Laboratory 9 - Calculation of the Age of the Universe

Materials Used: Spectral photos, vernier calipers, Excel spreadsheet.

Objectives: To study redshifts and recession velocity; to determine the Hubble Parameter; to determine the age of the Universe with a Hubble Plot.

Discussion: In the late 1920's astronomer Edwin Hubble discovered that galaxies were receding, i.e., from a reference frame in any galaxy all distant galaxies are moving away. A decade later Hubble and M. L. Humanson demonstrated that the more distant a galaxy the faster it is moving away with the discovery that distant galaxies have spectral lines that are strongly shifted toward the red (longer wavelength) end of the spectrum. This shifting of spectral lines is referred to as the *law of redshifts*. Hubble produced a plot of the recession velocity of galaxies vs. their distance from the Milky Way. This plot, known as a *Hubble Plot*, revealed that the more distant a galaxy the higher its *recession velocity* and that the increase in recession velocity is directly proportional to distance. This was the evidence needed to herald the end of the *Steady State* theory of the Universe and usher in the concept of an expanding Universe that began with the Big Bang.

In this procedure you will determine the Hubble Parameter, H_o, which is used to estimate the rate of expansion of the Universe, and produce a Hubble Plot to estimate the age of the universe. Many careful measurements and numerous calculations are required to determine the Hubble parameter and produce a Hubble plot. In this procedure you will take measurements and perform calculations just like any astronomer would to make the same determinations. A spreadsheet, Hubble_Plot.xls, has been created to aid you with the calculations required. Accurate measurements are crucial in obtaining a good value for the Hubble Parameter and a reasonable estimate for the age of the Universe. Take your time with each measurement and be as meticulous as you can.

Procedure

Calibrating the Spectrometer

Figure 1 contains images of five galaxies on the left with spectral data for each galaxy (and reference spectra above and below) on the right. The images of all five galaxies were acquired at the same magnification so the smaller the image of the galaxy the farther away it is. The galaxies are similar in shape and for this exercise we will assume that they all have a diameter of about 100,000 light years. The spectra of all five galaxies were obtained with the same diffraction grating. The reference spectra are included in order to calibrate the grating, which is accomplished by determining its *dispersion*. The reference spectral lines are labeled *a* through *g*, and their corresponding wavelengths (in angstroms) are:

a - 3889Å b - 3965Å c - 4026Å d - 4144Å e - 4472Å f - 4713Å g - 5016 Å

You will begin by measuring the distance between the "a" spectral line and all of the others. Observe that the reference spectra are identical in each image so it makes no difference which one you choose for your measurements. Accuracy, however, is very important. Use vernier calipers to measure the distance between the centers of the respective spectral lines (the *a* – *a* displacement will, of course, be zero). Measure each of the displacements and record them in the appropriate cells in the spreadsheet. You must convert your measurements from centimeters to millimeters. This is accomplished by shifting the decimal point in each measurement one position to the right. A reading of 5.25 cm, for instance, is 52.5 mm.

