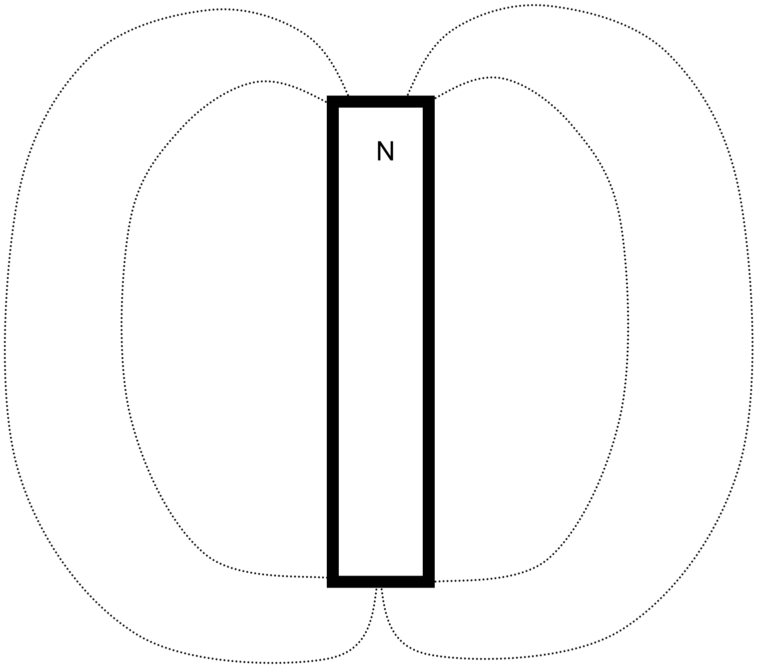
Magnetic Fields and Magnetic Forces

- Magnetic fields are created by charges in motion (currents)
- As electric fields are denoted with an ***E***, magnetic fields are denoted with a ***B***
- Electricity & magnetism are related (relativity is the bridge)
- Magnetic fields are force fields like electric fields but the magnetic force only acts on charges in motion
- All magnets have two “poles”, there are no magnetic monopoles
- The SI Unit of magnetic field strength is a Telsa (T). A 1 coulomb charge moving through a 1 Telsa $\vec{B}$ field with a velocity of 1 m/s ⊥ to the field experiences 1 Newton of force
- CGS unit of magnetism is a Gauss (G), $1T = 10^4G$



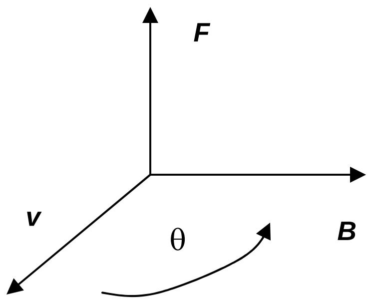
Bar Magnet

B field configuration

Properties of the *B* field

- Recall that ***E*** is defined as the electric force per unit charge acting on a test charge placed at that point in space
- ***B*** is defined similarly as the magnetic force that would be exerted on a moving charge *q* placed at a point in the field.
- The magnetic force is proportional to both charge and velocity
- The magnitude and direction of the magnetic force depend upon the velocity of the charged particle and the direction of the ***B*** field.
- When a charged particle moves parallel to magnetic field the magnetic force on the charge is zero due to the magnetic field
- When a charged particle moves along a path that is not parallel to the magnetic fields lines if feels a magnetic force due to the presence of the field. This force is proportional to the magnitude of the charge, the strength of the field and the velocity of the particle
- $\vec{F}_m = qv\vec{B}\sin\theta$

- For a positive charge moving in a magnetic field, the direction of the magnetic force is perpendicular to the plane containing ***v*** and ***B***, and may be determined via the right hand rule



right hand rule

- To apply the RHR, one points their fingers in the direction of the moving, positive charge, and curls them, towards the palm of the hand, in the direction of the magnetic field. The thumb points in the direction of the magnetic force.

