The Orbit of the Moon

Objective: To measure the eccentricity of the Moon's orbit.

Equipment: Drafting compass, protractor, vernier caliper, PC with spreadsheet.

Discussion: The Moon's orbit is nearly, but not quite, circular. The early Greeks were aware of a small but discernable change in the Moon's apparent diameter throughout the month. This apparent change in diameter is due to a slight variation in the Moon's distance from Earth due to the elliptical nature of its orbit (just as the planets circle the sun in elliptical orbits, the moon orbits the earth along an elliptical path). We can compute the *eccentricity* (deviation from a circular orbit) for the Moon by measuring the apparent change in its diameter throughout a monthly lunar cycle. Consider the Figure below. D is the apparent diameter of the Moon, d the distance to the Moon, and θ the angle subtended by the Moon.

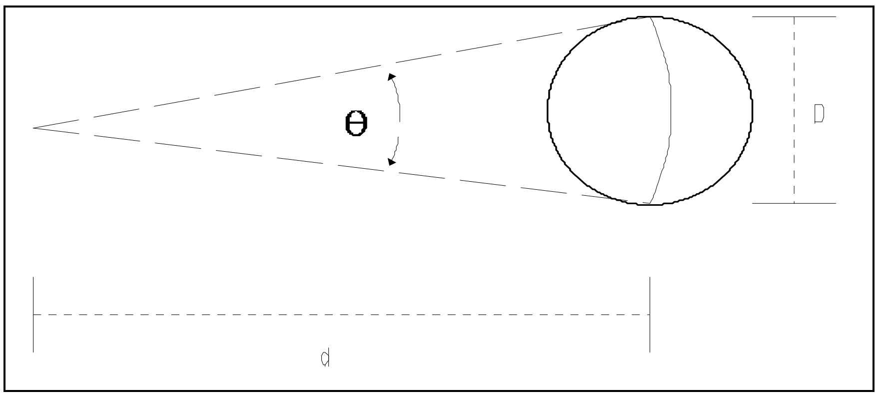

Figure 1.

Given the relationship:

$$d = \frac{D}{\theta}$$

one may compute the radius of the Moon's orbit at various points throughout the month by measuring the apparent diameter of the Moon from a sequence of photographs.

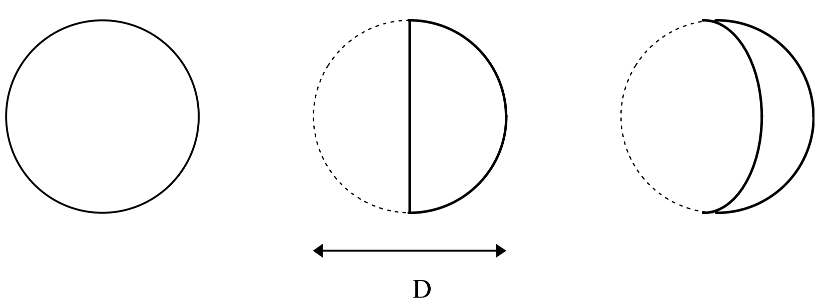

Figure 2. Measuring the diameter of the moon with vernier calipers.

Procedure

A sequence of Moon photographs is shown in Figure 3. Use the vernier calipers to measure at least two diameters from each photograph and find the average value (D_{ave}) of your measurements. We are using a peculiar but convenient method of determining the angle θ. A calibration bar is shown at the top of Figure 3. Measure the length of the calibration bar and divide the value of D_{ave} from each photograph by the length of the calibration bar. Multiply this result by .01 radians to obtain the angle θ subtended by the Moon for each photograph.

The images in Figure 3 are numbered 1 - 12 in rows from left to right. After measuring each image and making the necessary computations, you will need to make a plot of your data in order to determine the eccentricity of the Moon's orbit. It is not necessary to know anything about the actual distances involved, only the relative distance between Earth and the Moon at various times during the month. Hence, any convenient scale will suffice. By cleverly choosing the appropriate scaling factor one may generate a plot that will fit on a page in you lab notebook. A scale factor of 0.08 will work nicely. Divide each value of θ into 0.08.

Use a fresh page in you lab notebook and make a dot at its center. Draw a line through this dot perpendicular to the long axis of the page. Use a protractor and straight edge to construct straight lines from the dot to the edge of the paper at the azimuthal angles given in Table 1. Begin with the smallest angle (5^0) and work up to the largest angle (336^0). Use the vernier caliper to measure the scaled distance from the dot at the center of the plot for each azimuthal angle. In this manner you will construct a plot of the Earth (central point) Moon (radial points) system at 12 azimuthal angles during a lunar cycle.

