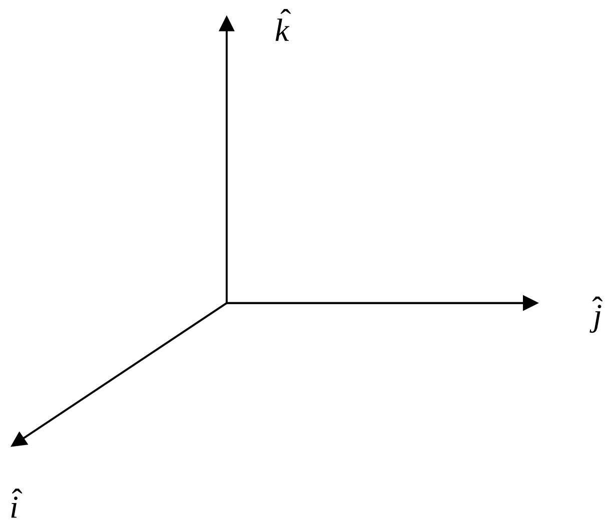
Vector Operations in R^3

Given: $\vec{A} = 2\hat{i} + 3\hat{j} + 0\hat{k}$, $\vec{B} = 2\hat{i} + 2\hat{j} + \hat{k}$

Find: $\vec{A} + \vec{B}$ $\quad \vec{A} \cdot \vec{B}$ $\quad \vec{A} - \vec{B}$ $\quad \vec{A} \times \vec{B}$

Recall:



Vector Addition: The vector sum of two vectors yields a resultant **vector** that points in the direction that the two individual vectors would if stacked "tip to tail" in R^3.

$$\vec{A} + \vec{B} = \left(2\hat{i} + 3\hat{j} + 0\hat{k}\right) + \left(2\hat{i} + 2\hat{j} + \hat{k}\right) = 4\hat{i} + 5\hat{j} + \hat{k} \therefore \vec{C} = 4\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{A} - \vec{B} = \left(2\hat{i} + 3\hat{j} + 0\hat{k}\right) - \left(2\hat{i} + 2\hat{j} + \hat{k}\right) = +\hat{j} - \hat{k} \therefore \vec{C} = +\hat{j} - \hat{k}$$

You should graph both of these operations to make sure that you understand how the resultant vector is related to the original vectors. Notice that this operation is not at all unlike the operations we've previously used to construct vectors from components. The difference here lies in the fact that the constituent vectors here do not have to lie along a coordinate axis. *Mathematical vector addition is merely an arithmetic method of lining up vectors "tip to tail."* Note that addition of vectors makes physical sense only of the vectors are alike, e.g., adding a displacement vector to another displacement vector to compute total displacement yields a physically meaningful quantity while adding a velocity vector to a displacement vector does not, even though both operations look the same mathematically.

Vector Multiplication: There are two distinct methods by which vectors may be multiplied. These methods are quite different both mathematically and in terms of their physical significance. One method of vector multiplication yields a **scalar** known as the *scalar product*. The other method of vector multiplication yields a **psuedovector** known as the *vector product* (http://terahertz.tn.tudelft.nl/kinsler/ircph/maze/vector.html).

The scalar product or *dot product* is a method of computing the magnitude of parallel components of non-perpendicular vectors. The dot product "picks off" the component of vector ***B*** that is parallel to vector ***A*** or vice versa. Scalar products have great significant in physics. *Work*, for instance is defined as the scalar product of *force* and *displacement*. Work, therefore, is not a vector quantity. The scalar product is physically meaningful only when carried out with dissimilar vectors, e.g., force and displacement (work). The scalar product of two velocity vectors, for instance, is not generally physically significant. The units of the scalar product will, therefore, be unique.

$$\vec{A}\cdot\vec{B} = \left(2\hat{i} + 3\hat{j} + 0\hat{k}\right)\cdot\left(2\hat{i} + 2\hat{j} + \hat{k}\right) = 4 + 6 + 0 = 10$$

Notice that the order of the scalar product operation is unimportant, i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

The vector product or *cross product* is a method of producing *axial* vectors, i.e., resultant vectors that lie in a plane perpendicular to the plane of the constituent vectors. These psuedo vectors are different from the polar vectors (such as force, velocity, acceleration, displacement, etc.) that you are used to so far. Axial vectors also have great significance in physics. Angular momentum and torque are examples of axial vectors. The cross product is physically meaningful only when carried out with dissimilar vectors, e.g., force and displacement (torque). The units of the vector product will, therefore, also be unique.

$$\vec{A}\times\vec{B}=\left(2\hat{i}+3\hat{j}+0\hat{k}\right)\times\left(2\hat{i}+2\hat{j}+\hat{k}\right)=\begin{array}{ccccc} i & j & k & i & j \\ 2 & 3 & 0 & 2 & 3 \\ 2 & 2 & 1 & 2 & 2 \end{array} = \left(3\hat{i}+0\hat{j}+4\hat{k}\right)-\left(6\hat{k}+0\hat{i}+2\hat{j}\right)$$

$$\vec{A}\times\vec{B}=3\hat{i}-2\hat{j}-2\hat{k}$$

Notice that the order of the operation is crucial here, i.e. $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$!