Summary

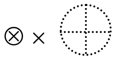
- The magnitude of the magnetic force on a moving charge is given by $\vec{F}_m = qv\vec{B}\sin\theta$
- The direction of the magnetic force is given by the right hand rule (for a positive charge)
- The magnetic force has a maximum value when $\theta = 90^0$ (the particle moves perpendicularly to the field lines)
- The magnetic force has a minimum value when $\theta = 0^0$ (the particle moves parallel to the field lines).
- The magnetic force *does no work* on a charge moving through a magnetic field. $W = (F_B)(s)\cos\theta$, and since the angle between the ***s*** and ***F***$_B$ vectors is always 90^0 the work done in displacing the charge is zero

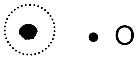
Comparison between the electric and magnetic forces

F$_E$	***F***$_B$
acts in the direction of the ***E*** field	acts perpendicular to the direction of the ***B*** field
depends on magnitude and polarity of the charge	depends on the magnitude, polarity and velocity of the charge
does work on charges turned loose in an electric field	does no work on charges moving in a magnetic field

Motion of charged particles in a *B* field

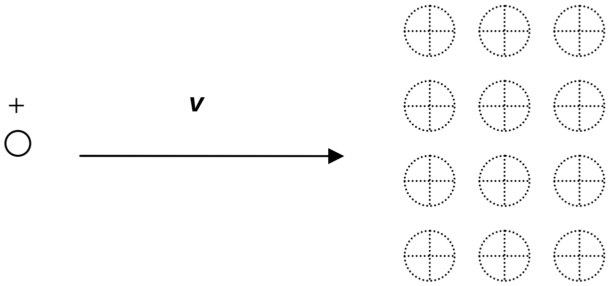
- Unless otherwise stated we will assume that the charged particle always enters a region of space containing a magnetic field <u>perpendicular</u> to the field lines and that the magnetic field is uniform and of large extent
- To represent any type of field lines in and out of the plane of a page we use the following symbols



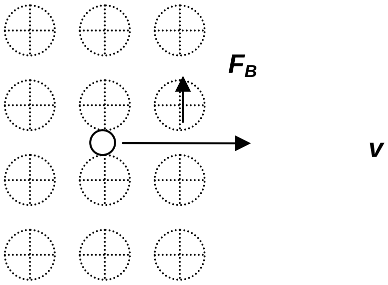


field into the page **field out of the page**

Consider a charge moving as shown below into a region of space containing the indicated uniform magnetic field

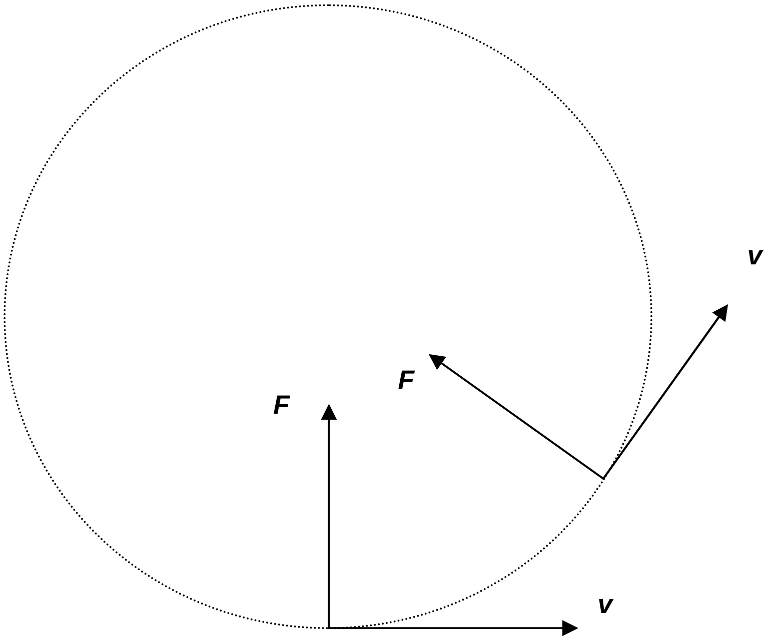


- This charge experiences a force of magnitude ***qvB*** in a direction given by the RHR. To apply the RHR one would point the fingers of their right hand in the direction of the velocity vector and curl their fingers towards their palm in the direction of the magnetic field lines which are into the plane of the page. In this case the thumb points up which is the direction of the force acting on the charge



- Since ***F*** and ***v*** are always perpendicular to each other, ***F*** and ***s*** are also always perpendicular to each other and the magnetic force does no work as it displaces the particle moving through the field

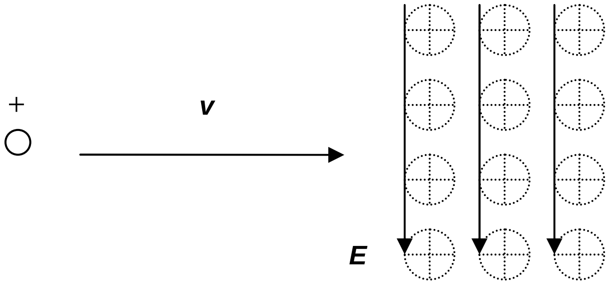
- If the magnitudes of ***v*** and ***F*** are constant and ***B*** is fixed, then the directions of **v** and ***F*** must be changing
- Since ***F*** is always perpendicular to ***v*** it is a centripetal force



