Laboratory 10 – Measuring the Properties of Binary Stars

Materials Used: Spectral data from the eclipsing binary star system JR-M 60; an Excel spreadsheet with system parameters, a light curve, velocity curves and spectral data; a Hertzsprung-Russell diagram.

Objectives: To study methods for determining the various properties of stars in binary systems.

Discussion: Measuring the physical properties of stars is an area of great interest in astronomy. Stars are the most numerous and important large objects in the Universe. Because of the great distances from us to all stars (except our own) it is only possible for us to observe and measure their properties from afar by indirect methods.

Binary systems – systems containing two stars that orbit around the same center of mass - are of considerable importance in the realm of stellar measurements. Because binary stars orbit each other their combined masses may be easily determined. All that is necessary to determine the mass of a binary system is to measure the *period* and *semimajor axis* of the system. If the orbital velocities of the individual stars are determined from spectral data (or some other means) the individual masses of the stars may then be computed with relative ease. The sizes, surface temperatures and luminosities of the individual stars and the distance of the system from us follow. Binary stars are quite common and their abundance allows astronomers to classify a wide variety of stars based on the study of binary systems.

A fundamental precept of astronomy is that what seems easy to accomplish in theory is almost invariably more difficult in practice and stellar measurements are no exception to this rule. Even the properties of stars in binary systems are not determined without fairly sophisticated methods. In this procedure we will concentrate on a particularly useful type of binary system known as an *eclipsing binary*. Eclipsing binaries are relatively rare (only a few thousand are known) but lend themselves to analysis due to their simple geometry and alignment with respect to our viewing angle.

Eclipsing binaries consist of two stars that are relatively close to each other – so much so that they often appear as a single point of light to observers on Earth. An eclipsing binary may, in fact, first appear to be a single variable star. Exceptionally large changes in apparent magnitude and cyclic Doppler shift data are indicative of a pair of closely spaced stars. Eclipsing binaries orbit in a plane nearly parallel to our line of sight and the brightness of the system changes periodically as the stars pass in front of each other. This variation in the brightness may be used to plot a *light curve* – a graph of brightness vs. orbital position for the system.

Because the stars in eclipsing binaries are generally close to each other their orbits tend to be very nearly circular. In this case any small inclination of the plane of their orbits with respect to our line of sight results only in a change in the amplitude, rather than the position, of the peaks and dips in the light curve - making it much easier to ascertain several important properties of the individual stars in those systems.

In this procedure you will examine data from a pair of stars in the fictional system JR-M 60 located about 30 parsecs from earth (1 parsec = 3.26 ly) – a distance we will take to be constant.

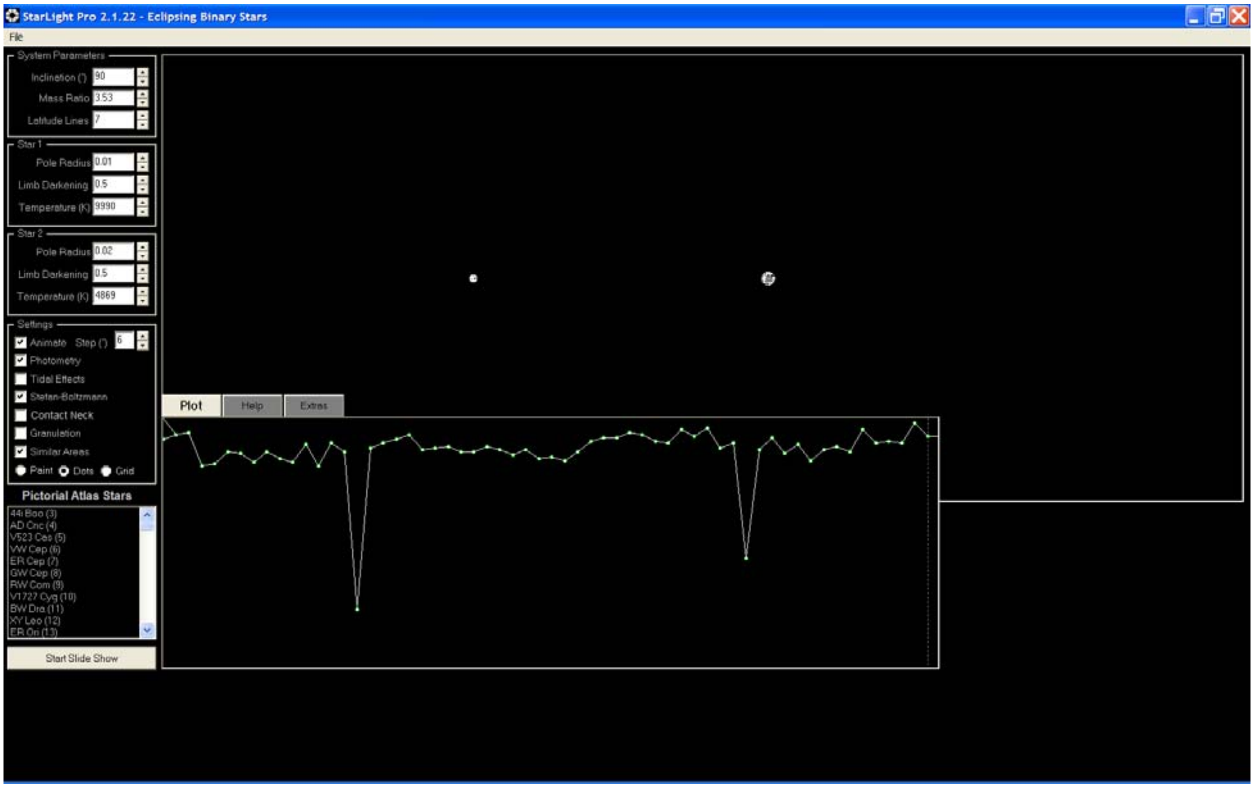
JR-M 60 is an eclipsing binary that orbits in a plane parallel to our line of sight and we will assume that the orbits of the individual stars are circular. You will use spectral data and a light curve from this system to determine its period and semimajor axis, the masses, sizes, temperatures, orbital velocities and spectral characteristics of each of the stars, and the approximate distance of the system from us.

