

PHYS 212 Examination 1

Name (print): Key 2-4-07

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Problem 1 _____

Problem 2 _____

Problem 3 _____

Problem 4 _____

Problem 5 _____

Problem 6 _____

Problem 7 _____

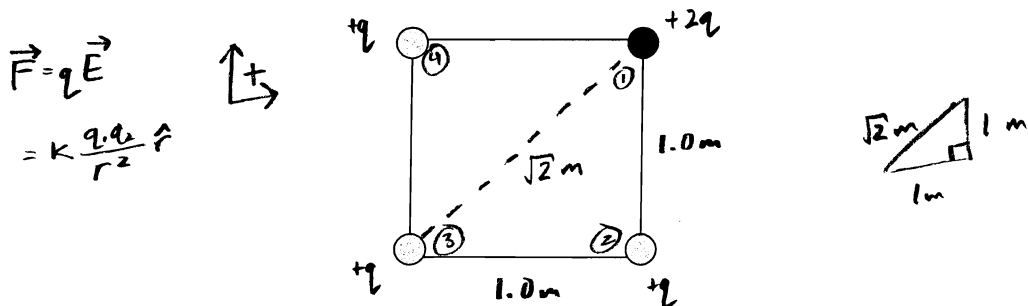
Problem 8 _____

Total _____

Directions: This exam is worth 71 points. Your solutions should be written as neatly as possible and arranged in a logical manner. Credit will be awarded on the basis of thought, compactness, and neatness of the written solution. Remember to use basic physical principles in solving the problems. Show all of your work. We will not award full points for a problem with a solution that we are unable to decipher even if the answer is correct.

An equation sheet has been provided. CRC or similar handbooks are allowed (but the proctor will check them) as well as Xeroxed integral tables. Calculator rule is in effect. If you need a physical constant that is not provided ask the proctor. Good Luck!

Problem 1. (8 points) Four point charges are arrayed on the corners of a square as shown below. Each charge has a magnitude of $+q$, except for the charge on the upper right which has a charge of $+2q$. If the charges are all 1.0 meter apart, what is the magnitude of the electric force on the charge on the upper right? What is the electric field at that point in space?



$$\bullet \vec{F}_{21} = K \frac{2q^2}{r^2} \hat{y} = \underline{2Kq^2 \hat{y}}$$

$$\bullet \vec{F}_{31} = K \frac{2q^2}{r^2} \hat{r} = Kq^2 \hat{r} = \underline{Kq^2 \cos 45 \hat{x} + Kq^2 \sin 45 \hat{y}}$$

$$\bullet \vec{F}_{41} = K \frac{2q^2}{r^2} \hat{x} = \underline{2Kq^2 \hat{x}}$$

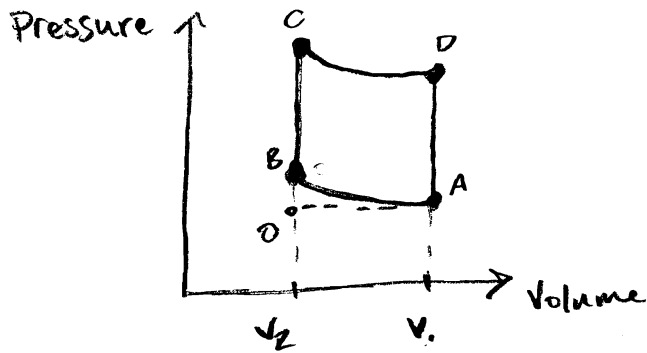
$$\vec{F}_{\text{net}} = (2Kq^2 + Kq^2 \cos 45) \hat{x} + (2Kq^2 + Kq^2 \sin 45) \hat{y}$$

$$(2.7Kq^2) \hat{x} + (2.7Kq^2) \hat{y} \Rightarrow |\vec{F}| = \boxed{3.8Kq^2}$$

$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{E} = (2.7Kq) \hat{x} + (2.7Kq) \hat{y}$$

$$\Rightarrow |\vec{E}| = \boxed{3.8Kq}$$

Problem 2. (6 points) Describe the Otto cycle.



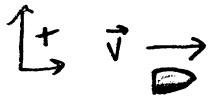
⇒ Otto cycle describes pressure and volume dynamics in a combustion engine.

- Intake: air @ atmospheric pressure is drawn in. Volume increases as piston moves downward. (isobaric)
- Compression: piston compresses air-fuel mixture (adiabatically). Volume decreases from V_1 to V_2 . Temperature increases. Work done is the area under the curve AB.
- Combustion: Occurring during BC. Q_h is heat released due to combustion. Pressure & temp increase rapidly but volume remains constant (isochoric). No work is done on the gas.
- Power stroke: C → D. Gas expands adiabatically. Volume changes from V_2 to V_1 . Temperature drops. Work done is area under curve CD.
- From D → A heat is extracted from the gas as pressure decreases, volume remains constant. No work done.
- Exhaust: A → B. Residual gases exhausted at atmospheric pressure, volume decreases from V_1 to V_2 (isobaric).

