You must show your work to get credit

FINAL Exam   Fall 2004   Physics 211

Professor: Philip L. Cole
Date: 13 Dec 2004

Test Time: 120 minutes
Possible Score: 100 points.

Write your name on each page. Do the easier problems first. If a problem seems too difficult, skip it, and return to it once you have completed all of the other problems first. You are required to answer problems 1 through 4, but you may choose between problems 5 or 5′. There are ten pages to this test. Good Luck!

Problem 1	________	(30 pts)
Problem 2	________	(15 pts)
Problem 3	________	(10 pts)
Problem 4	________	(20 pts)
Problem 5	________	(25 pts)
Problem 5′	________	(25 pts)
Total	________	(100 pts – MAX)

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Problem 1 A ball of clay of mass m is fired horizontally at a wooden block of mass M. The block is initially at rest and is situated away from the edge of a long table of 1.00-m height. Let $m = 50.0$ g and $M = 450.$ g. After this completely inelastic collision, the block/clay assembly travels 1.00 m along the surface of the table to the edge of the table. Upon reaching the edge, the block/clay assembly flies off the end of the table and lands 2.00 m away. You may assume that the acceleration due to gravity 9.80 m/s^2.

Useful Expressions

$$\vec{r} = \vec{r}_\circ + \vec{v}_\circ t + \frac{1}{2}\vec{a}t^2$$
$$\vec{v} = \vec{v}_\circ + \vec{a}t$$
$$v_y^2 = v_{y\circ}^2 + 2a_y\Delta y$$
$$v_x^2 = v_{x\circ}^2 + 2a_x\Delta x$$

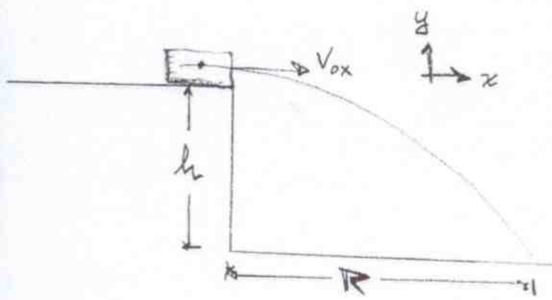
Part I (15pts)

(a) How long is the block/clay assembly in the air after it flies off the table?
(b) What is the speed of this assembly at the moment it leaves the table?
(c) When is linear momentum conserved?
(d) What is the initial speed of the ball of clay just before it strikes and sticks to the wooden block? Here, you may assume that the table possesses a smooth and frictionless surface.

Part II (15pts)

Now consider the case of a table with a rough surface, where the coefficient of kinetic friction between the block and the table is $\mu_k = 0.500$. The block/clay assembly leaves the table at the same speed as calculated in part (b), *i.e.* $v_{x\circ} = 4.43$ m/s.

(e) What is the Work-Kinetic Energy Theorem?
(f) Does friction do positive or negative work on the block as it slides along the table? Explain.
(g) What is speed of the block/clay assembly right at the moment when the ball of clay strikes and sticks to the wooden block? Hint: By means of the appropriate free-body diagram and the Work-Kinetic Energy Theorem, show that $\vec{f}_k \cdot \vec{d} = -\mu_k(m+M)gd = \Delta K = K_f - K_i$
(h) What is the initial speed of the ball of clay just before impact?



(a) $y = y_0 + V_0 t + \frac{1}{2}a_y t^2$

$0 = h + 0 + \frac{1}{2}(-g)t^2$

$\boxed{t = 0.452 s}$

$t = \left[\frac{2h}{g}\right]^{1/2} =$

$t = \left[\frac{2(1.00 m)}{9.80 m/s^2}\right]^{1/2}$

(b) $x = V_{ox} t \Rightarrow V_{ox} = \frac{R}{t} = \frac{2.00 m}{0.452 s}$

$\boxed{V_{ox} = 4.43 m/s}$

(c) We have by N II $\Sigma \vec{F}_{ext} = \frac{d\vec{p}}{dt}$. When the vector sum of the external forces is zero (i.e. $\Sigma \vec{F}_{ext} = 0$) then $\frac{d\vec{p}}{dt} = 0$.
This implies $\vec{p}$ = constant

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(d) The collision is completely inelastic $\vec{p}_i = \vec{p}_f \Rightarrow$

