## DC Resistive Circuits

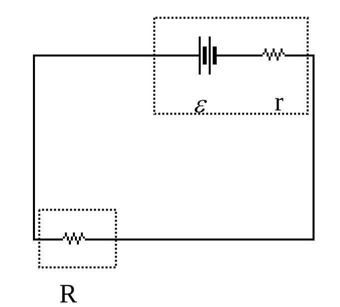
In our study of DC circuits we will begin by assuming that all DC circuits are in a *steady state* condition, i.e., currents do not vary with time.

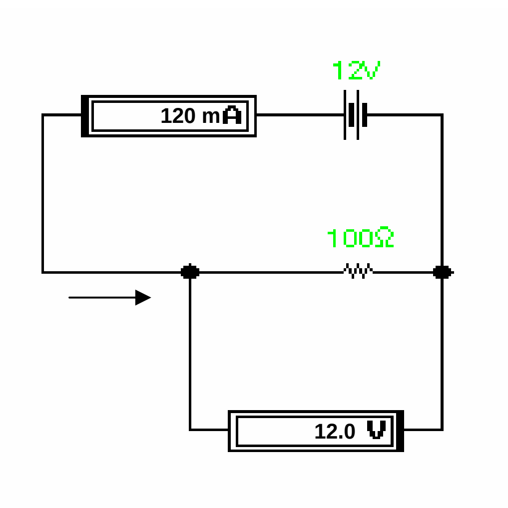
**EMF** - ***E***lectro***m***otive ***F***orce, **E**: a source of electrical potential energy ($\mathcal{E}$)

- batteries
- generators
- these behave differently

Consider the simple DC resistive circuit shown at right.

- The source of electrical potential energy, $\mathcal{E}$, is a battery.
- The internal resistance of the battery is *r*, and the circuit load is *R*.
- For the purpose of this discussion we consider the resistance of the connecting wires to be negligible compared to *R*.



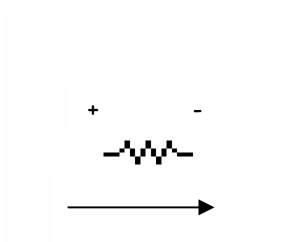

Consider a circuit containing a 12-volt power supply connected in series with a 100-ohm resistor as shown at the left.

- The *applied voltage* is 12 volts. This means that the electrical potential of the left terminal of the power supply is 12 *V higher* than the electrical potential of the right
- Generally this means that the high potential terminal is at 12 *V* and the low potential terminal is at 0 *V* but they may be at any respective values that are 12 *V* apart).
- As current flows around the circuit (counterclockwise in this case) the potential decreases as the current passes through the resistor.
- Ignoring the small amount of resistance in the connecting wires, the potential drop across the resistor in this (single resistor) circuit equals the applied voltage.

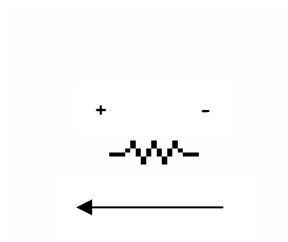
- The voltage drop would be the same regardless of the value of the resistor, i.e., the drop would be 12 volts if the resistor were 1 ohm or 10000000 ohms.
- This electrical energy is converted to heat by the resistor (or work in a load).
- Unlike electrical potential, which decreases as resistance is encountered in a circuit, current is not "used up" as it flows through a resistor. The current in Figure 1 is the same whether it is measured before or after the resistor.
- The amount of current flow is determined for a circuit by the amount of voltage available to the circuit (applied voltage) and the amount of resistance in the circuit.
- The relationship between voltage, current and resistance is quantified in Ohm's Law: $V = IR$. In this example the current is:

$$\frac{12V}{100\Omega} = 0.12A$$

- This is $120 \times 10^{-3}$ amperes or 120 mA. This is the amount of current that *any* 12 *V* circuit will draw with a load or resistance of 100-ohms.
- The potential across a resistor increases or decreases depending on the “polarity” of the resistor – generally based on the direction of current flow in a circuit.