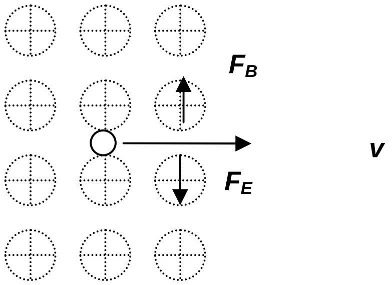
- Since $\vec{F} = m\vec{a}$ the acceleration is a centripetal acceleration of magnitude $a_c = \frac{v^2}{r}$
- Note that: $\vec{F} = q\vec{v}\vec{B} = m\vec{a} = m\frac{\vec{v}^2}{r} \rightarrow qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB}$, so the path of the charged particle moving in a uniform magnetic field is a function of velocity, charge and the field, as expected, and of mass which is required by Newton's second law.
- This assumes that the particle's initial velocity is ⊥ to the ***B*** field. If not the path is a helix rather than a circle.
- A magnetic field alone cannot alter the kinetic energy of a particle. A magnetic field can alter a particle's direction but not its speed
- A *mass spectrometer* is a device that uses magnetic fields to separate charged particles (ions) by mass

Lorentz Force Law

- Consider a charged particle moving through a region of space under the influence of uniform ***E*** and ***B*** fields. The total force acting on such a particle is the vector sum of the electric and magnetic forces or $\vec{F}_{tot} = q\vec{E} + q\vec{v}\vec{B}\sin\theta$. This is known as the Lorentz force law

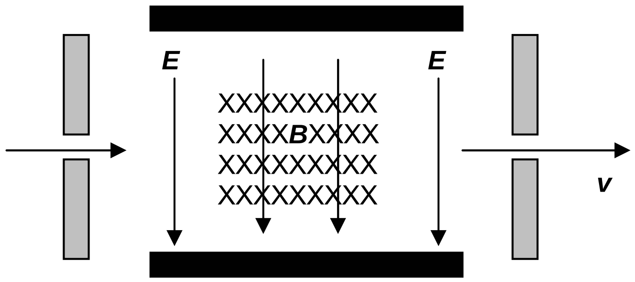


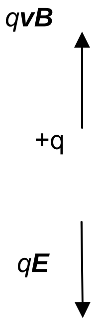
- The Lorentz force law is a vector equation but it is easier to use it to obtain the magnitude of the force and to determine the direction of the net force by the geometry of the situation.



- In the situation diagrammed above the electric and magnetic forces oppose each other. Their magnitudes may not necessarily be the same, but they are oppositely directed.
- Since, all things being equal, the magnetic force depends on velocity and the electric force does not the electric force will dominate at low velocities and the magnetic force will dominate at high velocities. It should be apparent that there is a velocity for which the electric and magnetic forces will be balanced for a given combination of field strengths: $F_B = F_E \rightarrow qE = qvB \rightarrow E = vB \because v = \frac{E}{B}$ and that at this velocity a particle of any charge or mass will move through the field undeflected (ignoring the effects of gravity)

Consider a crossed field velocity selector





A charged particle enters the velocity selector from the left through a device known as a *collimator* which is a slit designed to eliminate any particle that is not moving in the desired direction (along the +x axis). Inside the velocity selector there are two fields, a magnetic field directed into the plane of the page and an electric field directed downward. These fields produce forces on the charge in the directions shown at the right. Only particles of the desired velocity ***v*** will make it through the slit on the right side of the device

Example 1

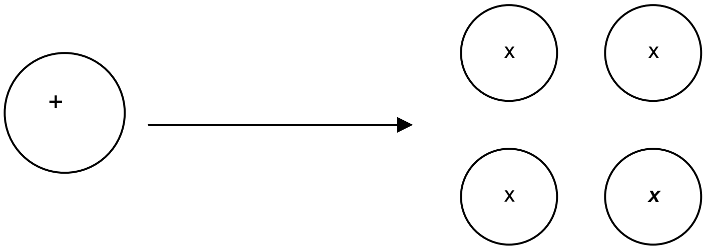
A velocity selector like that shown above contains fixed magnets capable of generating a magnetic field of ***B*** = .015T. What must the electric field strength be if 750eV protons are to be undeflected?