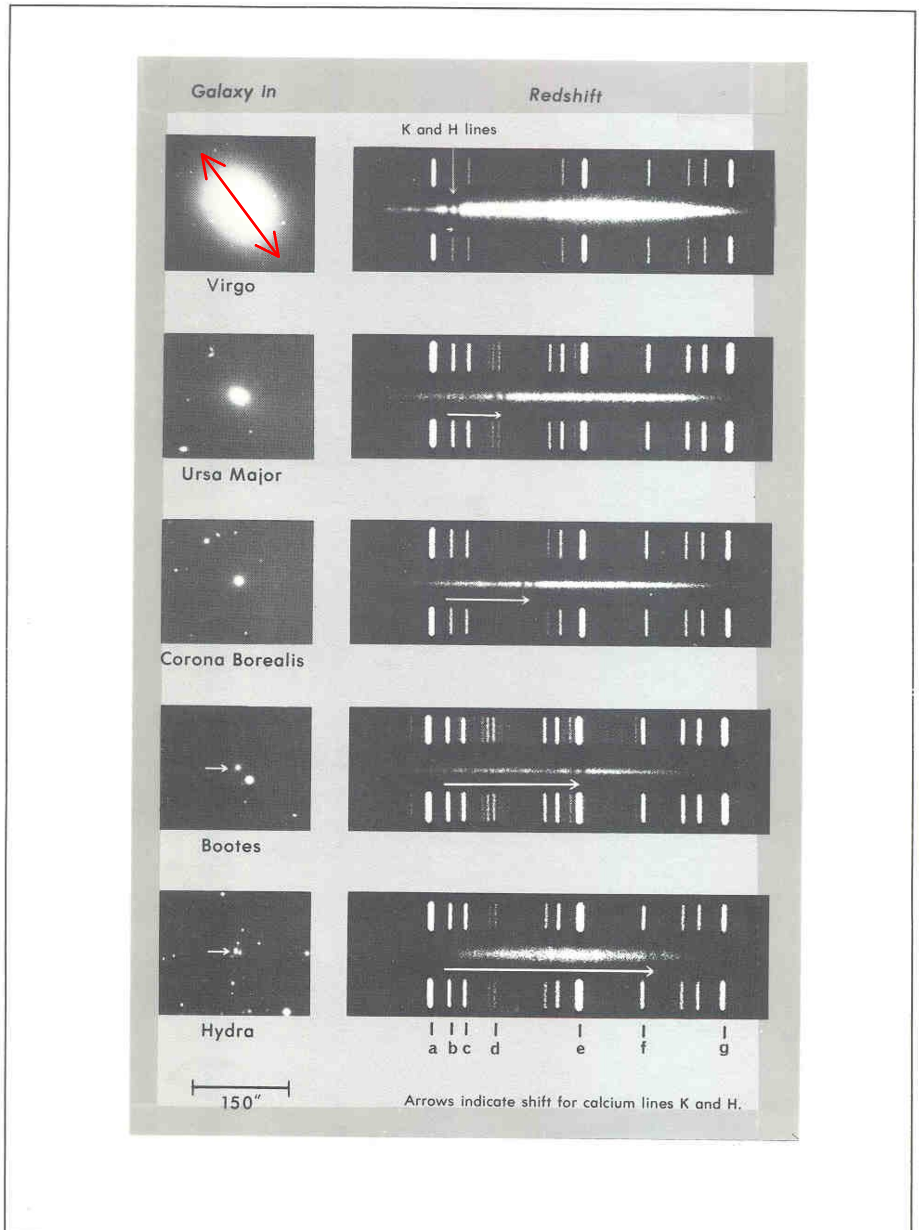

Figure 1. Images and spectra of five galaxies.

Once you have entered each of the line displacements for the reference spectrum the spreadsheet will produce a plot, similar to Figure 2 below, of wavelength vs. displacement for the grating. This is known as the dispersion curve. You may access this plot by clicking on the **calibration** tab at the bottom of the worksheet. Note that this "curve" is actually a straight line and that the slope and y-intercept information are displayed at the upper right of the graph. The slope is the value multiplied by x in the line equation. Click the **Data** tab on the worksheet to return to the spreadsheet and enter the slope (the grating dispersion) into the appropriate cell of the spreadsheet (G1).