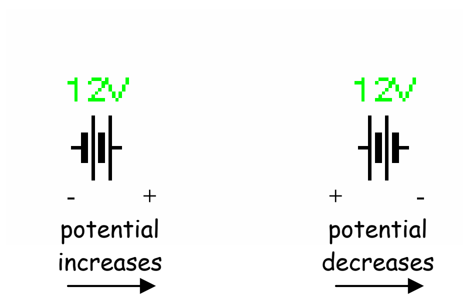


potential decreases

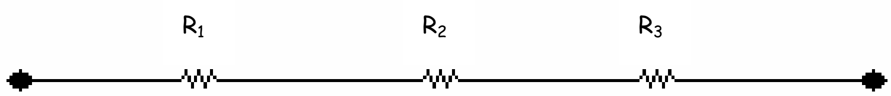


potential increases

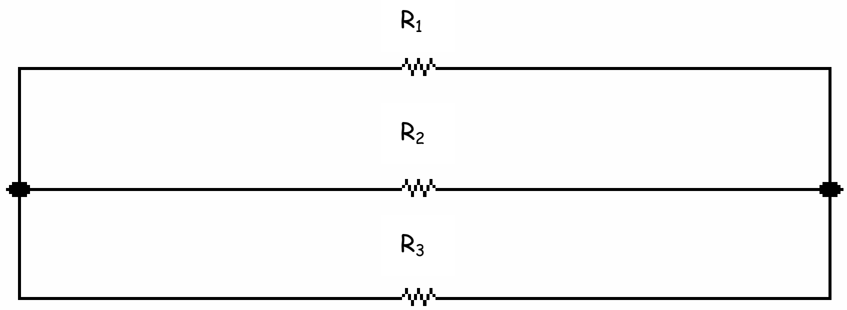
- The potential increase or decrease across any resistor is by a factor of *IR*.
- Potential increases or decreases across power supplies by a factor of $\mathcal{E}$.



Resistors in Series and Parallel




A series resistive circuit.

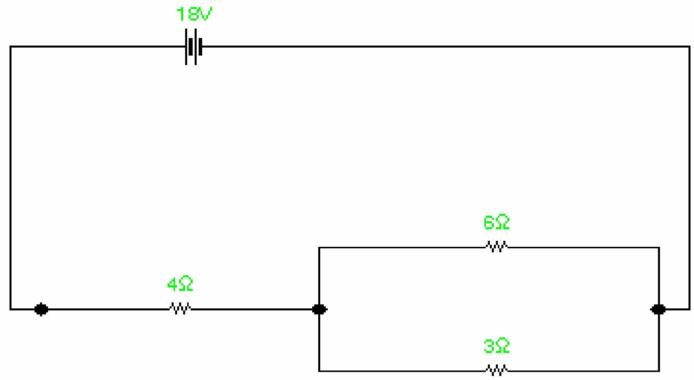



A parallel resistive circuit.

Rules for series and parallel resistive circuits.

- The current is the same for all resistors in series in the same branch of a circuit.
- The voltage is the same across all resistors in parallel branches of a circuit.
- The equivalent resistance for a series of resistors in parallel may be computed by adding the resistors as follows: $\frac{1}{R_1}+\frac{1}{R_2}+\ldots\frac{1}{R_i}=\frac{1}{R_{eq}}$
- The equivalent resistance for a series of resistors in series may be computed as follows: $R_1+R_2+\ldots R_i=R_{eq}$

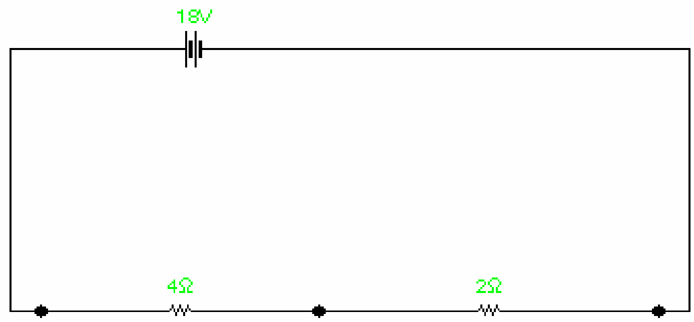
Example Ohm's Law applied to a simple series/parallel circuit