Procedure

Two pieces of software have been provided to assist you with this procedure. The first is a program called StarLightProject that creates an animation depicting the dynamics of JR-M 60 from a vantage point close to the system but along the line of sight from Earth. The second piece of software is an Excel spreadsheet, astlmsp.xls, which will perform all burdensome calculations for you. All that will be necessary for you to do is to analyze several graphs and extract data from them. Your TA will show you how to find and open all of the required programs on your computer.

Begin with the StarLightProject animation. Open the program, then under the file menu click "Load Star" and select the only star in the library. Make sure that the settings are selected as indicated below. If you cannot find a star already in the library just open the program and set the parameters as shown below to produce the desired animation.

Observe that both stars are in essentially circular orbits around the center of mass of the system. Note the light curve that is generated over the course of one full period.



System Parameters
Inclination (°) 90
Mass Ratio 3.53
Latitude Lines 7

Star 1
Pole Radius 0.01
Limb Darkening 0.5
Temperature (K) 9990

Star 2
Pole Radius 0.02
Limb Darkening 0.5
Temperature (K) 4869

Settings
☑ Animate Step (°) 6
☑ Photometry
☐ Tidal Effects
☑ Stefan-Boltzmann
☐ Contact Neck
☐ Granulation
☑ Similar Areas

Figure 1. The Binary System JR-M 60

Determining Orbital Speed

Open the Excel spreadsheet astlmsp.xls and click the "Velocity Curves" tab near the bottom left of the screen. This will display a graph similar to that shown below in Figure 2. The two curves are plots the Doppler shift of the Balmer Hα spectral line for each star through one complete orbit. During one half of each star's orbit its motion is predominantly toward us (blue Doppler shift) and the other half predominantly away from us (red Doppler shift). This is reflected in the negative and positive values along the vertical axis of the graph.

