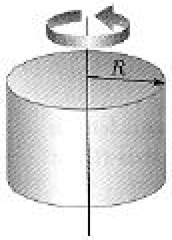
$$\mathrm{B}=(10)\log\frac{I}{I_0}$$

Hoop or
cylindrical shell
$I_{CM} = MR^2$

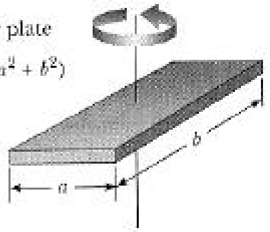
Hollow cylinder
$I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$



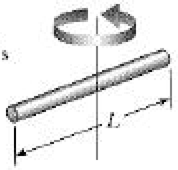
Solid cylinder
or disk
$I_{CM} = \frac{1}{2} MR^2$



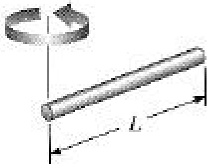
Rectangular plate
$I_{CM} = \frac{1}{12} M(a^2 + b^2)$



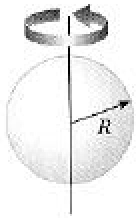
Long thin rod
with rotation axis
through center
$I_{CM} = \frac{1}{12} ML^2$



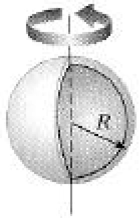
Long thin
rod with
rotation axis
through end
$I = \frac{1}{3} ML^2$



Solid sphere
$I_{CM} = \frac{2}{5} MR^2$



Thin spherical
shell
$I_{CM} = \frac{2}{3} MR^2$



Problem 1. (10 points) Choose any *two* of the following three

a) A particle of mass *m* travels in uniform circular motion in an x-y plane. If the radius of the path is *r*, and the particle's velocity *v*, find, with respect to the origin:

- The angular momentum $\vec{L}$ in terms of *r*
- The angular momentum L in terms of w

Is the angular momentum of this particle constant in terms of both magnitude and direction?

$$\vec{L} = \vec{r} \times \vec{p} = mvr \sin q \; = mvr$$
$$L = Iw$$

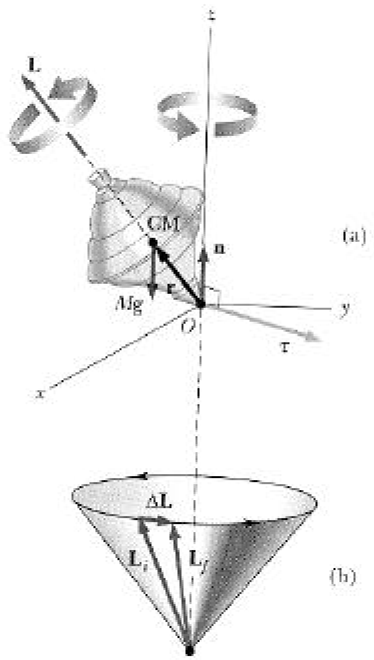
The angular momentum is constant. The quantities on the right side of both equations are constant and the direction, given by the rhr, is also constant.

b) The angular position of a reference line on a spinning wheel is given by $q = 3t^2 - 7t + 4$, where *t* is in seconds and q in radians. Find the instantaneous values for angular velocity and acceleration. Is the velocity constant? Is the acceleration constant? What is the velocity and what is the acceleration at *t* = 5s?

$$w = \frac{dq}{dt} = 6t - 7$$
$$a = 6$$

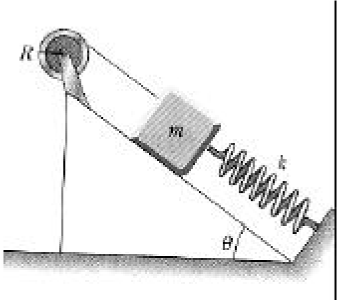
The acceleration is constant (6s^{-2}). The instantaneous velocity at *t* = 5s is 23s^{-1}.

c) Using the figure below. Explain gyroscopic precession.