Problem 3. (10 points) A 3g lead bullet (@ 0°C) is traveling at a speed of 240m/s when it comes to rest inside a very large block of ice (10kg) @ -5°C. What happens to the ice? The heat of fusion for ice is $3.33 \times 10^5 \text{ J/kg}$ and the specific heat capacity is $2090 \text{ J/kg}^\circ\text{C}$.

$$\frac{1}{2} m v_i^2 = m c \Delta T \Rightarrow \Delta T = 4.13 \times 10^{-3} \text{ }^\circ\text{C}$$

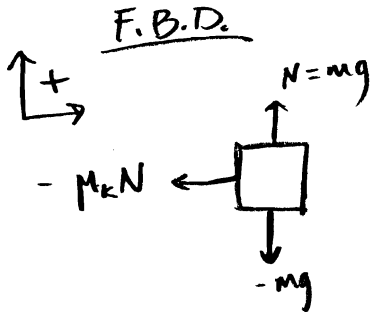
Now assume that the block of ice is initially at rest on a flat surface with a coefficient of friction of 0.05, how far does the ice travel after the collision before coming to rest? What heat is generated in this process? Assuming an adiabatic process how does this change the block of ice?



- $m_b = 0.003 \text{ kg}$ $v_i = 240 \text{ m/s}$
- $m_{ice} = 10 \text{ kg}$ $v_i = 0 \text{ m/s}$

$$m_b v_i + m_{ice} v_i = (m_b + m_{ice}) v_f \Rightarrow m_b v_i = (m_b + m_{ice}) v_f$$

$$v_f = \frac{m_b v_i}{(m_b + m_{ice})} = \underline{\underline{0.072 \text{ m/s}}}$$



$$\text{K.E.} = \frac{1}{2} m v_f^2 = \underline{\underline{0.026 \text{ J}}} \leftarrow \text{(heat generated)}$$

$$W = F \cdot d \rightarrow 0.026 \text{ J} = (0.05)(10.003 \text{ kg})(9.8 \text{ m/s}^2) d$$

$$\rightarrow d = \frac{0.026}{4.9} = \underline{\underline{0.005 \text{ m}}} \leftarrow \text{(distance traveled)}$$

• Mass of ice melted:

$$Q = m c \Delta T \rightarrow 0.026 \text{ J} = (10 \text{ kg})(2090 \text{ J/kg}^\circ\text{C}) \Delta T$$

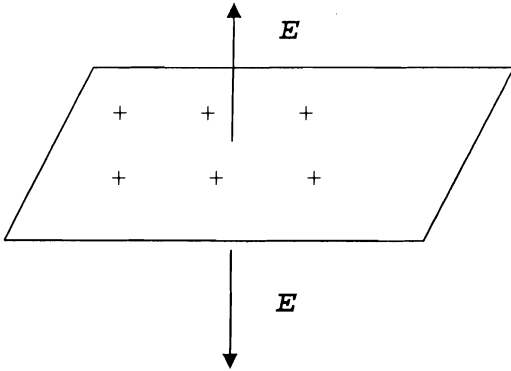
$$\Rightarrow \Delta T = \underline{\underline{1.24 \times 10^{-6} \text{ }^\circ\text{C}}}$$

The temperature of the ice increases $1.24 \times 10^{-6} \text{ }^\circ\text{C}$

Problem 4. (6 points) Describe the execution of a motorcycle turn.

- Countersteering helps the bike to lean in the direction of the turn. The lean of the bike causes a change in angular momentum that pulls the steering wheel to turn into the direction of the turn.

Problem 5. (10 points) Compute the electric field for an infinite sheet of charge as shown below.



$$dQ = \sigma L dx$$

$$dE = \frac{\sigma dx}{2\pi\epsilon_0 r}$$
$$= \frac{\sigma}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\sin\theta dx}{r}$$

$$\sin\theta = \frac{a}{r}$$

$$r^2 = a^2 + x^2$$

$$\Rightarrow E = \frac{\sigma a}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dx}{a^2 + x^2}$$

$$= \frac{\sigma a}{2\pi\epsilon_0} \left[\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \Big|_{-\infty}^{+\infty} \right]$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

(no dependence upon a)

Problem 6. (10 points) How much heat is needed to take ice of mass 1 kg at -10°C to a gaseous state at 115°C ?

$$c_{\text{ice}} = 2220 \text{ J/kg} \cdot \text{K}$$

$$c_w = 4190 \text{ J/kg} \cdot \text{K}$$

$$L_f = 333 \text{ kJ/kg}$$

• Heat to raise T to 0°C :

$$Q = mc\Delta T \rightarrow Q = (1\text{kg})(2220 \text{ J/kg} \cdot \text{K})(10\text{K}) \\ = \underline{2.22 \times 10^4 \text{ J}}$$