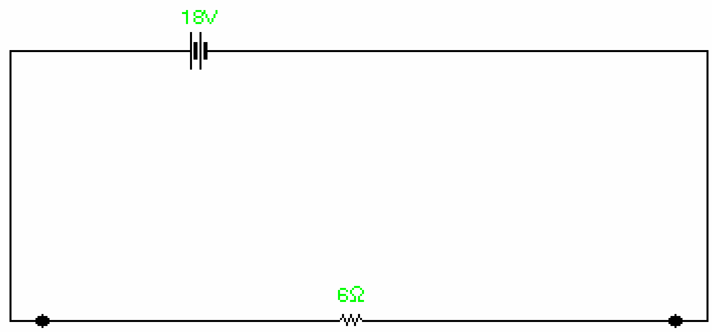

1. Reduce this circuit to a circuit with a single equivalent resistor.
2. Compute the current through and the potential drop across each resistor.

We begin by computing the equivalent resistance in the parallel branch of the first circuit:

$$\frac{1}{6\Omega}+\frac{1}{3\Omega}=\frac{1}{R_{eq}}\rightarrow R_{eq}=2\Omega$$

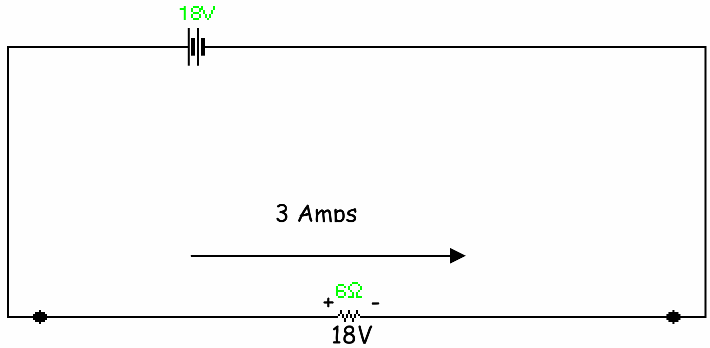



Next we add the two resistors in series to compute the equivalent resistance of the entire circuit: $4\Omega+2\Omega=6\Omega$

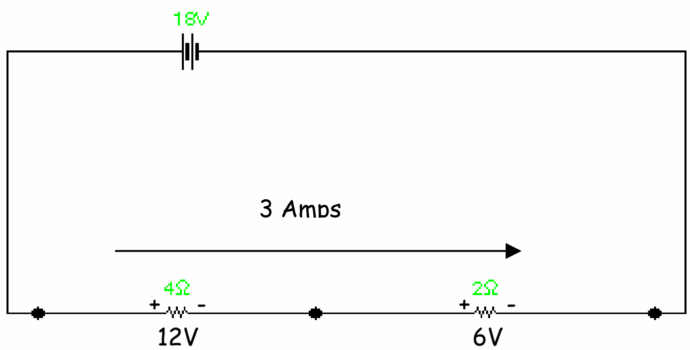



Ohm's Law now reveals the current that this circuit will draw since the equivalent resistance is merely the sum of the values of the individual resistors: $\frac{18V}{6\Omega}=3A$.

Now that we have established the total current drawn by the circuit we may begin computing the current through each resistor and the voltage across each resistor.

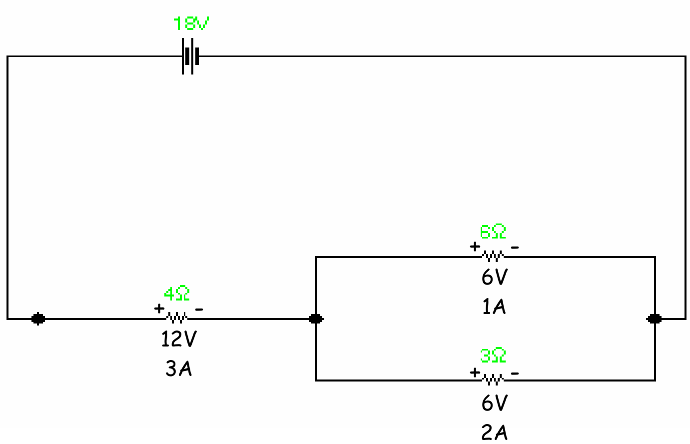



We begin by step by step reconstructing the original circuit. The first step is to substitute for the 6Ω resistor the two resistors in series it replaced. Armed with the knowledge that the current is the same for resistors in series (in this case 3 amps) we may compute the potential drop across each resistor:




$$V = (3A)(4\Omega) = 12V$$
$$V = (3A)(2\Omega) = 6V$$

Notice that the sum of the potential differences across each resistor equals the applied voltage.