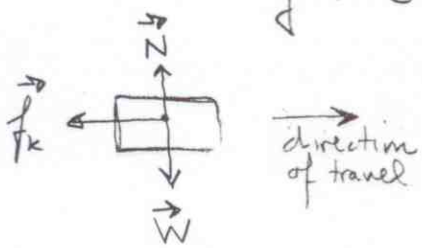
$$mV_1 = (m+M)V_{ox} \Rightarrow V_1 = \frac{(m+M)}{m}V_{ox} = \left(\frac{500g}{50g}\right)(4.43\,m/s)$$

$$\boxed{V_1 = 44.3\,m/s}$$

(e) $\boxed{W_{Net} = \Delta K}$

(f) Friction acts in such a manner as to oppose the motion. $\vec{f}_k$ will be antiparallel to $\vec{d}$, the dot product of which will be a NEGATIVE number. Moreover by work-kinetic energy theorem, the work done by friction must be negative when $K_f < K_i$.

(g)



$$f_k = \mu_k N = \mu_k W = \mu_k (m+M)g$$

$$W_{Net} = \Delta K$$

(1) $W_{Net} = \vec{f}_k \cdot \vec{d} = -\mu_k (m+M) g d$

(2) $W_{Net} = \Delta K = \frac{1}{2}(m+M)V_{ox}^2 - \frac{1}{2}(m+M)V_1^2$

Equating (1) & (2) and cancelling out the $m+M$ term yields

$$V_1 = [V_{ox}^2 + 2\mu_k g d]^{1/2} = [(4.43\,m/s)^2 + 2(0.500)(9.80\,m/s^2)(1.00\,m)]^{1/2}$$

$$\boxed{V_1 = 5.42\,m/s}$$

(h) Conservation of LINEAR MOMENTUM:

$$V_2 = \left(\frac{m+M}{m}\right)V_1 = \left(\frac{500g}{50g}\right)(5.42\,m/s)$$

$$\boxed{V_2 = 54.2\,m/s}$$

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Problem 2 A physically strong student sits on a rotating stool holding two 10.0-kg masses. When his arms are extended horizontally, the masses are 1.0 m from the axis of rotation, and he rotates with an angular speed of 0.50 rad/s. The moment of inertia of the student plus stool is 3.0 kg·m^2 and is assumed to be constant. The student then pulls the masses horizontally to 0.25 m from the rotation axis.

(a) When is angular momentum conserved?
(b) What is the mathematical definition of moment of inertia?
(c) What is the new angular speed of the student?

(a) $\sum \tau_{\phi} = \frac{\Delta L}{\Delta t}$ When the sum of the external torques is ZERO, then $\Delta L = 0$
$\underline{L_i = L_f}$ (about pt ϕ)

(b) $I = \sum m_i r_i^2$

(c) $I_i = I_o + 2mr_i^2$
$I_f = I_o + 2mr_f^2$

$\Delta L = 0 \Rightarrow L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f$

$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{I_o + 2mr_i^2}{I_o + 2mr_f^2}\omega_i$$

$$\omega_f = \frac{3.0\,\text{kg}\cdot\text{m}^2 + 2(10\,\text{kg})(1.00\,\text{m})^2}{3.0\,\text{kg}\cdot\text{m}^2 + 2(10\,\text{kg})(0.25\,\text{m})^2}(0.50\,\text{rad/s})$$

$$\omega_f = \left(\frac{23\,\text{kg}\cdot\text{m}^2}{4.25\,\text{kg}\cdot\text{m}^2}\right)(0.50\,\text{rad/s})$$