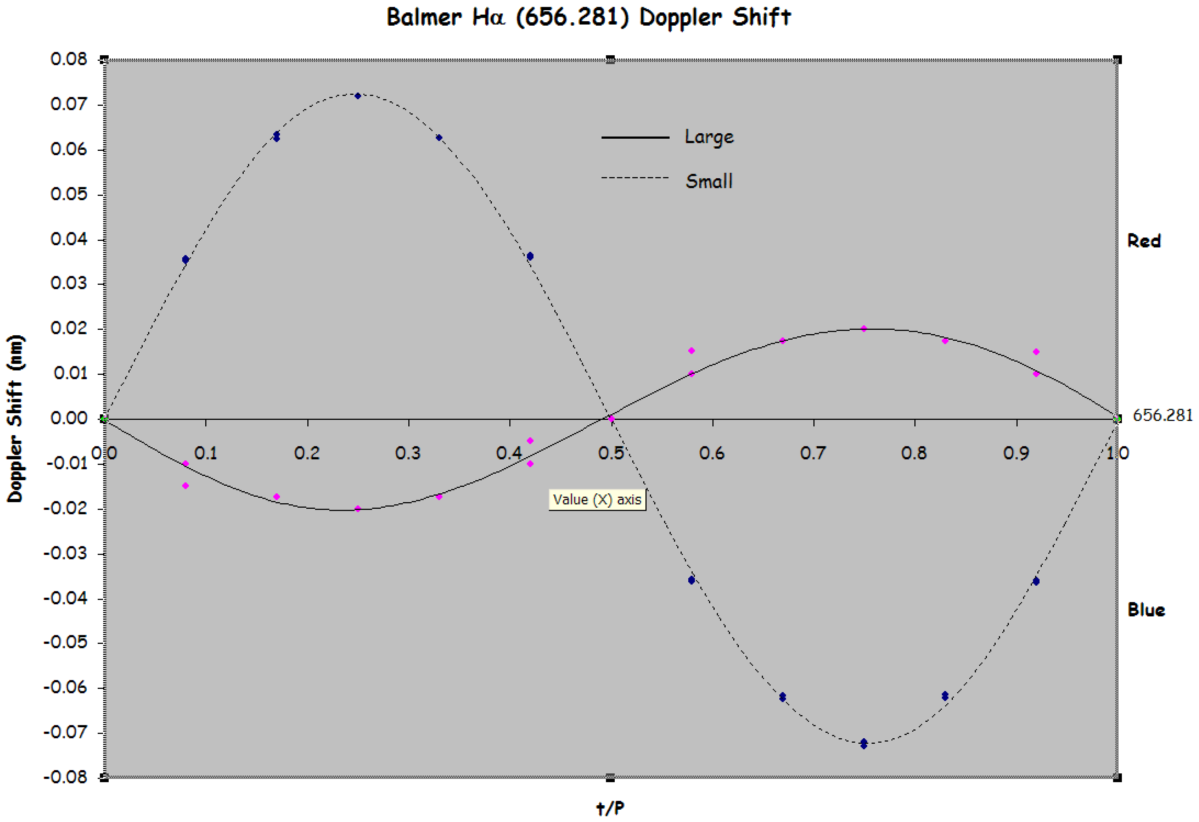


Figure 2. Velocity Curves for JR-M 60 from Doppler Shifts

In the first part of this exercise you are to determine the value of the maximum Doppler shift (which is related to orbital velocity) for each star. At what location in each orbit will this occur? When the star is closest to us, farthest from us, or at its maximum elongation?

Based on your answer, consider Figure 2. The vertical axis is scaled in nanometers (10^{-9}m) and represents the actual amount of Doppler shifting. The horizontal axis is scaled in units of phase and indicates where the star is in its orbit. The curves show that each star's Doppler shift varies sinusoidally over a full cycle (a 360^0 orbit). Notice that the amplitudes of the two curves are not the same. This indicates that the stars have different orbital velocities. We are interested in obtaining from this plot the maximum Doppler shift value for each star. What part of each curve yields each star's maximum Doppler shift?

Once you have identified the part of the plot that yields the needed data you'll next need to determine the value(s) at that point. Although it is possible to estimate this from just looking at the graph the easiest method is to move the mouse pointer close to the data point for which you wish to obtain a numeric value and have the spreadsheet display it for you. This takes some precision but with patience you should be able to obtain a display like that shown in Figure 3 below. The first value in parenthesis is the relative phase of the star and the second is the Doppler shift. In this particular example the Doppler shift is 0.072 nanometers.