Finally we replace the 2Ω resistor on the right with the original 6Ω and 3Ω resistors in parallel. Armed with the knowledge that the potential across resistors in parallel is the same (in this case 6 volts) we may use Ohm's law to compute the potential drop across each resistor: $\frac{6V}{6\Omega} = 1A$, $\frac{6V}{3\Omega} = 2A$. Again, notice that the sum of the currents across each branch is equal to the total current.

**Kirchhoff's Rules:**

It is not always possible to reduce a circuit to a single loop. For more complicated circuits Kirchhoff's rules may be used simplify circuit analysis.

- The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
- The algebraic sum of the changes in potential across all of the elements around any closed circuit loop must be zero.

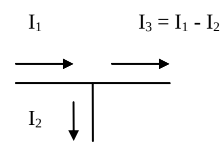
A *junction* is any point in a circuit where the current has a choice about which way to go. The first rule, also known as the point rule, is a statement of *conservation of charge*. If current splits at a junction in a circuit, the sum of the currents leaving the junction must be the same as the current entering the junction.

The second rule, also known as the loop rule, is a statement of *conservation of energy*. Recall that although charge is not "used up" as current flows through resistors in a circuit, potential is. As current flows through each resistor of a resistive circuit the potential drops. The sum of the potential drops must be the same as the applied potential.

To apply Kirchhoff's laws, one proceeds as follows:

1) Identify all of the junctions or branch points in the circuit.

2) Use the point rule to express the unknown currents as few terms as possible, e.g.:




In this case, $I_3$ may be expressed as the difference of $I_1$ and $I_2$. All three currents may then be expressed in terms of $I_1$ and $I_2$.

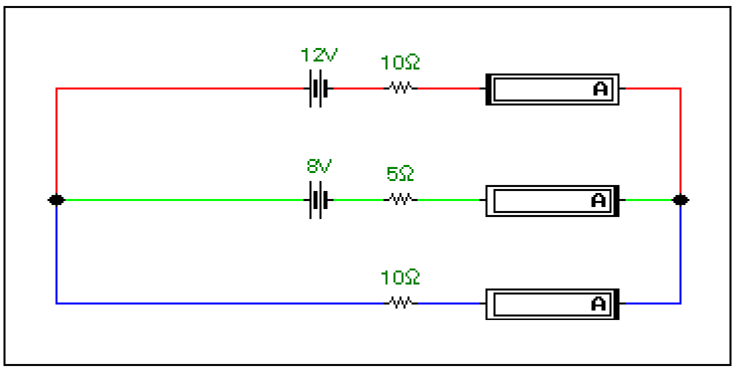
When applying the point rule it is necessary to assume a direction of current flow in each branch of the circuit. Label the terminals of each resistor with a + or - sign depending upon the direction of current flow (recall that we are using the positive test charge model of current , so current flows from the (+) to the (-) end of a resistor). Label each source of EMF as well. If your guess is incorrect for a particular branch, the value that you will eventually obtain for the current in that branch will contain a minus sign. A negative value for current is OK, and should be kept for any subsequent calculations.

3) Identify the current loops that exist in the circuit. Choose any loop and apply the loop rule. Remember that + → - is a potential drop while - → + is a potential gain. Write equations for each loop. Remember that the sums of the potential drops and gains must be zero.