$$\boxed{\omega_f = 2.7\,\text{rad/s}}$$

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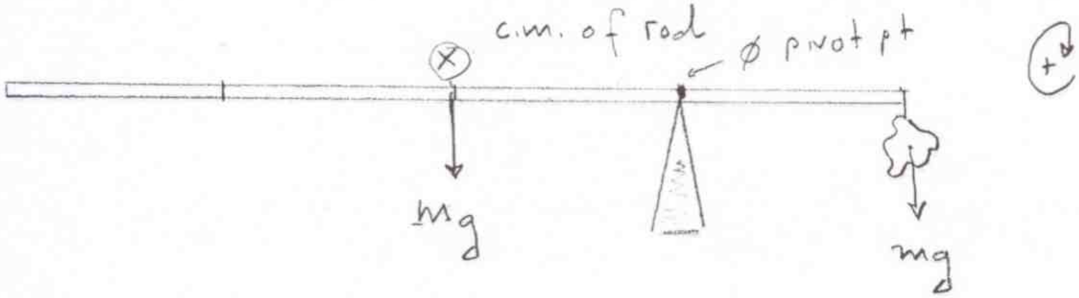
Problem 3 A 1.00-kg rock is suspended by a string from one end of a 1.00-m long measuring rod. The mass of the string is negligible. You may assume that rod is of uniform density.
(a) What conditions are necessary for an object to be in static equilibrium?
(b) What is the mass of the measuring rod if it is balanced by a support force at the 0.250-m mark?

(a) · $\sum \vec{F}_{ext} = 0$ & $\sum \vec{\tau}_{\phi} = 0$ about pivot pt ϕ



$\sum \tau_{\phi} = 0$

$+mg\ell - Mg\ell = 0 \Rightarrow \boxed{M = m}$

$\boxed{M = 1.00\,kg}$

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Problem 4 On planet Clair, where the acceleration due to gravity is always a pleasant 10.000 m/s^2, Dr. Aaron Aardvark decides to engage in some much needed water therapy. He steps off a 10.0-m high diving platform and begins to fall from rest. Dr. Aardvark's mass is 75 kg. Three (3.0) seconds after reaching the water, Dr. Aardvark comes to a rest.

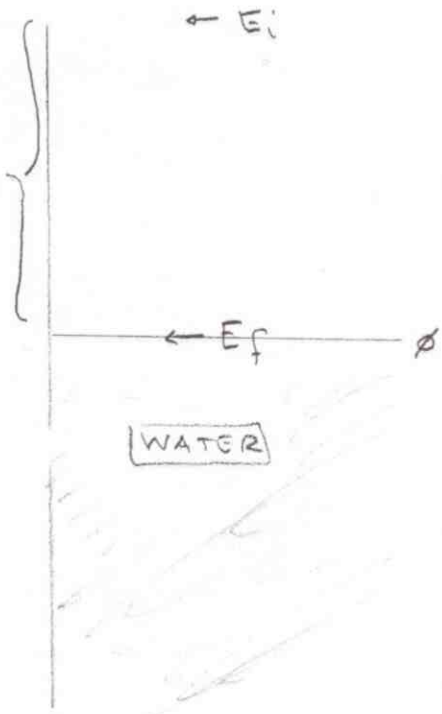
(a) What is the speed of Dr. Aardvark as he enters the water?
(b) Draw a free-body diagram of Dr. Aardvark when he is immersed in the water.
(c) What is the impulse-momentum theorem mathematically?
(d) What average force did the water exert on Dr. Aardvark between the time he enters the water and comes to a rest?

(a) $\Delta E = 0 = E_i = E_f$

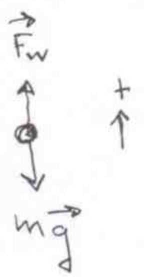
$mgh = \frac{1}{2}mv^2$

$v = [2gh]^{1/2} = [2(10.0\,m)(10.000\,m/s^2)]^{1/2}$

$\boxed{v = 14.1\,m/s}$



(b)



(c) $\Delta\vec{p} = (\Sigma\vec{F})\Delta t = \vec{J}$

(d) $F_W - mg = \frac{1}{\Delta t}(0 - (-mv))$

$F_W = mg + m\left(\frac{v}{\Delta t}\right) = m\left[g + \frac{v}{\Delta t}\right]$

$F_W = (75\,kg)\left[10.000\,m/s^2 + \frac{14.1\,m/s}{3.0\,s}\right]$

$\boxed{F_W = 1100\,N}$ (upwards)