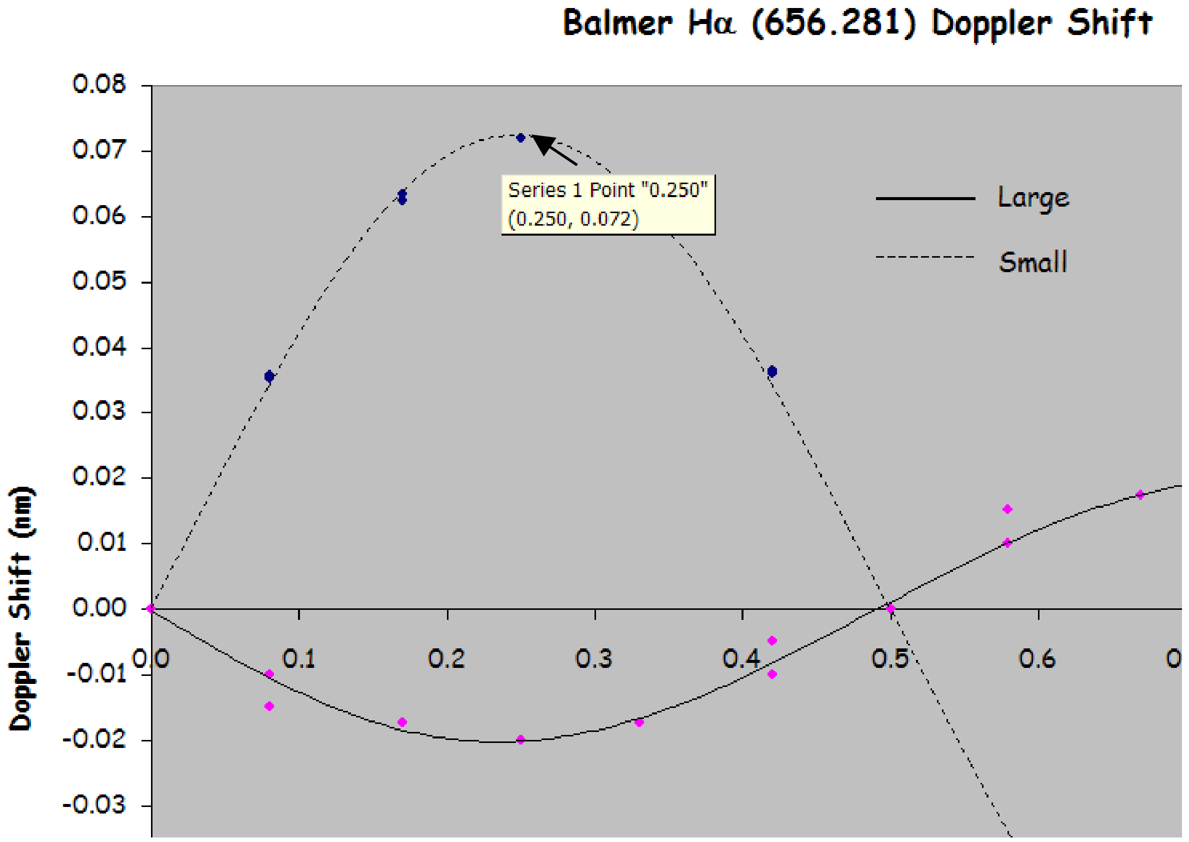


Figure 3. Determining Doppler Shift values from the Velocity Curve.

Note that both red and blue shifted values occur along each curve for each star. The maximum red shift may be used to compute the velocity of the star when it is moving directly away from us, a quantity known as a *radial velocity*. We are using a Doppler shift to acquire a "snapshot" of each star's orbital speed at a unique position in each orbit. If we assume approximately circular orbits (which we are) the velocity of each star should be relatively constant over a full orbit. The only thing that remains is to decide if more than one useful "snapshot" exists along each curve. Is there more than one place in each orbit where we may obtain a Doppler shift that might be particularly convenient to us in computing a radial velocity? If so, where would this second point be on each curve?

We need to determine the average value of both points for each curve using the absolute values of each point. Let's say that we determine, for instance, that one of the stars displays maximum red and blue Doppler shifts of 0.02 and –0.02 nm, respectively. To find the average value:

$$\text{Average Doppler shift} = \frac{|0.02nm| + |-0.02nm|}{2} = 0.02nm$$

Once you have determined the average Doppler shifts (Δλ) for each star, enter their values in cells D1 and D2, in the "System Parameters" sheet. Excel computes the respective orbital velocities using the relationship below and displays them in cells F1 and F2.

