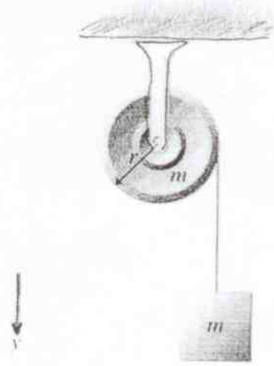
Name: ______________________

Problem 2



In a lab experiment to test the conservation of energy in rotational motion, we wrap a light, flexible cable around a solid cylinder of mass m and radius r. Here, $I = \frac{1}{2}mr^2$. The cylinder rotates with negligible friction about a stationary horizontal axis as depicted in the figure. We tie the free end of the cable to a mass m and release the the mass with no initial velocity a distance h above floor. As the mass falls, the cable unwinds – without stretching or slipping – thereby causing the cylinder to spin about its axis. (N.B. $K_{\text{rot}} = \frac{1}{2}I\omega^2$)

$$m = 2.00 \text{ kg} \qquad r = 0.500 \text{ m} \qquad h = 1.00 \text{ m}$$

PART I

(a) By means of the conservation of mechanical energy, calculate the speed of the falling mass just before it hits the ground.

(b) By means of the conservation of mechanical energy, calculate the angular velocity of the cylinder just before the falling mass hits the ground.

PART II

(c) Draw free-body diagrams for the hanging mass and the pulley and obtain the equations of motion for $\sum \tau_i = I\alpha$ and $\sum \vec{F}_i = m\vec{a}$.

(d) Does gravity provide a torque on the cylinder? Explain.

(e) Does the normal force from the strut upon the axle of the cylinder provide a torque on the cylinder? Explain.

(f) Does the cable provide a torque on the cylinder? Explain.

PART III   Hint: You may find the relationship $v^2 = v_\circ^2 + 2ah$ useful.

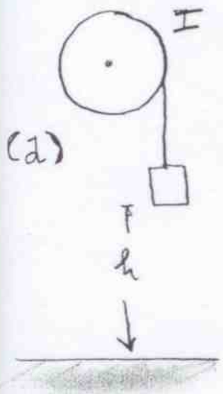
(g) Find the acceleration of the falling mass.

(h) Find the tension in the cable.

Name: ______________________

$I = \frac{1}{2}mr^2$

PART I



(a)

$E_i = mgh$

$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2$

$\Delta E_{mech} = 0 \Rightarrow E_i = E_f$

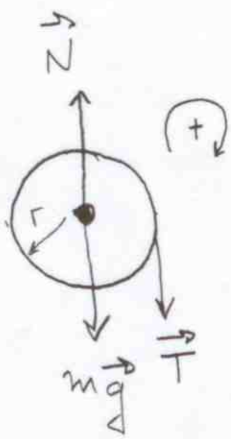
$mgh = \frac{3}{4}mv^2 \Rightarrow v = \left[\frac{4}{3}gh\right]^{1/2} = \left[\frac{4}{3}(9.80\,m/s^2)(1.00\,m)\right]^{1/2}$

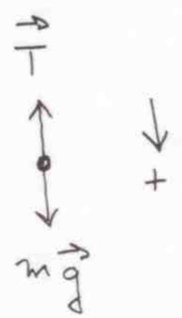
$\boxed{v = 3.62\,m/s}$

(b) $\omega = v/r = \dfrac{3.62\,m/s}{0.500\,m}$ $\boxed{\omega = 7.23\,rad/s}$

PART II

(c)





$\sum \tau_{cm} = Tr = I\alpha \quad (1)$

$\sum F_y = mg - T = ma \quad (2)$

(d) The cylinder is spinning about its center of mass.
Since $\vec{\tau} = \vec{r} \times \vec{F}_g$ and $|\vec{r}| = 0 \Rightarrow \boxed{\tau = 0}$
Ans: NO [The question, however, is ambiguous. If answered as in part (f), full credit will be awarded]

(e) Since the normal force passes through the axis of rotation, the moment arm is zero.
NO, $\vec{N}$ provides no torque.

(f) YES!

$|\vec{T}| \neq 0$, $r_\perp \neq 0$, and $\vec{T} \perp \vec{r}$

Hence $|\vec{\tau}| = r_\perp T \neq 0$.

Name: ______________________

$\boxed{\text{PART 3}}$ There are two ways to solve this problem.

A From Part I $v^2 = \frac{4}{3}gh$ since $v^2 = \cancel{v_0^2}^{\,0} + 2ah$,

(g) $a = \dfrac{v^2}{2h} = \dfrac{2}{3}g$ $\boxed{a = 6.53\,m/s^2}$

From eq. (2) in PART II $T = m(g-a) = m(g - \frac{2}{3}g)$

(h) $T = \frac{1}{3}mg = \frac{1}{3}(2.00\,kg)(9.80\,m/s^2) \Rightarrow \boxed{T = 6.53\,N}$

B Rewriting eq. (1) $Tr = (\frac{1}{2}mr^2)(\frac{a}{r}) \Rightarrow T = \frac{1}{2}ma$ (3)

Rewriting eq (2) $T = mg - ma$ (4)

Equating (3) and (4)

$$\frac{1}{2}ma = mg - ma \Rightarrow (\tfrac{1}{2}\cancel{m} + \cancel{m})a = \cancel{m}g$$

(g') $\Rightarrow \frac{3}{2}a = g$ or $\boxed{a = \frac{2}{3}g}$ ✓ checks.

Inserting this value of a into (4) gives

(h') $T = (1 - \frac{2}{3})mg$ or $\boxed{T = \frac{1}{3}mg}$ ✓ checks.