Use a compass centered on the central point (Earth) to construct the largest circle that just fits inside the points (r_i) and the smallest circle that just fits outside the points (r_o). Measure the radii of these two circles. The eccentricity (*e*) of the orbit is the difference of these radii divided by their sum, or:

$$e = \frac{r_o - r_i}{r_o + r_i}$$

The eccentricity of an ellipse is between zero and one, i.e., $0 < e < 1$. So you should arrive at a number between 0 and 1 (if you have done everything right). The actual value of *e* is quite small (<< 1). Why do you suppose that the value of *e* is so small?

The point at which the Earth and Moon are closest to each other is known as perigee, and the point at which they are farthest is known as apogee. With this in mind, what do the radii of the inner and outer circles represent in your Earth-Moon plot?

This exercise is straightforward and not too difficult. If you are careful with your measurements you should arrive at a value for that is very close to the actual value. The most common pitfalls encountered in this exercise are incorrect or poor measurements of the Moon's diameter and/or errors in the calculations. Try practicing with the example data provided in Table 1 before collecting your own data.

Table 1. An Example[1] Data Sheet.

Length of Calibration bar[2] (C) 6cm____

Image #	Diameters	D_{ave}	Subtended Angle ($\theta = D/C \times .01$)	Scaled Distance (cm) ($d = .08/\theta$)	Longitude φ
1					270^0
2					283^0
3					309^0
4					336^0
5					5^0
6					27^0
7					57^0
8					87^0
9					127^0
10					151^0
11	4.71, 4.75cm	4.73cm	4.73/6 x .01 = .0079	10.15 cm	176^0
12					212^0

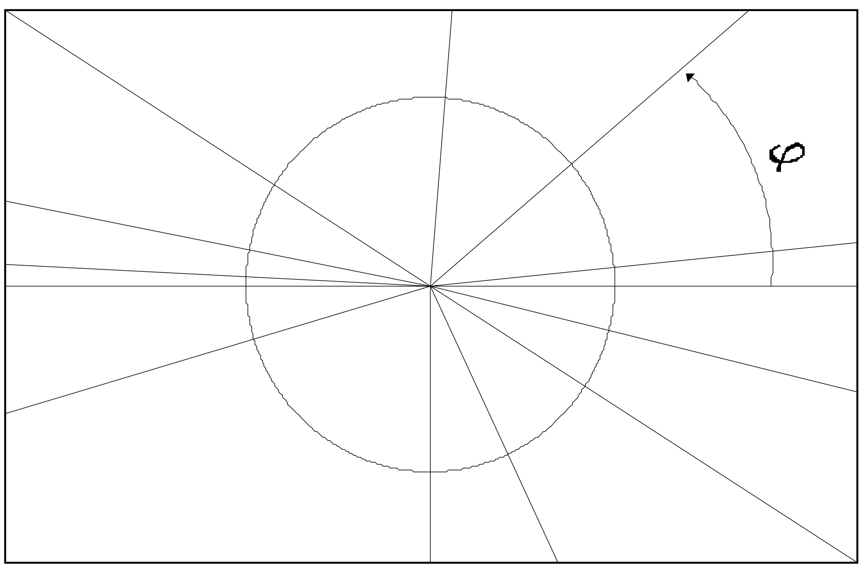

Figure 4. Example plot of the Earth-Moon System. Azimuthal angles φ are indicated by radial lines. The inner and outer radii are indicated by dashed lines. Data points are not shown.

Note: A spreadsheet (astmoon.xls) has been prepared to assist you with some of the repetitive calculations in this lab. Your lab instructor will show you how to access and use this spreadsheet

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[1] The values used are for example only.

[2] This is *not* the correct value for the length of the calibration bar. It is used as an example only.

Questions

1. To what is the apparent difference in the size of the moon during its 28 day cycle attributable?

2. What do the inner and outer circles on your moon plot represent?

3. Why is it necessary to take at least two diameter measurements from the Moon photographs?

4. How do vernier calipers differ from a ruler?

5. A satellite of a planet has a perigee distance of 350,000 km and an apogee distance of 375,000 km. What is the eccentricity of its orbit?

6. Why was the scaling factor of 0.08 used in this exercise?