Recall that 1 electron volt = 1.602 × 10^{-19} joules. This means that a proton of this kinetic energy has a velocity of: $(750)1.602\times10^{-19}\,J=\frac{1}{2}m_p v^2 \;\because\; v=\sqrt{\frac{(2)(750)(1.602\times10^{-19}\,J)}{1.672\times10^{-27}\,kg}}$

$$E = vB \to E = \left(\sqrt{\frac{(2)(750)(1.602\times10^{-19}\,J)}{1.672\times10^{-27}\,kg}}\right)0.015T = 5.7\times10^{3}V\cdot m^{-1}$$

Example 2

A 3 keV proton enters a ***B*** field of 1.0 T as shown below How far must the proton travel to be deflected a total of 90^0?



A proton of this energy in *eV* has energy in Joules of

$$3\times10^{3}eV\times\frac{1.602\times10^{-19}J}{eV}=4.80\times10^{-16}J$$

and is traveling with a velocity of

$$4.80\times10^{-16}J=\frac{1}{2}mv^{2}\because v=\sqrt{\frac{(2)(4.80\times10^{-16}J)}{(1.67\times10^{-27}kg)}}=7.6\times10^{5}m\cdot s^{-1}$$

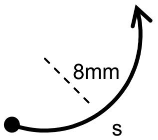
Next we must find the magnitude of the magnetic force acting on this proton

$$F=qvB=(1.602\times10^{-19}C)(7.6\times10^{5}m\cdot s^{-1})(1.0T)=1.2\times10^{-13}N$$

This is a *centripetal force* which causes the proton to move in a *circle* while in the field assuming that it enters the field perpendicularly. The radius of this circle is

$$qvB=m\frac{v^{2}}{r}\rightarrow r=\frac{mv}{qB}=7.9\times10^{-3}m$$

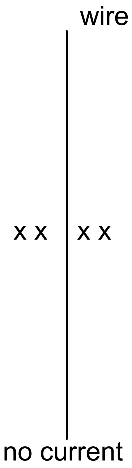
or approximately 8 mm. The final step is to find the arc length (*s*) along which the proton travels in being deflected 90^0.

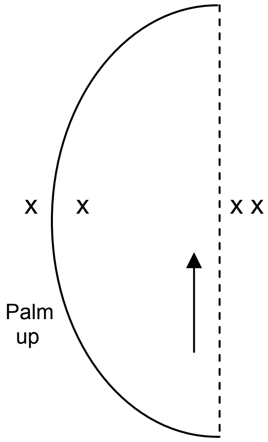


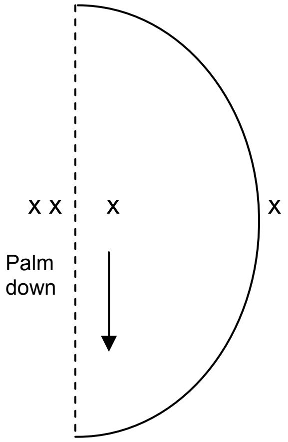
$$\begin{aligned}s&=r\theta\\&=(7.9\times10^{-3}m)(\frac{\pi}{2}rad)\\&=1.2\times10^{-2}m\end{aligned}$$

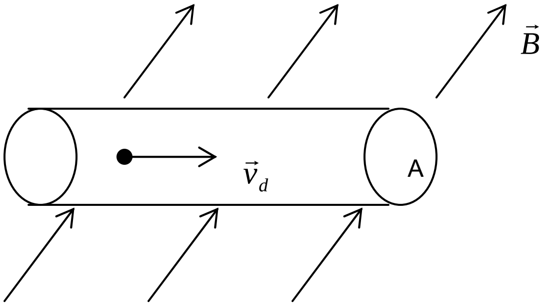
Magnetic force on a current carrying conductor

- Since current consists of moving charges, a current carrying wire also experiences a force when placed in a magnetic field.
- We consider current to consist of + charges
- Consider a wire fixed at each end in a magnetic field:
    - no current
    - a current running up the wire
    - a current running down the wire in









$\boldsymbol{v}_d$ = drift velocity

- Consider a segment of the wire of area *A*, length ℓ in an external field $\vec{B}$. The force on an individual charge moving through this segment of wire is $F = qv_d B\sin\theta$.