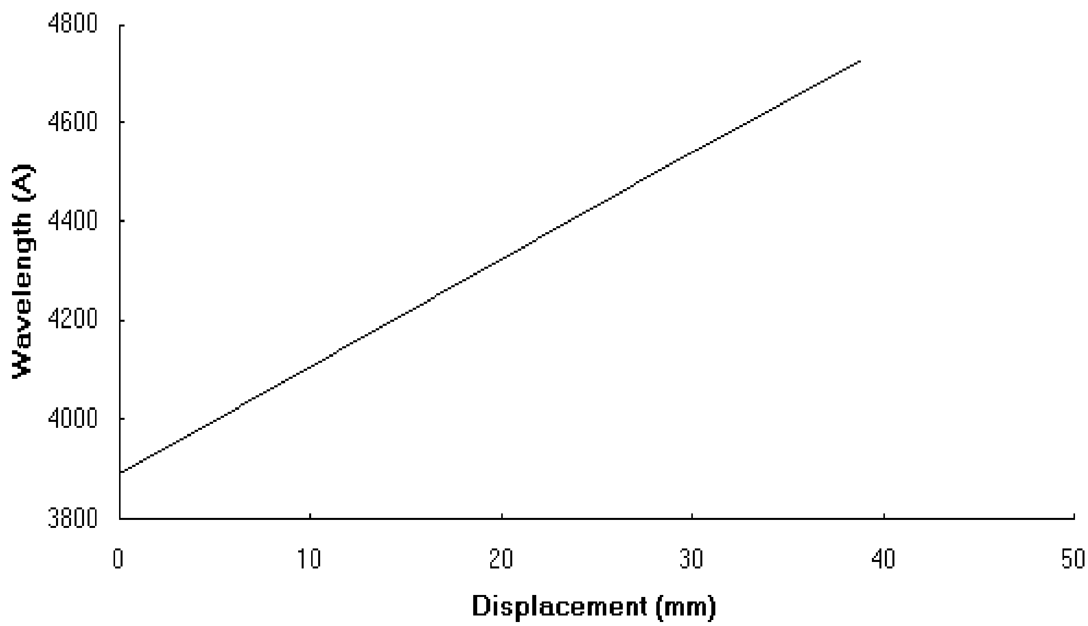

Figure 2. Grating dispersion curve.

Measurement of K and H Calcium Absorption Line Redshifts

The next step is to measure the displacement of the K and H calcium absorption lines in each galaxy. Recall that all of the spectral lines in a receding galaxy are displaced toward the red end of the spectrum by an amount proportional to the speed with which the galaxy is receding. By measuring the wavelength shift of the K and H lines in each galaxy in Figure 1 we can determine the recession velocity of each galaxy.

Refer to Figure 1. On the left, under the heading *Galaxy in* (this refers to the area of the sky where these galaxies are found), are five images of galaxies. On the right, under the heading *Redshift*, are five spectral line photos composed of a white streak (the actual line spectra of the galaxy) and reference spectra above and below the line spectra. The reference spectra mark the positions the indicated spectral lines would occupy in a stationary galaxy.

Examine the galaxy in Virgo line spectra closely. You will see two vertical dark lines in the white streak quite close together marked by a horizontal arrow. These are the calcium K and H absorption lines. The K line is the first line, the H line is the second. Examine each spectrum noting the changing position of these lines from image to image.

Use vernier calipers to measure from the center of the "e" spectral line to both the K and H lines in each spectrum. The e line is chosen mostly for convenience, but also to minimize any systematic errors that might crop up in acquiring your measurements. The e – K and e – H displacements are to the left of the g line in most of the measurements. Where this is so, record these measurements as negative numbers, in millimeters.

The spreadsheet takes each of your e – K and e – H measurements and multiplies them by the grating dispersion (Å/mm) you acquired from the calibration curve to compute a calibrated wavelength shift, in angstroms, for the K and H line in each galaxy from the reference e spectral line. The spreadsheet then adds the wavelength of the reference e line (4471.5Å) to each measurement to compute the actual redshifted wavelength of the K and H lines in each galaxy ($K\lambda$, $H\lambda$). These wavelengths represent the redshift of each galaxy.