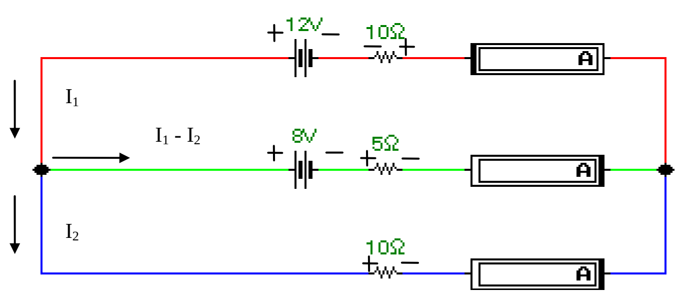
4) Reapply the loop rule as needed. For each unknown current you will need to write an equation. The fewer terms in which you express unknown currents, the fewer equations you have to write.

5) Solve the equations to determine the unknown currents.

Example   Analyze the following circuit with Kirchhoff's Rules.




- The branch points are identified by large dots. The point rule has been used to express the three independent currents in terms of two quantities, $I_1$ and $I_2$.
- Given the polarity of the power supplies let's assume that the current flows from right to left in the top branch (red),, and from left to right in the middle (green) and bottom (blue) branches.




- Indicate the polarity of the resistors accordingly.
- Apply the loop rule by choosing from three current loops: one that includes the upper (red) and middle (green) branches, one that includes the middle (green) and lower (blue) branches, and one that includes the upper (red) and lower (blue) branches.

- Because the three independent currents have been expressed in terms of two unknowns ($I_1$ and $I_2$), the loop rule only need only be applied twice. Any two of the three loops may be chosen to generate the two necessary equations.

Applying the loop rule to the red-green loop (counterclockwise):

$$(1) \qquad 12V - 8V - 5\Omega(I_1 - I_2) - 10\Omega(I_1) = 0$$

The first term (12 V) is positive because the current flows from low to high potential. Applying the loop rule to the red-blue loop (counterclockwise):

$$(2) \qquad 12V - 10\Omega(I_2) - 10\Omega(I_1) = 0$$

There are several methods that may be used to solve these two equations. Perhaps the most straightforward is to simply combine the equations in such a manner as to eliminate one of the unknowns. The first step in this process is to simplify both equations:

$$(3) \qquad 4V - 15\Omega(I_1) + 5\Omega(I_2) = 0$$
$$(4) \qquad 12V - 10\Omega(I_1) - 10\Omega(I_2) = 0$$

If the first equation is multiplied by 2, the third term in the first equation cancels the third term in the second:

$$(5) \qquad 8V - 30\Omega(I_1) + 10\Omega(I_2) = 0$$
$$(6) \qquad 12V - 10\Omega(I_1) - 10\Omega(I_2) = 0$$

Now if these two equations are added together:

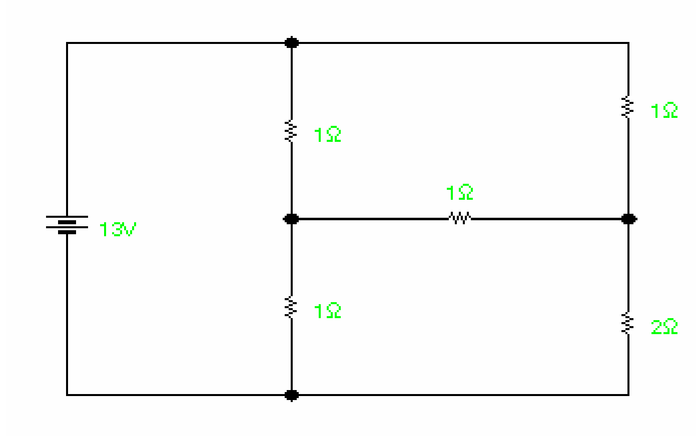
$$20V - 40\Omega(I_1) = 0$$

Or $I_1$ = .5 amperes (500 mA). We can plug the value of $I_1$ into either equation 5 or 6 and solve for $I_2$. Using equation 6:

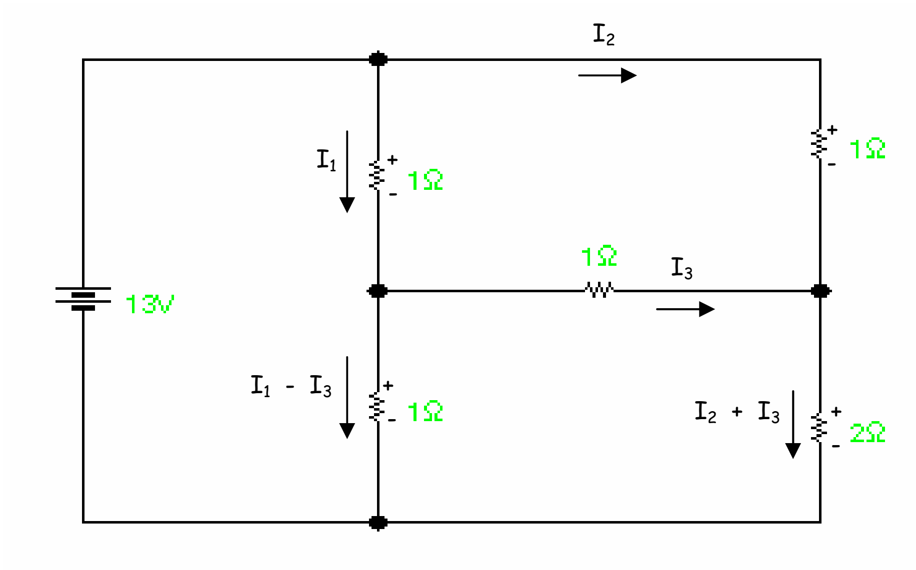
$$12V - 5V - 10\Omega(I_2) = 0$$

or $I_2$ = .7 amperes (700 mA). Using the point rule, $I_3$ is equal to $I_1$ - $I_2$ or -.2 amperes.

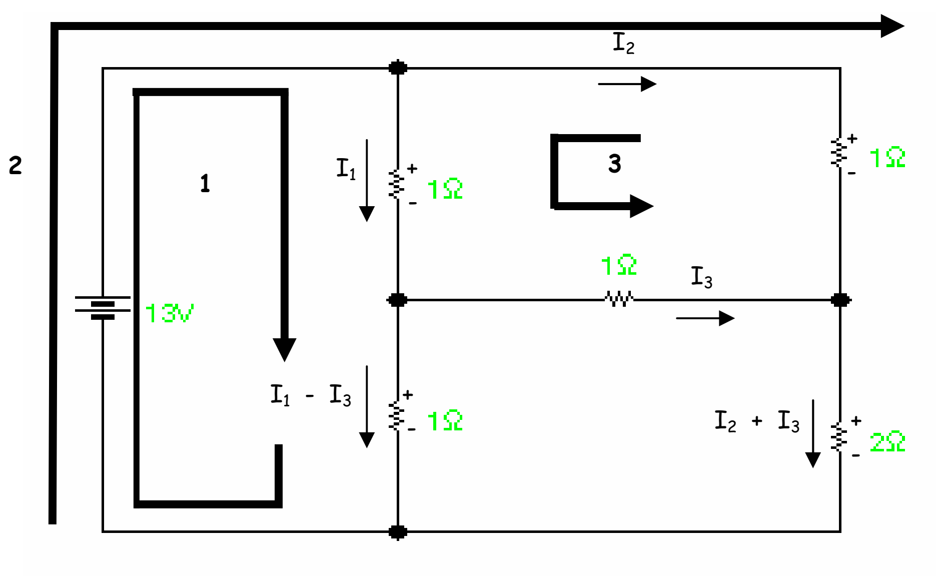
Example




Application of the point rule yields:




Application of the loop rule yields:



We have expressed all of the currents in terms of three unknowns: $I_1$, $I_2$, and $I_3$. We'll need three equations to find these values.

1. $13V - (I_1)(1\Omega) - [(I_1 - I_3)(1\Omega)] = 0$
   $13V - (I_1)(1\Omega) - (I_1)(1\Omega) + (I_3)(1\Omega) = 0$
   $13V - (I_1)(2\Omega) + (I_3)(1\Omega) = 0$*

2. $13V - (I_2)(1\Omega) - [(I_2 + I_3)(2\Omega)] = 0$
   $13V - (I_2)(1\Omega) - (I_2)(2\Omega) - (I_3)(2\Omega) = 0$
   $13V - (I_2)(3\Omega) - (I_3)(2\Omega) = 0$**