- To compute the total force on this wire segment we multiply the force on one charge by the number of charges moving through the segment. It can be shown that the number of charges moving through a segment of wire, length ℓ and area A is $nA\ell$

$$\vec{F}_{total} = \left(qv_d\vec{B}\right)nA\ell\sin\theta$$

- Recall from our model of current flowing through conductors that $I = nqv_dA$. This being the case the force may be expressed in terms of current rather than in terms of individual charges

$$\vec{F} = nqv_dAB\ell \rightarrow F = I\ell B\sin\theta$$

- This result applies only to straight wires and ***<u>ignores the field produced by the motion of the current itself.</u>***
- It may be shown the for a closed current carrying loop in a magnetic field the total magnetic force on the loop is always zero. This does not mean that no magnetic forces act on such a loop, just that their vector sum is zero. Can you prove this geometrically?

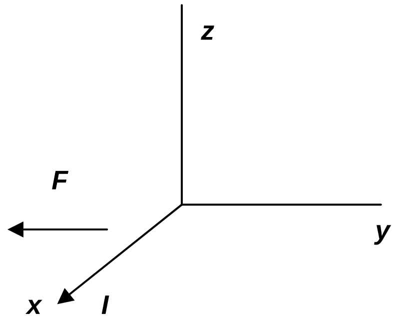
Example 3

A current of 15 amps flows along a wire as shown in the diagram below. If the force on the wire is 0.63 Newtons per meter in the direction shown, what is the strength of the magnetic field? In what direction does the magnetic field point?

$$F = I\ell B\sin\theta = I\ell B$$

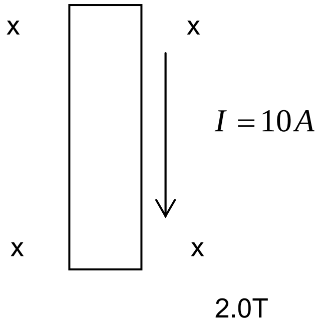
$$\frac{0.63N}{\ell} = IB \rightarrow 0.63N \cdot m^{-1} = (15amps)B \rightarrow B = 0.042T$$

The RHR shows that the ***B*** field must point along the +z axis.



Example 4

A conducting rod carries a current of 10A in a magnetic field of 2.0T as shown below. What force does the conductor experience?



RHR gives direction of force to the right

$$F = I\ell B\sin\theta = I\ell B$$

$$\frac{F}{\ell} = (10A)(2.0T) = 20A\frac{N}{A\cdot m} = 20N\cdot m^{-1}$$

If the rod is 1 meter long, how long does it take to accelerate it to 25 m/s if it has a mass of 1 kg?

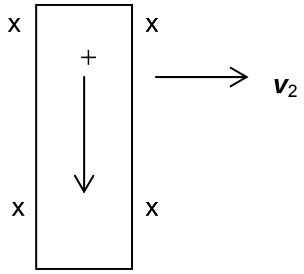
From dynamics:

$$\frac{F}{m} = a \rightarrow \frac{20N}{1kg} = 20m\cdot s^{-2}$$

From kinematics:

$$\frac{v}{a} = t = 1.25s$$

What keeps the rod in the previous example from accelerating (to infinity) with a small fixed field?



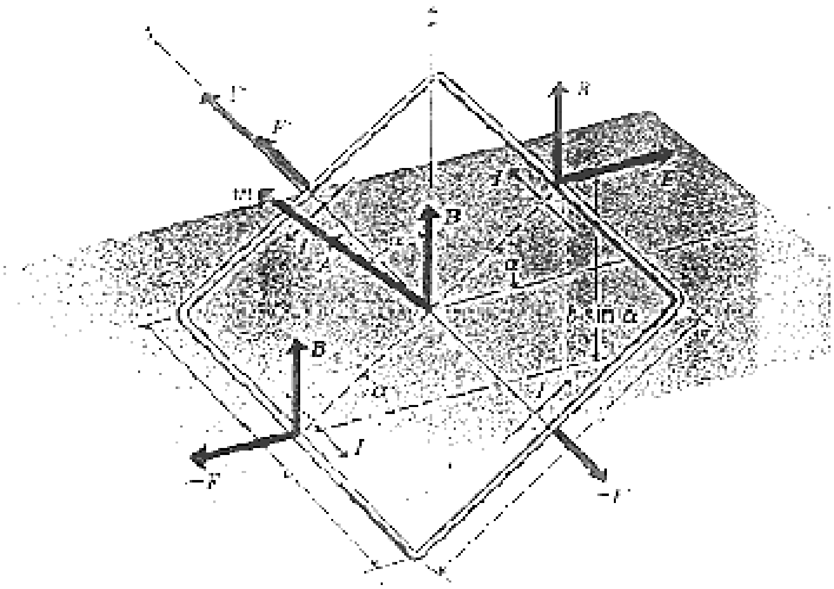
Notice that as the rod begins to move to the right the moving charges in the rod experience two components of motion, one down (the *in situ* motion) and one to the right.