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Problem 5 A cylinder having an initial volume V contains a sample of gas at pressure p and temperature T. You may assume that the parameters of an ideal gas are described by the equation

$$pV = nRT,$$

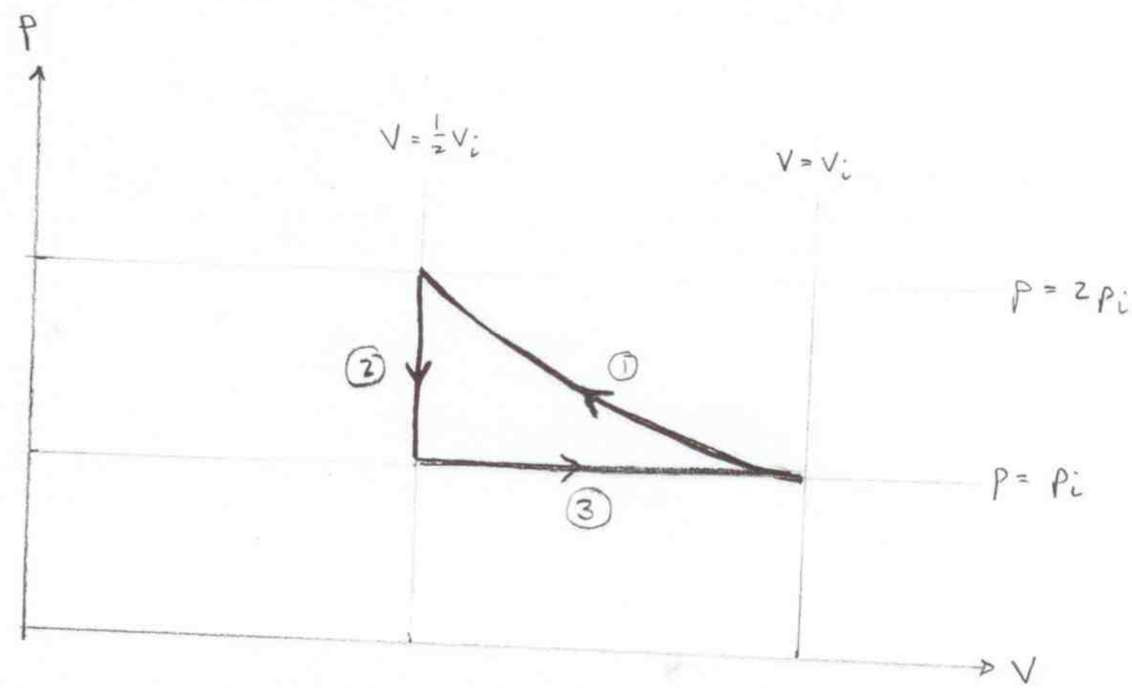
where R is the universal gas constant (8.31 J/mol K) and p, V, n, and T are the **state variables.**

A gas characterized by the initial state variables p_i, V_i, and T_i undergoes the following "cycle." You may assume that n is held fixed. As a reminder: *isothermal process* – the temperature remains constant, *isochoric process* – the volume does not change, and *isobaric process* – the pressure stays the same.

1. The gas undergoes *isothermal* compression: the volume is halved and the pressure is doubled.
2. After the gas reaches the volume $\frac{1}{2}V_i$ and pressure $2p_i$, it is cooled *isochorically* to its original pressure p_i.
3. The gas then expands *isobarically* to its original volume V_i and pressure p_i.

(a) On a pV diagram, draw and label each of these three segments of the cycle detailed above.
(b) For the isothermal compression, what is the change in the internal energy of the gas? Explain.
(c) For the isothermal compression, how is the work W and heat transfer Q related?
(d) For the isochoric process, in terms of the variables p_i and V_i, what work is done on the gas?
(e) For the isochoric segment of the cycle, in terms of the T_i, what is temperature of the gas?
(f) For the isobaric process, in terms of p_i and V_i, what is the work done during this process?
(g) At the end of the isobaric process, in terms of T_i, what is the final temperature?

(a)



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(b) if $\Delta T = 0 \Rightarrow \Delta E_{th} = 0$ $\boxed{\text{NO CHANGE}}$

(c) $\Delta E_{th} = Q + W \Rightarrow$ $\boxed{W = -Q}$

(d) $\Delta V = 0$ $\boxed{W = 0}$

(e) $nR = \text{const}$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \Rightarrow T_2 = \frac{p_2 V_2}{p_1 V_1} T_1$$

$$T_2 = \frac{(p_i)(\frac{1}{2}V_i)}{(2p_i)(\frac{1}{2}V_i)} T_i$$

$\boxed{T = T_i/2}$

(f) $$W = -\int_{V_2}^{V_3} p\,dV = -p_i(V_3 - V_2)$$

$$= -p_i\left(V_i - \frac{1}{2}V_i\right) = -\frac{1}{2}p_i V_i$$

$\boxed{W = -\frac{1}{2}p_i V_i}$

$\boxed{T = T_i}$

(g) $p_i V_i = (nR)T$

$$T = \frac{p_i V_i}{nR} = T_i$$

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Problem 5′

(a) Define mathematically the first law of thermodynamics and state which conservation principle this law reflects.
(b) Define in words what is meant by the second law of thermodynamics and give an example of a clear violation of this law.