$$v_r = \frac{\Delta\lambda}{\lambda} c$$

Analyzing The Light Curve

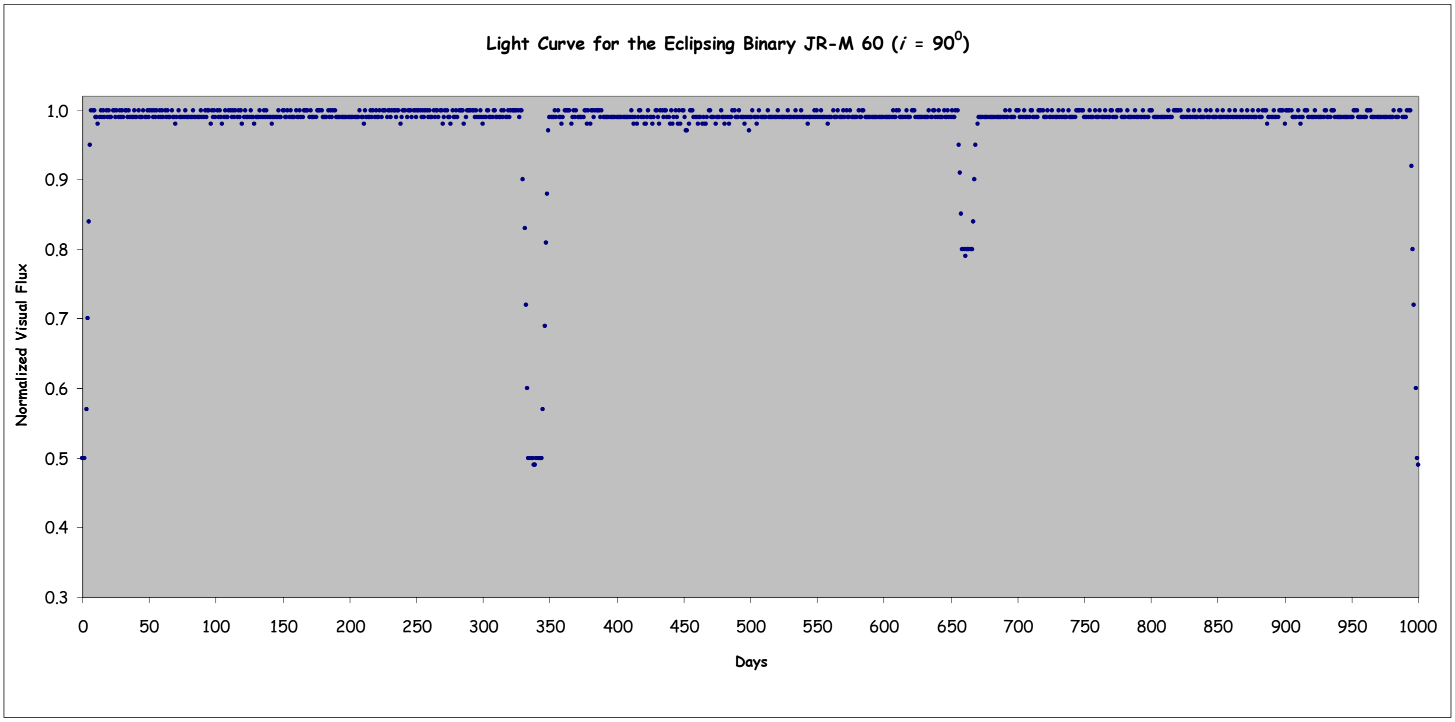


Figure 4. Light Curve for JR-M 60

The next step is to examine the light curve for our system. Recall that this curve is produced as the stars eclipse each other over the course of one full orbital period. The animation that you viewed earlier indicated the relative positions of the stars as the light curve was being produced. Based on this animation, what are the relative positions of the stars in our system at a time of about 340 days based on the light curve in Figure 4?

We'll use data from the light curve for JR-M 60 to determine the period of the system and then, when combined with the orbital data we've already computed, estimate the size of each star, the semimajor axis of each star's orbit, and the mass of each star.

Click on the "Light Curve" tab in the spreadsheet. The first value you are to obtain from the light curve is the system's period. Note that the period (in days) is displayed along the horizontal axis of the system. Convert this value to years and enter it into cell B1 of the "System Parameters" sheet. Excel converts this value to seconds for you and displays it in cell B2.

Next you need to determine the time at which the larger star begins to eclipse the smaller. By determining the interval of time that elapses from the beginning of this eclipse to maximum occlusion, then the interval from the beginning to the end of maximum occlusion, we may determine the radii of both stars.

The data point that indicates the beginning of the eclipse is shown near the top of Figure 5. As before you may estimate this value by just examining the graph but the easiest method for determining it is to place the cursor near the point and let Excel display it for you. This time the parenthetic values are: (days, normalized intensity) and you are interested in the former. Record this value in cell B3 of the "System Parameters" sheet.