The RHR establishes that the motion of the charges in the rod to the right results in a force (***F***$_2$) that acts upward on the charges moving downward in the rod of magnitude $F_2 = qv_2B\sin\theta$. This force counters the flow of current through the rod.

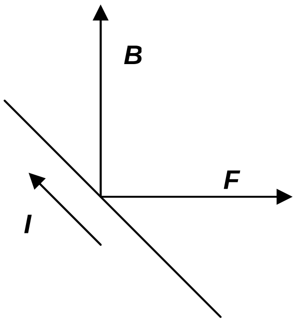
Force and torque on a current loop

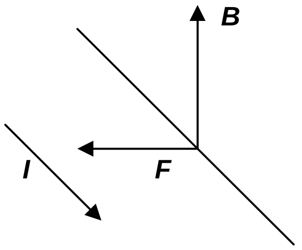
We can represent current carrying loops as a series of straight line segments. Even curved segments can be approximated as straight line segments. Recall that while the total force on any current carrying loop is zero, the force on any particular segment of such a loop is $F = I\ell B\sin\theta$

Consider:



Let’s examine the force on right side (ℓ = *a*) of the loop. The RHR gives the direction and the magnitude is $F = IaB\sin\theta = -IaB$



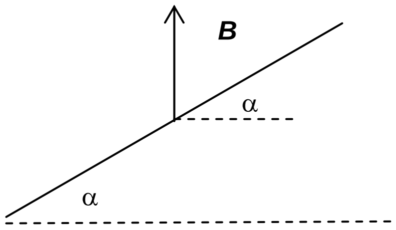


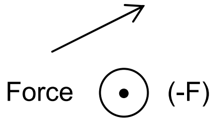
On the left side of the loop, *F* = *IaB* but in the opposite direction

It is clear from looking at both of the “a” sides of the loop that $-F_a + F_a = 0 = F_{a\,net}$

Now let’s consider the “b” sides of the loop (top and bottom where ℓ = b)

