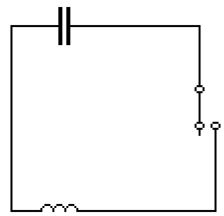
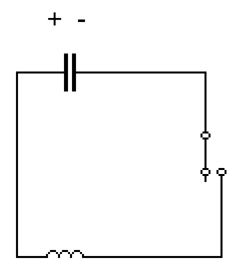
LC Circuits

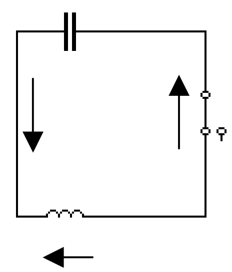
RL or RC circuits approach, exponentially, steady state values of voltage, charge and current. LC circuits, by contrast, are characterized by oscillating current, charge and voltage.



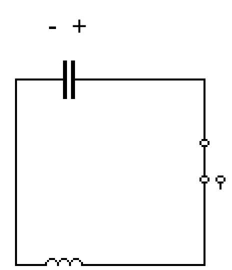


Consider a circuit like that shown above containing a switch, a capacitor (fully charged) and an inductor. We will ignore the small amount of resistance in the connecting wires.

1. The circuit is initially in the state above right with the capacitor charged and a potential V_{ab} established across the capacitor as shown.

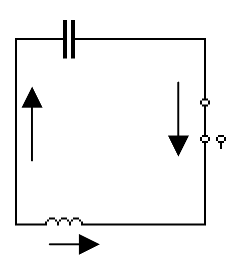


2. As soon as the switch is closed the capacitor discharges. Initially the inductor opposes the growth of the current in the circuit by creating a change in magnetic flux that slows the growth of the current.



3. When the capacitor has completely discharged the field around the inductor begins to collapse. This produces a current in the same direction as the *in situ* current thus charging the capacitor again but with the opposite polarity

4. The cycle repeats itself in the opposite direction. In the absence of any energy loss the oscillations may continue back and forth indefinitely.



- The oscillations involve a transfer of energy back and forth from the electric field of the capacitor to the magnetic field of the inductor.
- The total energy in the circuit remains constant.
- Analogous to the transfer of energy from potential to kinetic in a mass-spring oscillator

Compare the LC oscillator with a Mass-Spring oscillator

Mass-Spring	**LC circuit**
x – displacement	q – charge
v – velocity	i – current
m – mass	L – inductance
k – spring stiffness	C^{-1}
c - damping factor	R - resistance
F – force	V – potential difference
KE = $\frac{1}{2}mv^2$	magnetic energy = $\frac{1}{2}Li^2$
PE = $\frac{1}{2}kx^2$	electrical energy = $q^2/2C$
$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$	$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$
$x = A\cos\omega t$	$q = q\cos\omega t$
$v = -\omega A\sin\omega t$	$i = -\omega Q\sin\omega t$

Both systems are conservative in the absence of damping or frictional forces. The angular frequencies are given by $\omega = \sqrt{\frac{k}{m}}$ and $\omega = \frac{1}{\sqrt{LC}}$ and the linear frequencies are given by $f = \frac{\omega}{2\pi}$.

- The capacitor in a LC circuit acts like a stretched (or compressed) spring in a mechanical oscillator
- The inductor in a LC circuit acts like a mass in a mechanical oscillator with that opposes any change in inertia.

Energy in LC circuits

$$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$$

The first term represents the instantaneous energy stored in the inductor, the second the instantaneous energy stored in the capacitor and the third the total energy.

From this expression we may derive an expression for instantaneous current as a function of charge:

$$Li^2 + \frac{q^2}{C} = \frac{Q^2}{C}$$

$$i^2 = \frac{Q^2}{LC} - \frac{q^2}{LC}$$

$$i^2 = \frac{1}{LC}\left(Q^2 - q^2\right)$$

$$i = \pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q^2}$$