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What if the axis of rotation moves?

When this happens, the motion of the body is a combination of

1) translational motion of the center of mass

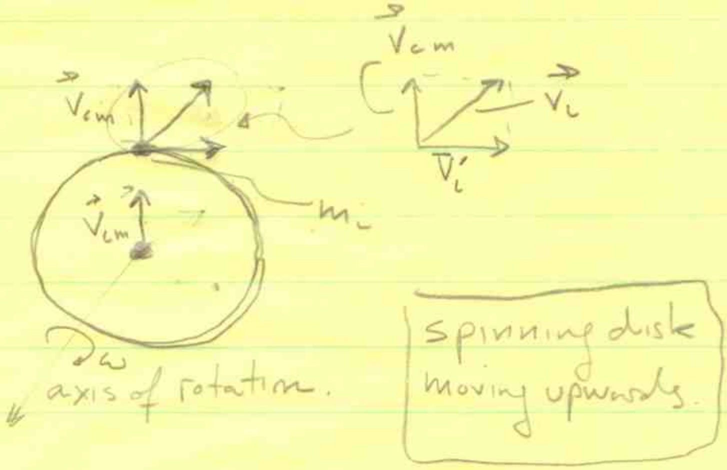
2) Rotation about an axis through the center of mass.

The <u>total kinetic energy</u>, then, is a sum of of K_{trans} and $K_{rotational}$.

$$K = K_{trans} + K_{rotational}$$

$$(*) \quad \boxed{K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2}$$

Let's prove (*)



$$\vec{V}_i = \vec{V}_{cm} + \vec{V}_i'$$

The kinetic energy of the i^{th} particle of mass m_i is

$$K_i = \frac{1}{2} m_i V_i^2 = \frac{1}{2} m_i (\vec{V}_{cm} + \vec{V}_i') \cdot (\vec{V}_{cm} + \vec{V}_i')$$

$$= \frac{1}{2} m_i (V_{cm}^2 + 2 \vec{V}_{cm} \cdot \vec{V}_i' + V_i'^2)$$

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$K_{TOT} = \sum_i K_i$ is the total kinetic energy of of the all the particles making up the body.

$$= \sum \tfrac{1}{2} m_i \left(v_{cm}^2 + 2\vec{v}_{cm}\vec{v}_i' + v_i'^2 \right)$$

$$= \tfrac{1}{2} \sum m_i v_{cm}^2 + \sum m_i \vec{v}_{cm} \vec{v}_i' + \tfrac{1}{2} \sum m_i v_i'^2$$

$$= \tfrac{1}{2} \left(\sum m_i \right) v_{cm}^2 + \vec{v}_{cm} \sum_i m_i \vec{v}_i' + \tfrac{1}{2} \sum m_i v_i'^2$$

<u>The First term</u>

$$\tfrac{1}{2} \left(\sum m_i \right) v_{cm}^2 = \tfrac{1}{2} M v_{cm}^2$$

<u>The second term.</u>

Note that $\sum m_i \vec{v}_i' = M \vec{v}_{cm}'$ is the velocity of the cm wrt the cm.

$$\vec{v}_i = \vec{v}_{cm} + \vec{v}_i'$$

$$m_i \vec{v}_i = m_i \vec{v}_{cm} + m_i \vec{v}_i'$$

$$\sum m_i \vec{v}_i = \sum m_i \vec{v}_{cm} + \sum m_i \vec{v}_i'$$

$$M \vec{v}_{cm} = \vec{v}_{cm} \left(\sum m_i \right) + M \vec{v}_{cm}'$$

$$M \vec{v}_{cm} = M \vec{v}_{cm} + M \vec{v}_{cm}'$$

$$\Rightarrow \quad \vec{v}_{cm}' = \emptyset.$$

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The third term

$$\frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\sum m_i r_i^2 \omega^2$$

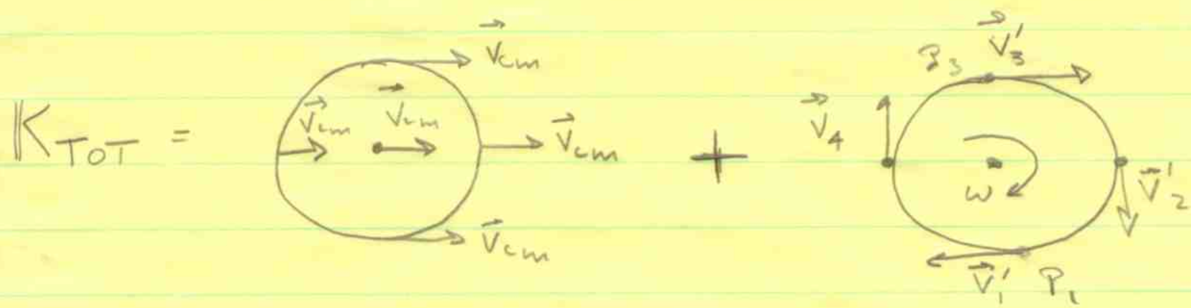
$$= \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$$= \frac{1}{2} I_{cm}\omega^2$$

Therefore

* $$K_{TOT} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$$

The condition for ROLLING WITHOUT SLIPPING.



If the ball is not slipping $\vec{V}_1' = -\vec{V}_{cm}$

i.e. it must be instantaneously at rest @P_1 so that it is not moving wrt point 1 in an inertial frame fixed wrt EARTH

$$\vec{V}_1 = \vec{V}_{cm} + \vec{V}_1' \; -$$

$$0 = \vec{V}_{cm} + \vec{V}_1' \Rightarrow \cancel{\vec{V}_{cm}} = -\vec{V}_1'$$

$$\vec{V}' = -\vec{V}_{cm}$$