• Heat to melt 1kg of ice:

$$Q = mL_f \rightarrow Q = (1\text{kg})(3.33 \times 10^5 \text{ J/kg}) \\ = \underline{3.33 \times 10^5 \text{ J}}$$

• Heat to raise temp. to 100°C :

$$Q = mc\Delta T \rightarrow Q = (1\text{kg})(4190 \text{ J/kg} \cdot \text{K})(100\text{K}) \\ = \underline{4.19 \times 10^5 \text{ J}}$$

• Heat to boil 1kg H_2O :

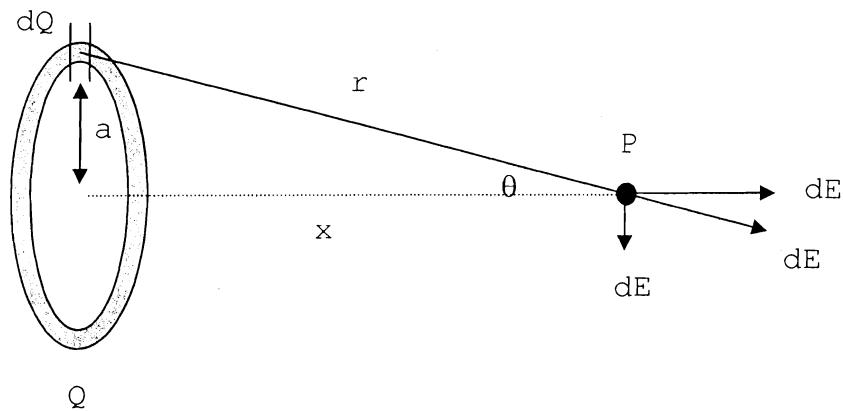
$$Q = mL_v \rightarrow Q = (1\text{kg})(22.6 \times 10^5 \text{ J/kg}) \\ = \underline{22.6 \times 10^5 \text{ J}}$$

• Heat to raise temp. to 115°C :

$$Q = mc\Delta T \rightarrow Q = (1\text{kg})(2010 \text{ J/kg} \cdot \text{K})(15\text{K}) \\ = 3.02 \times 10^4 \text{ J}$$

$$\text{Total: } \boxed{3.1 \times 10^6 \text{ J}}$$

Problem 7. (10 points) Consider a ring of charge, radius a , as shown below. Compute the electric field at point P .



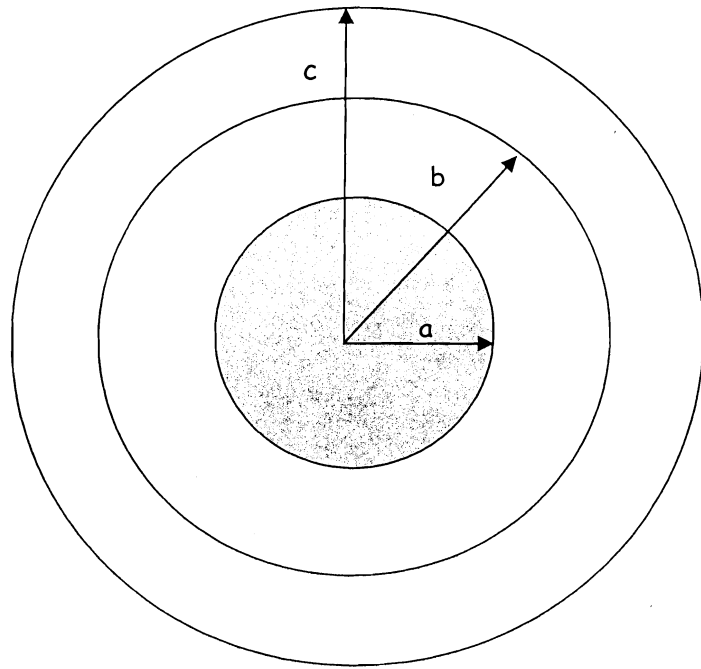
$$E_y = 0$$

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dQ}{x^2 + a^2} \cos\theta = \frac{1}{4\pi\epsilon_0} \frac{x dQ}{(x^2 + a^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{x dQ}{(x^2 + a^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x Q}{(x^2 + a^2)^{3/2}}$$

Problem 8. (10 points) A solid conducting sphere of radius a carries a net positive charge of $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge of $-Q$. Find the electric field in the regions:



- 1 - $r < a$
- 2 - $a < r < b$
- 3 - $b < r < c$
- 4 - $r > c$

$$1. \vec{E} = 0. \quad q_{enc} = 0.$$

$$2. \vec{E} = K \frac{2Q}{r^2} \hat{r}$$

$$3. \vec{E} = 0. \quad (\text{by induction})$$

$$4. \vec{E} = K \frac{Q}{r^2} \hat{r}$$