Frictional Forces

- There are two types of frictional forces: those arising from *static* friction and those arising from *kinetic* friction.

- Static friction is the friction that results between two relatively smooth, flat objects in contact that are at rest with respect to each other

- Kinetic friction is the friction that results between two relatively smooth, flat objects in contact that are in motion with respect to each other.

- The coefficient of friction between two surfaces is a measure of the degree to which two surfaces resist moving with respect to each other.

- Coefficients of friction depend on the mechanical, electrical, chemical properties of the surfaces and the temperature of the surfaces.

- There are two coefficients for any two surfaces in contact with each other, the coefficient of static friction (μ_s) and the coefficient of kinetic friction (μ_k).

- The frictional force between two surfaces is proportional to this coefficient, e.g.

$$\vec{F}_k = \mu_k N$$
$$\vec{F}_s = \mu_s N$$

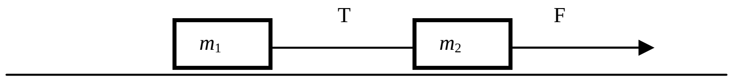
where N is the normal force with the surface in question.

- Note that the force of friction between any two surfaces depends directly on:

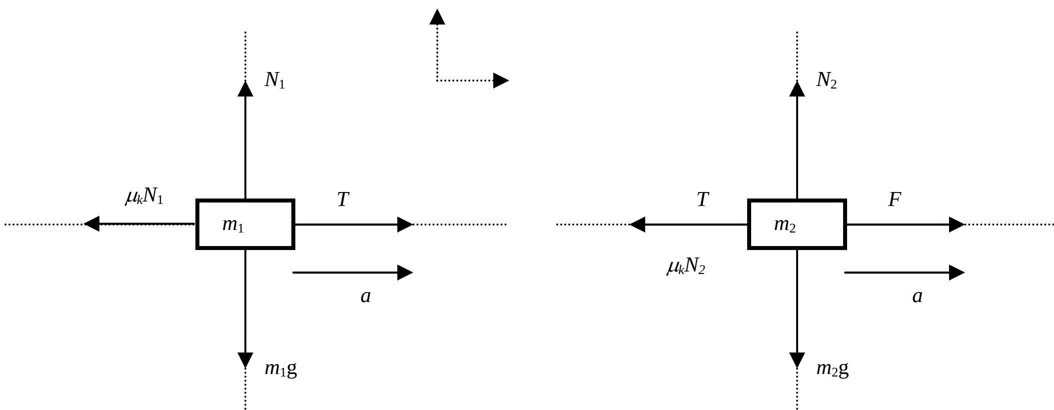
    - the force holding the two surfaces in contact with each other
    - the mechanical properties of the two surfaces (the coefficient of friction)
    - whether or not the surfaces are at rest with respect to each other

but does not depend on directly on surface area!

Example 1 Consider a system consisting of two blocks (masses = m_1 & m_2) attached by a light cord on a rough, flat table pulled to the right by a force *F*. The coefficient of kinetic friction between the blocks and table is μ_k . Find the tension in the connecting cord and the acceleration of the system.



The FBD's for this system:



FBD$_1$

$$\sum F_y = N_1 - m_1 g = 0$$
$$\therefore N_1 = m_1 g$$
$$\sum F_x = T - \mu_k m_1 g = m_1 a \qquad (1)$$

FBD$_2$

$$\sum F_y = N_2 - m_2 g = 0$$
$$\therefore N_2 = m_2 g$$
$$\sum F_x = F - T - \mu_k m_2 g = m_2 a \qquad (2)$$

Now if we combine equations (1) and (2):

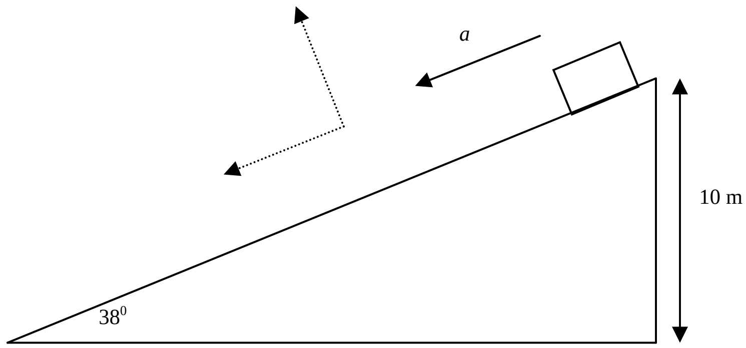
$$m_1 a + \mu_k m_1 g = T = F - m_2 a - \mu_k m_2 g \;\therefore\; m_1 a + m_2 a + \mu_k m_1 g + \mu_k m_2 g = F$$

$$(m_1 + m_2)a + \mu_k (m_1 + m_2)g = F$$

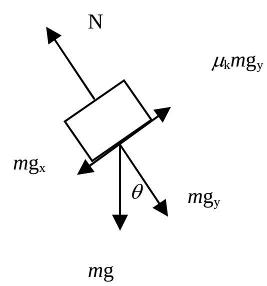
$$a = \frac{F - \mu_k (m_1 + m_2)g}{(m_1 + m_2)}$$

$$T = \mu_k m_1 g + m_1 \left(\frac{F - \mu_k (m_1 + m_2)g}{(m_1 + m_2)} \right)$$

Example 2 A 10 kg block slides down a rough surface (μ_k=0.2). It is released from rest at the top of an incline (38^0 to the horizontal) at a height 10 meters. What is its speed at the bottom of the incline?