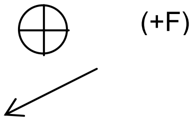
Side b view





$$F_b = IBb\sin(90° - \alpha)$$

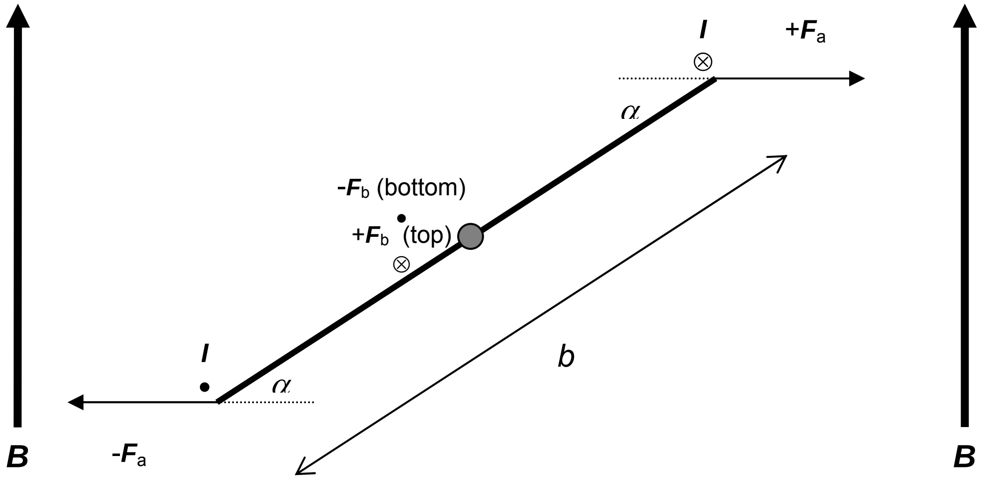
$$-F_b = IBb\cos\alpha$$



$$+F_b = IBb\cos\alpha$$

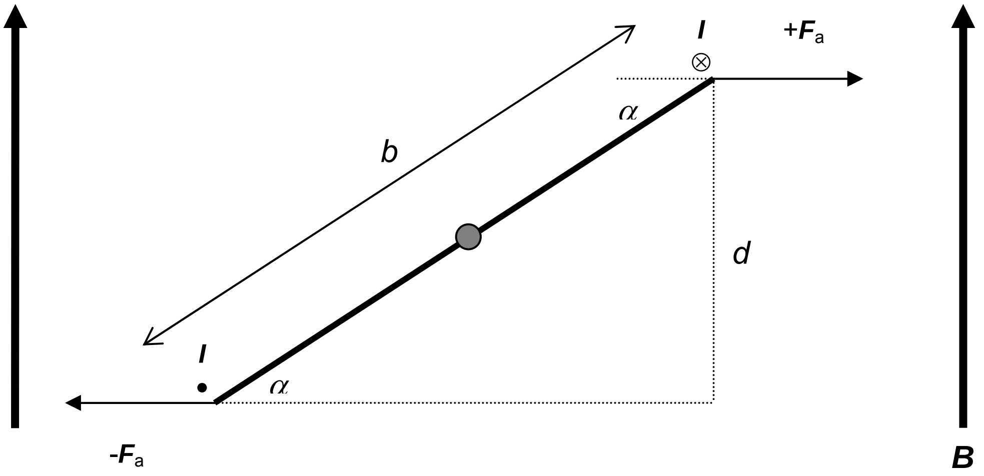
$$F_b + -F_b = 0 = F_{b\,net_b}$$

The first condition of equilibrium is met for both sides of the loop but the second condition of equilibrium is not because while **F**$_b$ and –**F**$_b$ lie along the same line **F**$_a$ and –**F**$_a$ do not.



Consider the sketch above. The perspective is the plane of the loop viewed from the bottom

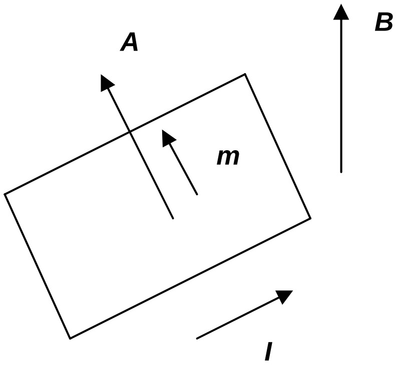
- The current is coming out of the page on the lower left side of the loop
- The current goes from lower left to upper right along the bottom side of the loop (view)
- The current goes into the page at the upper right side of the loop
- The current goes from upper right to lower left along the top of the loop (hidden)
- **F**$_a$ and –**F**$_a$ are equal and opposite everywhere along side *a* but *do not* share a common line of action
- **F**$_b$ and –**F**$_b$ are equal and opposite everywhere along side *b* and *do* share a common line of action
- Recall that when the second condition of equilibrium is not met that a torque is produced
- Γ = ***F*** × ***distance***
- **F**$_a$ and –**F**$_a$ produce a torque about the rotational axis of the loop



- The torque produced by F_a and $-F_a$ about the rotational axis of the loop is equal to either *both* forces multiplied by ½ *d* or either force multiplied by *d*. The torque will have a maximum value when the loop is vertical and a minimum value when the loop is horizontal (as *d* → 0). Be sure to verify this for yourself.
- Recalling that the force on either side *a* of the loop is ***IℓB*** or in this case ***IaB***

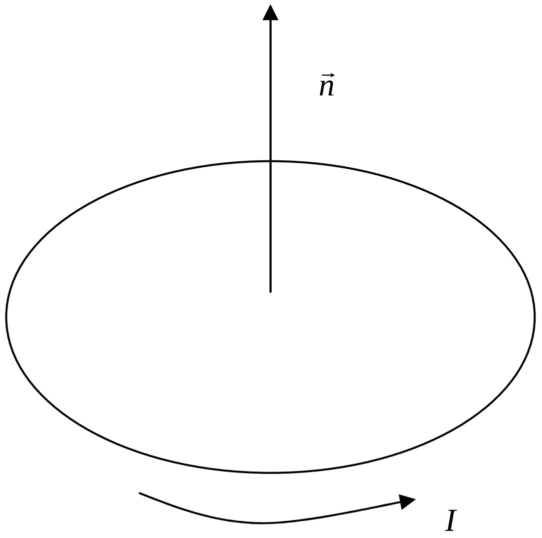
$$\sin\alpha = \frac{d}{b} \to d = b\sin\alpha \to \Gamma = Fb\sin\alpha = IBab\sin\alpha$$

- The torque has a maximum value when $\alpha = 90^0$ (plane of the loop parallel to ***B***) and a minimum value when $\alpha = 0^0$ (plane of the loop perpendicular to ***B***)
- The torque tends to rotate the loop in the direction of decreasing α
- Since the area of the loop is *ab*, *A* = *ab* and $\Gamma = IAB\sin\alpha$
- The quantity **I*A*** is known at the *magnetic moment* of the loop, $\vec{m} = I\vec{A} \to \Gamma = \vec{m}\vec{B}\sin\alpha$. The magnetic moment is a vector that is parallel to ***A***.