Recession Velocity

To find the galaxy's recession velocity we need to know how much the K and H lines in each galaxy are shifted from a set of K and H lines that are stationary in the frame of reference of the observer. The values for the unshifted K line and H lines are 3933.7Å and 3968.5Å, respectively, in a stationary frame. The spreadsheet will calculate a *wavelength difference* ($\Delta\lambda$) for each K and H line, i.e., the difference between the shifted and unshifted wavelengths

The next calculation is the fractional wavelength difference ($\Delta\lambda/\lambda$) for each line. Note that the result of this calculation is a unitless number. Fractional wavelength difference, which is always less than 1.0, is the recession velocity of each galaxy expressed in % of the speed of light. The speed of light (c) has a value of about 3×10^8 meters per second. A galaxy with an average fractional wavelength difference of .75 would be moving at 75% of the speed of light away from us.

The K and H recession velocities are averaged in column M and converted to a recession velocity for each galaxy, in km/sec, in column O.

What is implied by the fact that the average fractional wavelength difference for each galaxy is always less than 1.0?

A set of sample calculations for the galaxy in Hydra has been provided below to illustrate the thread of calculations:

Hydra K line: 4726.2Å   Unshifted K line: 3933.7Å
Hydra H line: 4771.1Å   Unshifted H line: 3968.5Å

Wavelength Difference ($\Delta\lambda$):

4726.2Å - 3933.7Å = **792.5Å** (K)
4771.1Å - 3968.5Å = **802.6Å** (H)

Fractional Wavelength Difference ($F\Delta\lambda$):

792.5Å ÷ 3933.7Å = **0.201Å** (K)
802.6Å ÷ 3968.5Å = **0.202Å** (H)

Average $F\Delta\lambda$: (0.201 + 0.202) ÷ 2 = **0.202** = **20.2%** of *c*. The recession velocity for Hydra is roughly 0.2 or 20% of *c*.

Your spreadsheet displays the values for the recession velocities of all of the galaxies in Figure 1. You should notice a pattern in the measurements, which is that the recession velocities increase as the distance to the galaxies increases.

Calculation of Distances to the Galaxies

We are now ready to calculate distances to the galaxies in Figure 1. Distances are the second set of values we need to determine the Hubble Parameter and to construct a Hubble Plot.

We must first arrive at a calibrated value for the size of the images of each galaxy just as we did for their spectra. The reference line at the bottom left of Figure 1 represents 150 arc seconds. Use vernier calipers to measure the length of this line (in millimeters), divide it into 150 arcseconds, and record the result in cell R1 of the spreadsheet. This is the photographic dispersion in arcseconds/mm.

Next use vernier calipers to measure the average diameter of each galaxy from the images on the left of Figure 1. Take two measurements for each galaxy and enter them in the spreadsheet in the columns labeled D_1 and D_2. Be sure to measure each galaxy along its long and short dimension (where applicable) and be sure to include the halo surrounding each galaxy (an arrow has been included in the first image to indicate the approximate edges of the halo). It's difficult to determine exactly where the halo ends in some of the photographs so just make your best careful estimate. The spreadsheet will compute the average of your two measurements for each galaxy.

The spreadsheet computes the angular size of each galaxy by multiplying the value you acquired for photographic dispersion (in cell R1) by the average diameter of each galaxy. Note that the angular size is another unitless number, a radian. The distance to each galaxy is then determined by dividing the angular size into 100,000 LY (our assumption of the diameter of each galaxy) and is given in LY. These distances are converted from LY to Megaparsecs in column U.

The spreadsheet produces a plot of recession velocity (km/sec) vs. distance (Mpc), a Hubble Plot, based on your measurements. You may view this plot by clicking the **Hubble Parameter** tab at the bottom of the worksheet. This plot is a straight line and the slope is displayed at the upper right of the plot. The slope of the Hubble Plot is the Hubble parameter, H_o. Click the **Data** tab at the bottom of the page to go back to the spreadsheet and enter this value in cell F11. The current best value for H_o is about 72 km/sec/Mpc ±10%.

The spreadsheet also creates a graph of distance vs. recession velocity (% of the speed of light) like the one in Figure 3 below. You may access this plot by clicking the **Age of the Universe** tab at the bottom of the worksheet. Note that this plot is very similar to the Hubble Plot.

