



If the collision is elastic, how high does the center of mass of m_2 rise after the collision?

When m_1 collides with m_2 it has speed:

$$m_1 g h_1 = \frac{1}{2} m_1 v_{ii}^2 \Rightarrow (1) \quad v_{ii} = \sqrt{2gh_1}$$

Now, in a collision momentum is conserved: $P_i = P_f$

$$m_1 v_{ii} = m_1 v_{if} + m_2 v_{2f}$$

$$(2) \quad v_{2f} = \frac{m_1}{m_2} (v_{ii} - v_{if})$$

$$(3) \quad v_{2f}^2 = \left(\frac{m_1}{m_2}\right)^2 (v_{ii} - v_{if})^2$$

If the collision is ELASTIC, $K_i = K_f$

$$\frac{1}{2} m_1 v_{ii}^2 = \frac{1}{2} m_1 v_{if}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$(4) \quad v_{2f}^2 = \frac{m_1}{m_2} (v_{ii}^2 - v_{if}^2) = \frac{m_1}{m_2} (v_{ii} - v_{if})(v_{ii} + v_{if})$$

Equating (4) and (3)

$$\frac{m_1}{m_2} (v_{ii} - v_{if})(v_{ii} + v_{if}) = \left(\frac{m_1}{m_2}\right)^2 (v_{ii} - v_{if})^2$$

$$v_{ii} + v_{if} = \frac{m_1}{m_2} (v_{ii} - v_{if})$$

② Regrouping like terms,

$$V_{if} \left(1 + \frac{m_1}{m_2}\right) = V_{ii} \left(\frac{m_1}{m_2} - 1\right)$$

$$V_{if} (m_1 + m_2) = V_{ii} (m_1 - m_2)$$

$$(5) \quad V_{if} = V_{ii} \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

Inserting (5) \rightarrow (2) yields

$$\begin{aligned} V_{2f} &= \frac{m_1}{m_2} \left[V_{ii} - V_{ii} \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \right] \\ &= \frac{m_1}{m_2} \left[\frac{m_1 + m_2 - m_1 + m_2}{m_1 + m_2} \right] V_{ii} \end{aligned}$$

$$(6) \quad V_{2f} = \left[\frac{2m_1}{m_1 + m_2} \right] V_{ii} =$$

Q: What happens when $m_1 = m_2$?
 $V_{if} = 0$ $V_{2f} = V_{ii}$

By conservation of M.E.

$$m_2 g h_2 = \frac{1}{2} m_2 V_{2f}^2$$

$$= \frac{1}{2} m_2 \left[\frac{2m_1}{m_1 + m_2} \right]^2 V_{ii}^2$$

$2gh_1$ from (1)

$$g h_2 = \frac{1}{2} \left[4 \left(\frac{m_1}{m_1 + m_2}\right)^2 \right] 2g h_1$$

$$h_2 = 4 \left(\frac{m_1}{m_1 + m_2}\right)^2 h_1$$

If $m_1 = m_2$

$$h_2 = h_1 \quad \checkmark$$

$$V_{if} = 0 \quad \text{from (5)}$$

$$V_{2f} = V_{ii} \quad \text{from (6)}$$