Define vector ***A*** perpendicular to the plane of the loop. The magnitude of ***A*** is the area of the loop and direction is determined by RHR

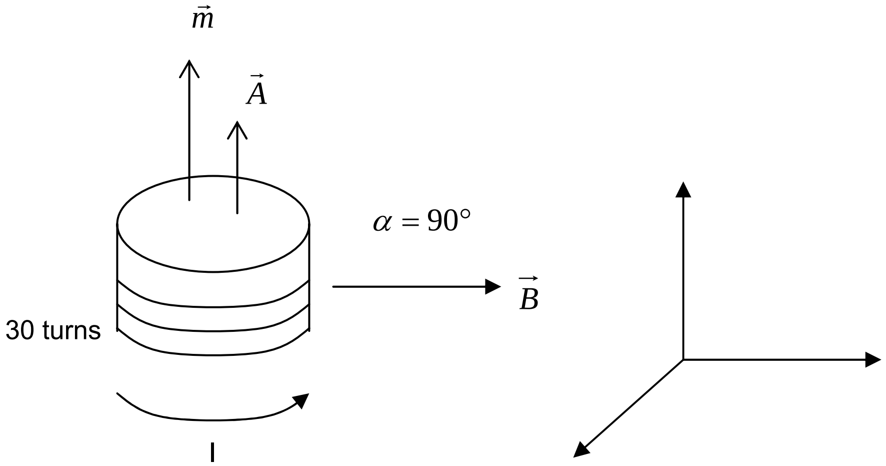
- For a circular loop of radius ***r*** it can be shown that $\Gamma = IAB\sin\alpha = I\pi r^2 B\sin\alpha$



- For multiple loops of any geometry the total torque is equal to the sum of the torque contributions from the individual loops, hence Γ = n***IAB***sinα where n is the number of loops.

Example 5

A *solenoid* of 30 turns with a radius ***r*** = .05m has a 5 amp current in its coils. It is placed in a ***B*** field of 1.2T as shown below. Find the magnetic moment and the torque generated by the ***B*** field on this coil.



To find the magnetic moment

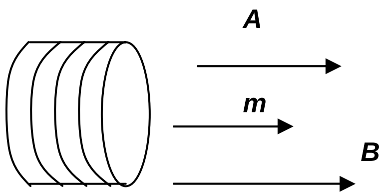
$$A = \pi r^2 = 7.85 \times 10^{-3} m^2$$

$$m = IA = 3.93 \times 10^{-2} A \cdot m^2$$ 1 loop

$$m = nIA = 1.13 A \cdot m^2$$ 30 loops

$\vec{m} = 1.18 A \cdot m^2$ in the +z direction

To find the torque



Rotates the loop as shown

$$\alpha = 90° \qquad \left(\vec{B} \perp Area\right)$$

$$\Gamma = nIBA \sin\alpha$$

$$\Gamma = (30)(5A)(1.2T)\left(7.85 \times 10^{-3} m^2\right)(1) = 1.41 N \cdot m$$

or we can use the magnetic moment

$$\Gamma = mB \sin\alpha$$

$$\Gamma = \left(1.18 A \cdot m^2\right)(1.2T)(1) = 1.41 N \cdot m$$

What force would be required to hold the coils ⊥ to the field if applied to opposite edges of the coil?

$$\Gamma = (2)(0.05m)(F) \rightarrow 1.41 N \cdot m = (.1)(F) \rightarrow F = 14.1 N$$

An interesting way of looking at this current carrying loop in a magnetic field is from the perspective of energy. Is there any change in kinetic or potential energy here? Even though magnetic forces do no work in displacing particles traveling in ***B*** fields, here work is obviously done because the loop has a higher potential energy when $\alpha = 90^0$ than it does when $\alpha = 0^0$.

We will define the potential energy of any current carrying loop in a magnetic field to be:

$$U = -mB\cos\alpha$$

So for this particular loop

$$U_i = -\left(1.18A \cdot m^2\right)\left(1.2T\right)\left(\cos 90°\right) = 0$$

$$U_f = \left(-1.18A \cdot m^2\right)\left(1.2T\right)\left(\cos 0°\right) = -1.41J$$

$$\therefore \Delta U = -1.41J$$