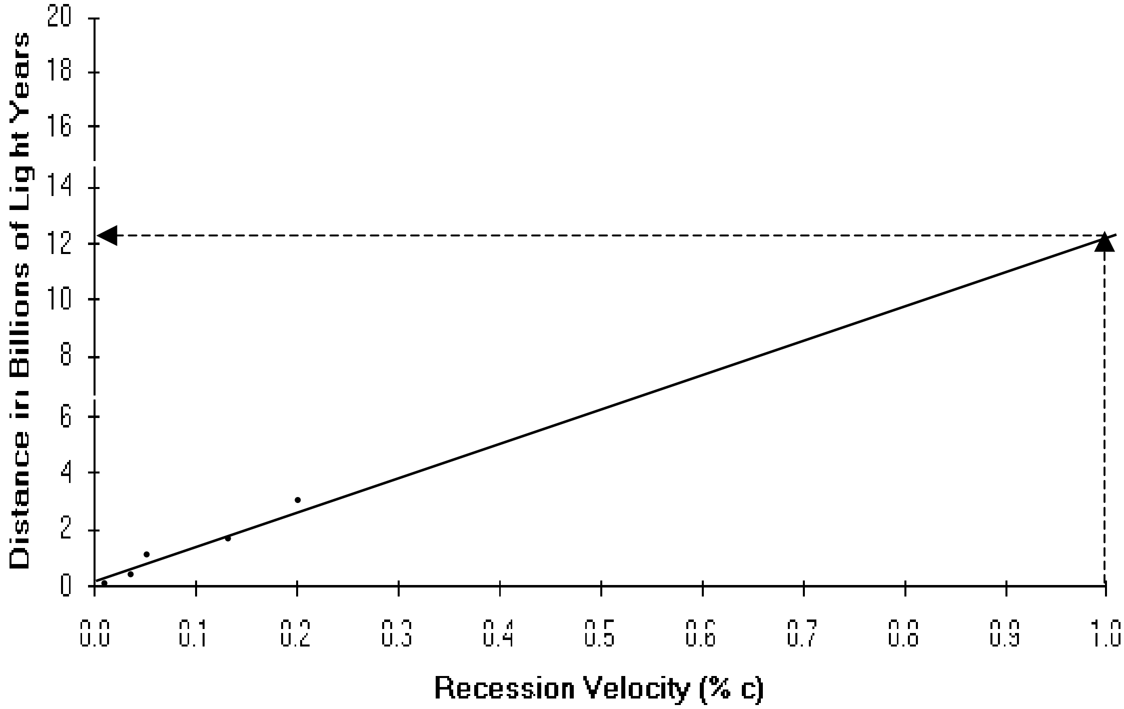

Figure 1. The age of the Universe.

Calculation of the Age of the Universe

To estimate the age of the universe from the Hubble Plot one simply draws a vertical line from 1.0 c on the recession velocity axis to the straight line fit of the data points, then a horizontal line from this point to the distance (vertical) axis. The value for the age of the Universe is read from the vertical axis. The spreadsheet does this for you and displays the result in cell F12.

The age of the Universe is given in years because we are assuming that light from the farthest visible objects has traveled *x* billions of years at the speed of light - thus giving units of years instead of light years. We also assume that the earliest observable material in the universe will be moving away with a velocity of *c*. Thus, the total elapsed time since the creation of the material is the total distance traveled divided by the speed of light.

The current best estimate of the age of the universe from the Hubble parameter is between 12 and 15 billion years.

Exercises

1. Why is the recession velocity of a galaxy always less than the speed of light?

2. What is a redshift?

3. What spectral lines are the measurements in this exercise based on?

4. Why are the K and H spectral lines in different places in the spectra of the different galaxies?

5. A galaxy is observed to have a K line at 4740Å and an H line at 4762Å. Compute its recession velocity.

6. Why was it necessary to determine the grating dispersion in order to begin this procedure?