Note: $\sin 38^0 = \frac{10m}{hyp} \Rightarrow$ the length of the incline is 16.2 meters. In the coordinate system we've chosen, the FBD is:



Based on this FBD: $\sum F_x = mg\sin\theta - \mu_k mg\cos\theta = ma \therefore a = g\sin\theta - \mu_k g\cos\theta$

The first term is the same expression as we got when working this example without accounting for the presence of friction. The second term is a component that reduces the overall acceleration due to the presence of kinetic friction.

In this case:

$$a = (9.8m\cdot s^{-2})(0.616) - (0.2)(9.8m\cdot s^{-2})(0.788) = (6.03m\cdot s^{-2}) - (1.54m\cdot s^{-2}) = 4.5m\cdot s^{-2}$$

By determining the acceleration we've solved the *dynamics* part of the problem and with this information we can proceed to solve the *kinematics* part of the problem.

$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow v = \sqrt{2a(x - x_0)} = \sqrt{(2)(4.5m\cdot s^{-2})(16.2m)} = 12.1m\cdot s^{-1}$$

Example 3 In the previous example, what would be the effect if the "frictional" term in the acceleration equation were greater than the regular term? What if the two terms are equal?

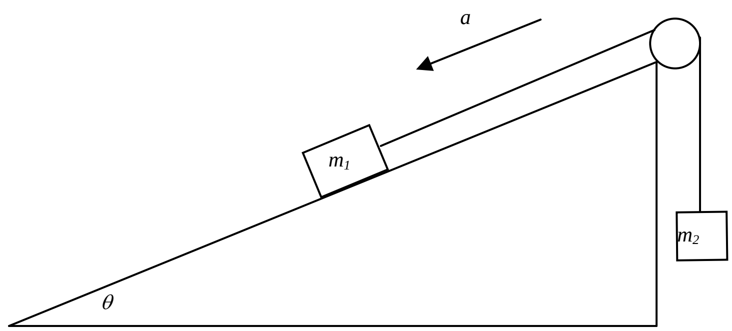
If the frictional term were larger than the regular term it would imply that the block accelerates back up the plane under its own volition! Clearly this is a physically nonsensical case.

If the two terms are equal the block is in equilibrium. Because of the fact that it takes more force to overcome static friction and get the block moving than it does to keep it moving, this statement implies that the block is motionless.

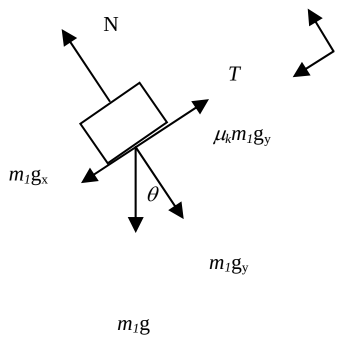
$$g\sin\theta = u_k g\cos\theta \therefore \frac{\sin\theta}{\cos\theta} = \mu_k \therefore \tan\theta = \mu_k$$

In this case, μ_k = 0.2 so θ = 11.3^0. This is (approximately) the maximum angle for which this block will resist sliding down the incline.

Example 4 Consider the system below. If $m_1 > m_2$ and the surface is rough find the acceleration of the system.

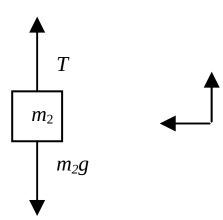


FBD #1



$$\sum F_y = N - m_1 g\cos\theta = 0 \therefore N = m_1 g\cos\theta$$
$$\sum F_x = m_1 g\sin\theta - T - \mu_k m_1 g\cos\theta = m_1 a \qquad (1)$$

FBD #2



$$\sum F_x = 0$$
$$\sum F_y = T - m_2 g = m_2 a \qquad (2)$$

$$m_1 g \sin\theta - \mu_k m_1 g \cos\theta - m_1 a = m_2 a + m_2 g$$

$$(m_1 + m_2)a = (m_1 \sin\theta - \mu_k m_1 \cos\theta - m_2)g$$

$$a = \frac{(m_1 \sin\theta - \mu_k m_1 \cos\theta - m_2)g}{(m_1 + m_2)}$$