See Fig. 11.19, p 345 in your textbook.

Problem 2. (20 points) Consider the system below.



The pulley is a solid disk of mass = 2.0 kg, radius = 0.3 m. The mass of the block is 0.5 kg. The stiffness of the spring is 50.0 N/m. The incline is smooth and at an angle of 37^0 to the horizontal. If the pulley is wound so that the spring is stretched 0.2 meters, then released from rest, find the angular displacement, angular velocity and angular acceleration when the spring returns to it's equilibrium position.

$$E_i = E_f$$

$$\frac{1}{2}kx^2 + mgh = \frac{1}{2}Iw^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}kd^2 + mgd\sin q = \frac{1}{2}Iw^2 + \frac{1}{2}mv^2$$

$$\frac{1}{2}kd^2 + mgd\sin q = \frac{1}{2}Iw^2 + \frac{1}{2}mR^2w^2$$

$$\frac{1}{2}kd^2 + mgd\sin q = \frac{1}{2}w^2\left(I + mR^2\right)$$

$$w = \sqrt{\frac{2mgd\sin q + kd^2}{I + mR^2}} = w = \sqrt{\frac{2mgd\sin q + kd^2}{\frac{1}{2}MR^2 + mR^2}} = \sqrt{\frac{2mgd\sin q + kd^2}{R^2\left(\frac{1}{2}M + m\right)}}$$

$$w = \sqrt{\frac{2(0.5kg)(9.8m/s^2)(0.2m)(\sin 37^0) + (50N/m)(0.2m)^2}{(0.3m)^2\left(\frac{1}{2}(2) + 0.5\right)kg}} = \sqrt{\frac{1.18 + 2}{0.11}} = 4.85s^{-1}$$

$$s = Rq \therefore \frac{s}{R} \to q = \frac{0.2m}{0.3m} = 0.67rad$$

$$\Gamma = Ia \therefore \frac{\Gamma}{I} = a \to \frac{(mg\sin q)R}{\frac{1}{2}MR^2} = \frac{2mg\sin qR}{MR^2} = \frac{2mg\sin q}{MR} = 9.82s^{-2}$$

(Note: The spring exerts no force and therefore no torque at the equilibrium position).

Problem 3. (15 points) Three objects of uniform density, a solid sphere, a solid cylinder, and a hollow sphere are placed at the top of an incline. If they are released from rest at the same time and roll without slipping, determine:

- The angular velocity of each at the bottom of the incline
- The linear velocity of each at the bottom of the incline
- The angular acceleration of each
- The order in which they reach the bottom of the incline

For a Solid Cylinder

$$PE_i = KE_{f_{rot}} + KE_{f_{trans}}$$

$$mgh = \frac{1}{2}Iw^2 + \frac{1}{2}mv_{cm}{}^2$$

$$mgh = \frac{1}{2}I\left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2}mv_{cm}{}^2$$

$$mgh = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2}mv_{cm}{}^2$$

$$gh = \frac{1}{2}\left(\frac{1}{2}R^2\right)\frac{v_{cm}{}^2}{R^2} + \frac{1}{2}v_{cm}{}^2 \rightarrow gh = \frac{1}{4}v_{cm}{}^2 + \frac{1}{2}v_{cm}{}^2 \therefore v_{cm} = \sqrt{\frac{4}{3}gh}$$

Solving for angular velocity.

$$mgh = \frac{1}{2}\left(\frac{1}{2}mR^2\right)w^2 + \frac{1}{2}m(Rw)^2$$

$$gh = \frac{1}{2}\left(\frac{1}{2}R^2\right)w^2 + \frac{1}{2}(Rw)^2 \rightarrow gh = \frac{1}{4}R^2w^2 + \frac{1}{2}R^2w^2 \therefore gh = \frac{3}{4}R^2w^2 \therefore w = \sqrt{\frac{4}{3}\frac{gh}{R^2}}$$