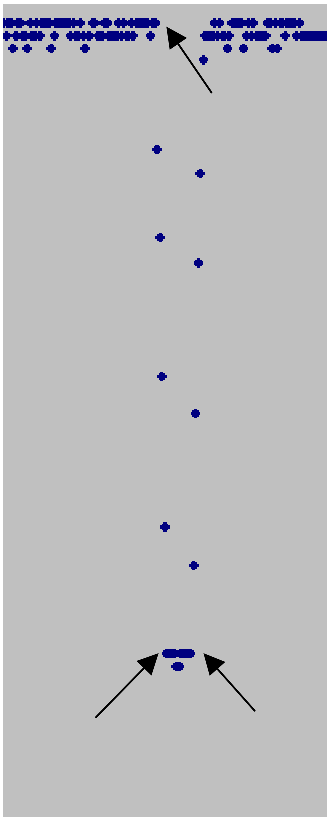


Figure 5.

Next record the values indicated at the bottom left and right of Figure 5 in cells B4 and B5 of the "System Parameters" sheet. The spreadsheet uses these values (along with the orbital data acquired earlier) to compute the mass and radius of each star, the semimajor axis of each star's orbit and the semimajor axis of the entire system. These are displayed in the labeled cells in columns H – P of the "System Parameters" sheet.

Spectral Data

Next we'll examine the spectral data acquired from JR-M 60. This may be obtained by clicking on the "Radiation Curve" tab, which displays a graph of intensity as a function of wavelength for each star in a manner similar to that shown in Figure 6 below. The axis units of this plot are Flux (in Watts · $m^{-2} \cdot nm^{-1}$) vs. wavelength (nm). Each individual data point represents the intensity of the star at that particular wavelength. The visible spectrum is shown along the horizontal axis for reference.