3. $(I_2)(1\Omega) - (I_1)(1\Omega) - (I_3)(1\Omega)$***

The results are:

1. $13V - (I_1)(2\Omega) + (I_3)(1\Omega) = 0$
2. $13V - (I_2)(3\Omega) + (I_3)(2\Omega) = 0$
3. $0 = (I_2)(1\Omega) - (I_1)(1\Omega) - (I_3)(1\Omega)$

Now we need to solve these equations. Cramer's Rule, matrix manipulation or simple algebra will all work. We'll employ the latter here and begin by solving equation 3 for $I_2$.

$$(I_2)(1\Omega) = (I_1)(1\Omega) + (I_3)(1\Omega) \rightarrow I_2 = I_1 + I_3$$

We'll substitute this result into equation 2.

$13V - (I_1 + I_3)(3\Omega) + (I_3)(2\Omega) = 0$

2. $13V - (I_1)(3\Omega) - (I_3)(5\Omega) = 0$

Now compare equations 1 and our reformulated equation 2.

1. $13V - (I_1)(2\Omega) + (I_3)(1\Omega) = 0$
2. $13V - (I_1)(3\Omega) - (I_3)(5\Omega) = 0$

Notice that if we multiply equation 1 through by 5, then add the two equations together the third terms cancel.

$$\begin{array}{r} 65V = (I_1)(10\Omega) + (I_3)(5\Omega) = 0 \\ +13V = (I_1)(3\Omega) - (I_3)(5\Omega) = 0 \\ \hline 78V = (I_1)(13\Omega) + 0 \end{array}$$

Solving this equation yields $I_1 = 6A$. Substitution into the original equations yields $I_2 = 5A$, $I_3 = -1A$

## The Matrix Method

Rearrangement of the three equations we developed using the loop rule yields:

$$13V - (I_1)(2\Omega) + (I_2)(0) + (I_3)(1\Omega) = 0 \rightarrow 13V = (I_1)2\Omega - (I_2)(0) - (I_3)(1\Omega)$$
$$13V - (I_1)(0) - (I_2)(3\Omega) - (I_3)(2\Omega) = 0 \rightarrow 13V = (I_1)(0) + I_2(3\Omega) + I_3(2\Omega)$$
$$0 = -(I_1)(1\Omega) + (I_2)(1\Omega) - (I_3)(1\Omega)$$

Those of you familiar with linear algebra will recognize this as a vector and a matrix:

Vector $\quad 13 \quad 13 \quad 0$

Matrix $\quad \begin{array}{ccc} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 1 & -1 \end{array}$

Dividing the vector by the matrix:

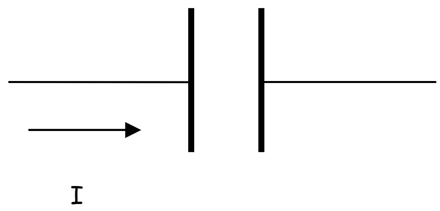
$$6 \quad 5 \quad -1$$

So $I_1 = 6A$, $I_2 = 5A$, $I_3 = -1A$

Displacement Current

So far we have considered simple circuits consisting of capacitors and resistors, analyzing the flow of current through latter with some rigor.

An obvious question arises when analyzing the flow of current through a device such as a capacitor in a circuit,




i.e., how does the current manage to flow across the non-conducting space between the plates of the capacitor? A *conduction current* flows onto the left plate but in the absence of any conducting material within the capacitor how is a current supposed to flow from the right plate?

Maxwell formulated an alternative definition of current such that we can equate any change in the electric field within the capacitor with an effective current density.

For a parallel plate capacitor: $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

As current flows onto one plate of the capacitor and it charges in some interval of time dt:

$$dQ = I_C dt$$

The corresponding change in E is:

$$dE == \frac{dQ}{\varepsilon_0 A} = \frac{I_c dt}{\varepsilon_0 A} \rightarrow \frac{dE}{dt} = \frac{I_C}{\varepsilon_0 A}$$

Given: $J_D = \varepsilon_0 \frac{dE}{dt}$

Displacement Current: $I_D = J_D A = \varepsilon_0 \left( \frac{dE}{dt} \right) A = I_C$