Solving for angular acceleration.

$$\sum F_x = mg\sin q - f_s = ma_{cm}$$

$$\sum \Gamma = Rf_s = Ia$$

$$f_s = \frac{mR^2a}{2R} = \frac{1}{2}mRa$$

$$\sum F_x = mg\sin\theta - \frac{1}{2}mR\alpha = mR\alpha$$

$$g\sin\theta = \frac{3}{2}R\alpha$$

$$\alpha = \frac{2g\sin\theta}{3R}$$

The same analysis for the solid sphere yields:

$$v_{cm} = \sqrt{\frac{10}{7}gh}$$

$$\omega = \sqrt{\frac{10}{7}\frac{gh}{R^2}}$$

$$\alpha = \frac{5g\sin\theta}{7R}$$

The same analysis for the hollow sphere yields:

$$v_{cm} = \sqrt{\frac{6}{5}gh}$$

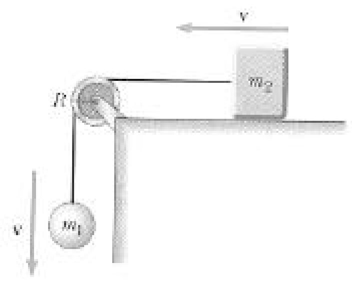
$$\omega = \sqrt{\frac{6}{5}\frac{gh}{R^2}}$$

$$\alpha = \frac{3g\sin\theta}{5R}$$

One may look at kinematics or the percentage of energy put into translation to determine:

(1) Solid sphere, (2) disk, (3) hollow sphere.

Problem 4. (20 points) A 10 kg block rests on a flat table. The coefficient of kinetic friction between the block and the table is 0.2. A light cord runs from this block, around a pulley to a solid sphere suspended beneath the pulley. The pulley has a mass of 2.5 kg, a radius of 20 centimeters, and consists of a solid disk with a narrow groove milled for the cord. If the system is released from rest it accelerates at a rate of 4 m/s^2. What is the mass of the suspended sphere?



Up, left, ccl (+)

Block: $$\sum F = T_2 - m_k m_2 g = m_2 a_2$$

Sphere: $$\sum F = T_1 - m_1 g = -m_1 a_1$$

Pulley: $$\sum \Gamma = T_1 R - T_2 R = Ia = \frac{1}{2} M R^2 a$$

Note: $a_1 = a_2 = a_t = Ra$

1. $T_2 - m_k m_2 g = m_2 a$
2. $T_1 - m_1 g = -m_1 a$
3. $T_1 - T_2 = \frac{1}{2} M a$

Multiply equation #2 by -1 and add all three together:

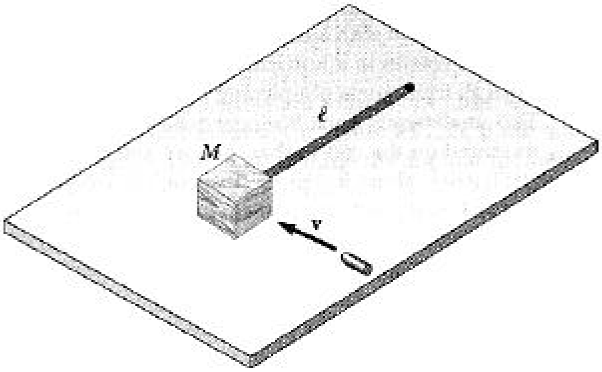
$$T_2 - T_1 + T_1 - T_2 = m_k m_2 g + m_2 a - m_1 g + m_1 a + \frac{1}{2} M a$$

$$m_1 g - m_1 a = m_2 a + \frac{1}{2} M a + m_k m_2 g$$

$$m_1 (g - a) = m_2 a + \frac{1}{2} M a + m_k m_2 g$$

$$m_1 = \frac{m_2 a + \frac{1}{2} M a + m_k m_2 g}{(g - a)} = 11.1 kg$$

Problem 5. (15 points) Consider the system below.