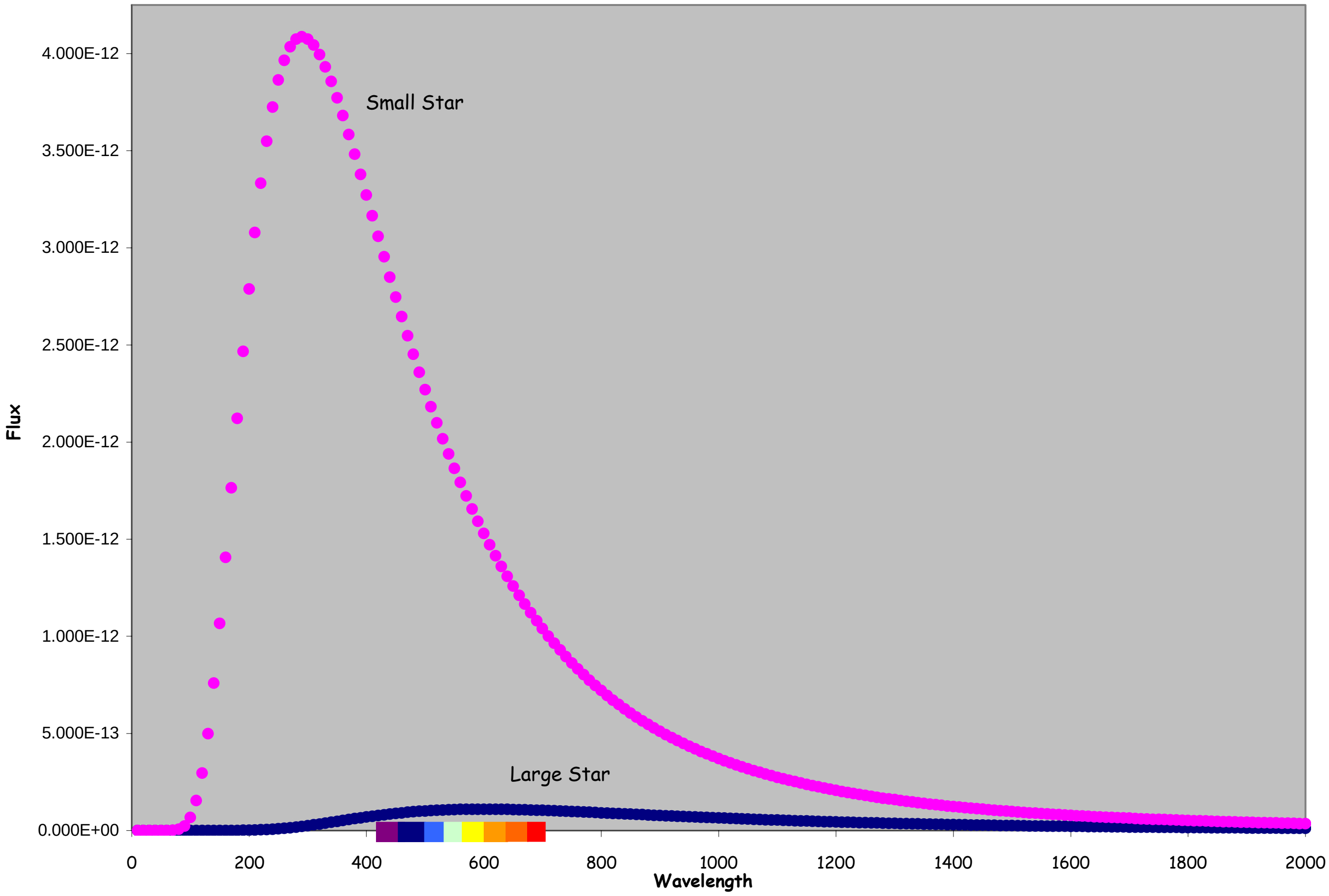


Figure 6. Spectral data for JR-M 60

Based on the spectral data for JR-M 60 what is the color of the larger star? What about the smaller star? Which is hotter?

As you should be aware from the lecture, all stars behave as *blackbodies*. This means, among other things, that the surface temperature of any star may be computed by determining the wavelength of maximum intensity in the star's radiation curve – a relationship known as Wien's Law. We are using a particular formulation of Wien's Law designed to make these calculations simple:

$$T_k \approx \frac{3 \times 10^6}{\lambda_{\max}}$$

Your next task is to determine the wavelength, in nanometers, of each star's peak intensity from the radiation curves. Record these values in cells R1 and R2 of the "System Parameters" sheet. Excel computes the surface temperature of each star and displays these values in cells T1 and T2. Do the computed temperatures match your estimates for the relative temperatures of both stars?

The *luminosity* or total radiated power (flux) of any spherical blackbody may be computed with the Stefan-Boltzmann Law:

$$L = \sigma T^4 4\pi R^2$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$, *T* is the temperature of the star (in Kelvin) and *R* is it's radius. Since we have already computed each of the needed values on the right side of the equation above we may determine the individual luminosities of each star. Excel computes these values and displays them in cells V1 and V2.

The measured flux of a system is the intensity as measured by an observer on Earth. Because stars radiate their energy away in all directions most of it does not come toward us. The farther away the star, the less energy we intercept. Our sun, for instance radiates about 4×10^{26} Watts of total power but at our distance from it, 150 million kilometers, a satellite orbiting the earth would intercept only about 1400 Watts of this power for every square meter of detector area.

We will use a geometric relationship known as the inverse square law to compare the amount of power (flux) we receive from JR-M 60 to its actual radiated power. This will allow us to determine the distance to the system. The inverse square law is:

$$R = \sqrt{\frac{L}{4\pi I}}$$

where *L* is the total radiated power (flux) of the system (computed with Stefan-Boltzmann), *I* is the total measured flux (acquired from the spectral distribution) and *R* is the distance to the system.

In order to determine the total measured flux of each star across its entire spectrum, the *bolometric flux*, we must sum the measured flux at each wavelength across all of the wavelengths under each curve. The exact method by which this is done is fairly complex and a detailed discussion of it would add little insight to the current discussion. The spreadsheet does the calculation for you and enters the value in column V3. This value contains the contributions to the bolometric flux of the entire system from both stars and represents the power of the system as measured here on Earth.

The spreadsheet uses the inverse square law to estimate the distance to JR-M 60, in light years, based on the values in cells V1, V2 and V3 and displays the value in cell Z1.

Classification of the Stars in JR-M 60

The last step in this exercise is to determine the spectral class of each star in the JR-M 60 system and plot their positions a Hertzsprung-Russell diagram.

