See Section 14-2
(pages 372 - 377)

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Quiz 7 **Physics 211** **Spring 2008** **Prof. Philip Cole**
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Worth 10 pts

Imagine there is a mass m at the end of a spring (force constant k), which is executing Simple Harmonic Motion.

Note that Newton's Second Law can be written as $\sum F_i = m\frac{d^2x}{dt^2}$. ← 1 Dimension

1. Show that when $F = -kx$, you get the equation $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$
2. What is ω_o in terms of k and m? What is this ω_o called and what are the units associated with it?
3. Show that $x(t) = A_o\cos(\omega_o t + \varphi)$ is a solution to the equation in part 1.
4. What is a_{max} and v_{max}?
5. If the phase angle, φ, is 0, plot the displacement, velocity (dx/dt), and acceleration (d^2x/dt^2) as a function of time for at least one period of its motion. Identify the Amplitude and the Period T. **You should use three separate plots.**

1. If $\sum F = m\frac{d^2x}{dt^2}$ and $\sum F = -kx$ (Hooke's Law) then $-kx = m\frac{d^2x}{dt^2}$ and it ensues that $\frac{d^2x}{dt^2} + \omega_o^2 x = 0$ (1). Here $\omega_o^2 = \frac{k}{m}$

2. $\omega_o = \sqrt{\frac{k}{m}}$ This is the angular frequency. It has units of rad/s.

3. $x = A_o\cos(\omega_o t + \varphi)$ (3a) | $\frac{d^2x}{dt^2} = -A_o\omega_o^2\cos(\omega_o t + \varphi)$ (3c)

$\frac{dx}{dt} = -A_o\omega\sin(\omega_o t + \varphi)$ (3b)

Inserting (3a) and (3c) into (1) yields

$$\underbrace{-\omega_o A_o^2\cos(\omega_o t + \varphi)}_{\frac{d^2x}{dt^2}} + \omega_o^2 \underbrace{A_o\cos(\omega_o t + \varphi)}_{x} = 0$$

Yes! $x = A_o\cos(\omega_o t + \varphi)$ is indeed the soln to the equation in part 1 (SHO).

4. $a_{max} = \left|\frac{d^2x}{dt^2}\right| = \left|-A_o\omega_o^2\cos(\omega_o t + \varphi)\right| = A_o\omega_o^2$ ← [m/s²]

$V_{max} = \left|\frac{dx}{dt}\right| = \left|-A\omega_o\sin(\omega_o t + \varphi)\right| = A_o\omega_o$ ← [m/s]

5. See Fig 14.8 on p. 374