Answer the following questions within the framework of the kinetic theory of gases. You may assume that n is fixed. By the Equipartition Theorem, the translational degrees of freedom:

$$\frac{1}{2}M(v_x^2)_{\text{avg}} = \frac{1}{2}M(v_y^2)_{\text{avg}} = \frac{1}{2}M(v_z^2)_{\text{avg}} = \frac{1}{2}k_B T$$

$$[\text{ N.B. } (v_x^2)_{\text{avg}} = (v_y^2)_{\text{avg}} = (v_z^2)_{\text{avg}} \text{ and } v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}} = \sqrt{3(v_x^2)_{\text{avg}}}\ ]$$

where Boltzmann's constant has the value of $k_B = 1.38 \times 10^{-23}$ J/K.

(c) If you doubled the temperature of a monatomic gas, how would the average translational kinetic energy change?
(d) If you reduced the speed of every molecule in a monatomic gas by a factor of two, how would the temperature of the gas change?

Given that hydrogen molecules have a mass of $2u$ and oxygen molecules have a mass of $32u$, where u is defined as the atomic mass unit ($u = 1.661 \times 10^{-27}$ kg), answer the following:

(e) At what gas temperature T would the average translational kinetic energy of a hydrogen molecule be equal to that of an oxygen molecule at 320 K?
(f) At what gas temperature T would the root-mean-square (rms) speed of a hydrogen molecule be equal to that of an oxygen molecule at 320 K?

For an isothermic process, the temperature of an object remains constant as it exchanges heat with the surroundings. The change in entropy is given by

$$\Delta S = \frac{Q}{T}$$

(g) In a well-insulated calorimeter, 1.0 kg of water at 20°C is mixed with 1.0 g of ice at 0°C. What is the *net* change in entropy ΔS_{sys} of the system at the very moment the ice completely melts? The heat of fusion of ice is $L_f = 3.33 \times 10^5$ J/kg. Express your answer numerically in joules per kelvin.

(a) $\Delta E_{th} = Q + W$ (conservation of energy)

(b) The ENTROPY of an isolated system never decreases. The entropy either increases, until the system reaches equilibrium, or, if the system began in equilibrium, stays the same.

Example An example of a clear violation of the 2nd Law of Thermodynamics: HEAT energy transferring from an ice cube into the water it is immersed in making the ice cube colder and turning the water into steam.

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(c) Since $K = \frac{3}{2}k_B T$ $\quad T \to 2T \Rightarrow \boxed{K \to 2K}$

(d) $V_{rms}^2 = \frac{3k_B T}{m}$ $\quad V_{rms} \to \frac{1}{2}V_{rms} \Rightarrow \boxed{T \to \frac{1}{4}T}$

(e) Kinetic Energy is a function only of Temperature, it is not a function of molecular specie.

$\boxed{T = 320K}$ ← At a given temp T, all molecular species have the same Kinetic Energy

(f) $V_1^2 = \frac{3k_B T_1}{m_1}$ & $V_2^2 = \frac{3k_B T_2}{m_2}$

$V_1^2 = V_2^2$ when $T_2 = \frac{m_1}{m_2}T_1 = \frac{2u}{32u}(320k)$ $\quad \boxed{T_2 = 20K}$

(g) The ice will melt once it absorbs sufficient energy to bring it to 0°C and then absorb enough $Q = mL_f$ to bring it to a liquid state. Since the ice cube is already at $T = 0°C$

$$Q_1 = (0.001kg)(3.33 \times 10^5 J/kg) = 333 J$$

$$\Delta S_1 = \frac{Q_1}{T} = \frac{333J}{273K} = 1.22 J/k$$

$$\Delta S_2 = \frac{-333J}{293k} = -1.14 J/k$$

$$\Delta S_{sys} = \Delta S_1 + \Delta S_2 = \underline{+0.083 J/k}$$