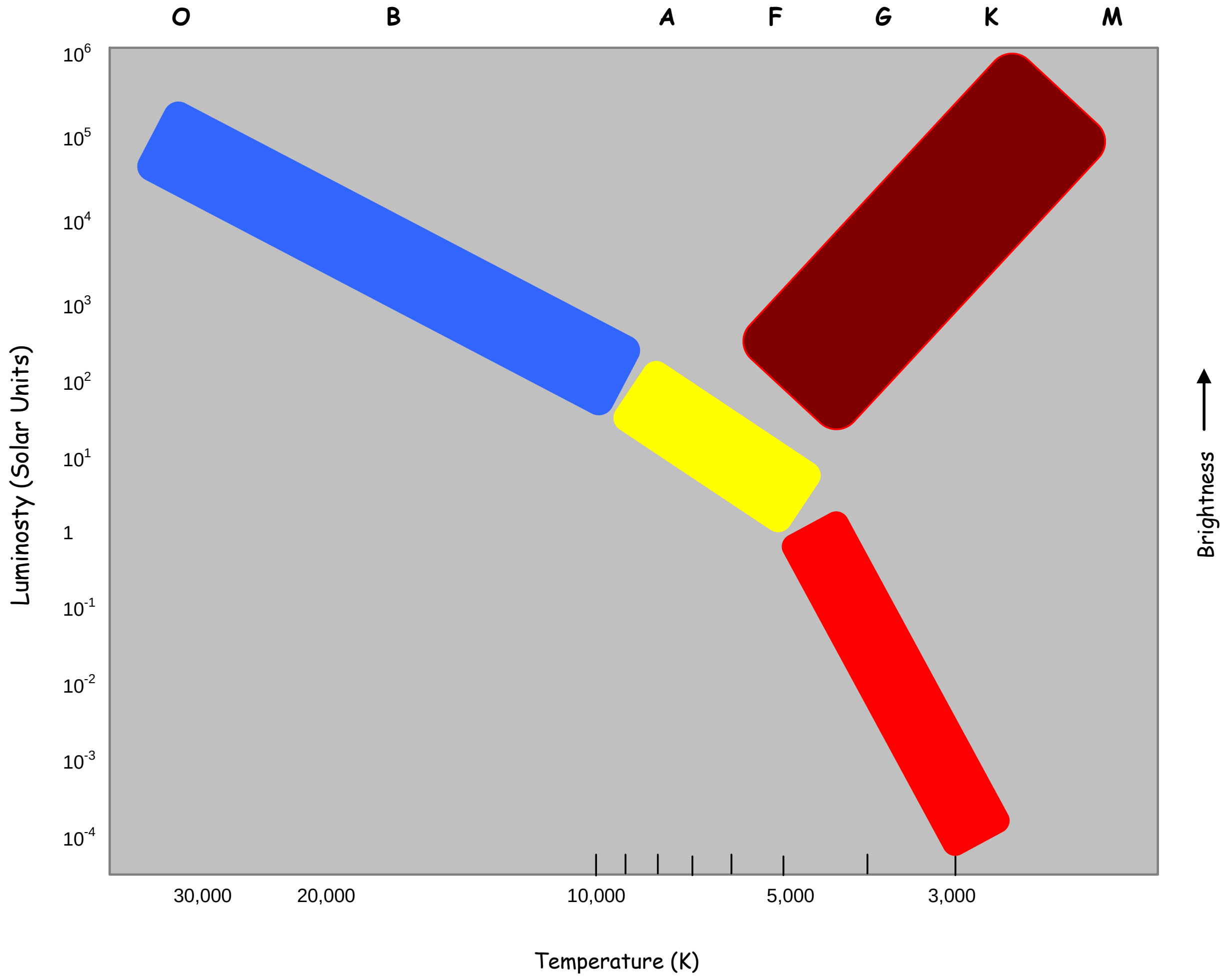


Figure 7. Hertzsprung-Russell Diagram.

The temperature (K) of each star is given in cells T1 and T2 of the "System Parameters" spreadsheet and the luminosities, in solar units, in cells X1 and X2. Based on these values place each star on the H-R diagram in Figure 7. What types of stars inhabit the system JR-M 60? Note: before attempting to answer this question you may want to refer to our earlier exercise on H-R diagrams.

Many thanks to Kirsten Bernabee, Casey White, Sam Plesner, Dayton Syme, Emily Schreiner, Eric Ivie, Cody Womack and Anthony Andrews who rendered invaluable assistance in crafting and troubleshooting this lab.

Exercises

1. What is a radial velocity?

2. What is a redshift?

3. What about the orbit of our system made it convenient to study?

4. What is the difference between total radiated flux and total measured flux?

5. What three relationships are used to relate the temperature of a star to its radiated power to its measured power to its distance from us?

6. What are the masses, radii, temperatures, spectral classes, color and radiated power of the stars in JR-M 60? What types of stars are these? What is the approximate distance, in light years to this system?