The system consists of a wooden block (*M*) and a bullet (*m*) and a slender rod of length ℓ. The surface is frictionless. If the bullet collides inelastically with the block normal to the rod and parallel to the surface of the table what is the angular momentum of the system? What fraction of the original kinetic energy is lost in the collision?

Since momentum is conserved $L_i = L_f$

$$L_i = mv\ell$$
$$L_f = (m+M)v_f\ell$$

Either expression is correct.

$$K_i = \frac{1}{2}mv^2$$
$$K_f = \frac{1}{2}(M+m)v_f{}^2$$

where

$$v_f = \frac{m}{m+M}v$$

The fraction of Kinetic Energy lost is: $\dfrac{K_i - K_f}{K_i} = \dfrac{M}{m+M}$.

Problem 6. (20 points) A projectile of mass *m* is fired from the ground with an initial speed v_0 and an initial angle q_0 above the horizontal. (a) Find an expression for the angular momentum about the firing point as a function of time. (b) Find the rate at which the angular momentum changes with time. (c) Evaluate the magnitude of $\vec{r} \times \vec{F}$ directly and compare the result with (b).

Recall:

$v = v_0 + at$

$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\vec{r} = V_{0x} t \hat{i} + (V_{0y} t + \frac{1}{2} g t^2) \hat{j}$

$\vec{r} = V_0 \cos\theta_0 t \hat{i} + (V_0 \sin\theta_0 t + \frac{1}{2} g t^2) \hat{j}$

$\vec{v} = V_x \hat{i} + V_y \hat{j}$

$= V_0 \cos\theta_0 \hat{i} + V_0 \sin\theta_0 + gt \hat{j}$

$\ell = m(\vec{r} \times \vec{v})$

$$\vec{r} \times \vec{v} = \begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ V_0\cos\theta_0 t & V_0\sin\theta_0 t + \frac{1}{2} g t^2 & 0 & V_0\cos\theta_0 t & V_0\sin\theta_0 t + \frac{1}{2} g t^2 \\ V_0\cos\theta_0 & V_0\sin\theta_0 + gt & 0 & V_0\cos\theta_0 & V_0\sin\theta_0 + gt \end{array}$$

$= [(V_0\cos\theta_0 t)(V_0\sin\theta_0 - gt)]\hat{k} - [(V_0\cos\theta_0)(V_0\sin\theta_0 t - \frac{1}{2} g t^2)]\hat{k}$

$= (V_0^2 \cos\cancel{\theta_0} \sin\theta_0 t - V_0\cos\theta_0 g t^2) - V_0^2 \cos\cancel{\theta_0} \sin\theta_0 T + \frac{1}{2} V_0 \cos\theta_0 g t^2$

$= -\frac{1}{2} V_0 \cos\theta_0 g t^2 \hat{k}$ $\therefore$ $\ell = -m \frac{1}{2} V_0 \cos\theta_0 g t^2 \hat{k}$

(b) $\frac{d\ell}{dt} = -V_0 m g t \cos\theta_0 \hat{k}$

(c) $\vec{r} \times \vec{F} = (v_0\cos\theta_0 t)\hat{i} + (v_0\sin\theta_0 t + \frac{1}{2}gt^2)\hat{j} \times (-mg)\hat{j}$

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ v_0\cos\theta_0 t & v_0\sin\theta_0 t + \frac{1}{2}gt^2 & 0 & v_0\cos\theta_0 t & v_0\sin\theta_0 t + \frac{1}{2}gt \\ 0 & -mg & 0 & 0 & -mg \end{array}$$

$(v_0\cos\theta_0 t)(-mg)\hat{k} \Rightarrow \vec{r} \times \vec{F} = -v_0 mgt\cos\theta_0\hat{k}$