Name: Key

Student ID #: ____________________

Examination 3 Spring 2008 Physics 211 Professor Philip L. Cole
Date April 4, 2008

Test Time: 50 minutes

Write your name on each page. Do the easier problems first. If a problem seems too difficult, skip it, and return to it once you have completed all of the other problems first. There are five pages to this test. You will find a formula sheet on the last page. An extra sheet is provided for scratch.

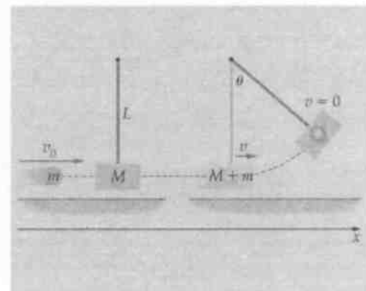
You must show all work. No credit will be assigned for answers without the appropriate work. Be very careful with significant figures.

Problem 1	____________	**(30 pts)**
Problem 2	____________	**(40 pts)**
Problem 3	____________	**(25 pts)**
Problem 4	____________	**(15 pts)**
Maximum	____________	**(100 pts)**

Name: ______________________

Problem 1. Ballistic Pendulum

In a ballistic pendulum, an object of mass m is fired with an initial speed of v_0 at the bob of a pendulum. The bob has a mass M and is suspended by a rod of negligible mass. After the collision, the object and the bob stick together and swing through an arc, eventually gaining a height h.



1. In the absence of external forces what quantity must be conserved?

$\Sigma \vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \Delta\vec{p} = 0$ Hence LINEAR momentum IS CONSERVED.

2. What is the momentum just before and the momentum just after the collision?

(1) $P_i = mV_0$

(2) $P_f = (M+m)V$

3. An object of mass 10.g is fired into this ballistic pendulum formed of a 1.000-kg bob attached to the ceiling by a light rod of length 1.00 m. The object stops completely in the bob as depicted in the figure above. The pendulum then rises to a height of 0.051 m. Find the speed of the bullet before impact. Assume that the mechanical energy is conserved throughout the process of swinging.

$E_i = \frac{1}{2}(m+M)V^2$ $\qquad E_f = (m+M)gh$

Since $\Delta E = 0$ $\quad E_i = E_f \Rightarrow \frac{1}{2}(m+M)V^2 = (m+M)gh$

(3) $V = \sqrt{2gh}$

Equating (1) and (2) since $\Delta p = 0 \Rightarrow V_0 = \left(\frac{m+M}{m}\right)V$ (4)

Inserting (4) into (3)

$$V_0 = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

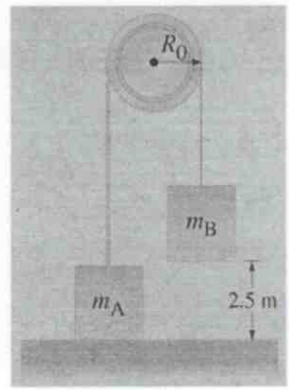
$$= \left(\frac{1.01}{0.010}\right)\left[2(9.80\,m/s^2)(0.051\,m)\right]^{1/2}$$

$\boxed{V_0 = 100\,m/s}$ (to two sig figs)

Name: ______________________

Problem 2. <u>Atwood's Machine</u>

Atwood's machine consists of two masses, m_A and m_B, which are connected by a massless inelastic cord that passes over a pulley. The pulley has radius R_0 and moment of inertia I about its axle. You may assume that the cord does not slip. The masses of m_A and m_B, are 45.00 kg and 50.00 kg, respectively. The pulley is a uniform cylinder of radius 0.2001 m and mass 10.00 kg and rotates about an axle passing through its center of mass. Mass m_A is initially on the ground and mass m_B rests 2.5 m above the ground.



1. From your Free Body Diagrams for m_A and m_B, find the sum of the forces acting on m_A and m_B, respectively.

m_A: T_A, $m_A g$, $\downarrow +$

$\Sigma F = m(-a)$

$m_A g - T_A = -m_A a$

(1) $T_A = m_A g + m_A a$

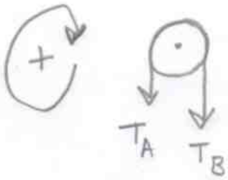
m_B: T_B, $m_B g$, $\downarrow +$

$\Sigma F = m_B a$

$-T_B + m_B g = m_B a$

(2) $T_B = m_B g - m_B a$

2. Find the sum of the torques acting on the pulley.



$\Sigma \tau_{cm} = I_{cm}\alpha = I \frac{a}{R_0}$

$\Sigma \tau_{cm} = RT_B - RT_A$

$\Rightarrow$ (3) $T_B - T_A = I \frac{a}{R_0^2}$

3. Determine the acceleration of the masses m_A and m_B, in terms of m_A, m_B, and I.
After you have done this algebraically, find the acceleration numerically.

Inserting (1) & (2) into (3)

$(m_B g - m_B a) - (m_A g + m_A a) = I \frac{a}{R_0^2}$

$a(m_A + m_B + I/R_0^2) = (m_B - m_A) g$

$a = \left(\frac{m_B - m_A}{m_A + m_B + I/R_0^2}\right) g$

Since $I = \frac{1}{2} M R_0^2$

$a = \left(\frac{m_B - m_A}{m_A + m_B + \frac{M}{2}}\right) g$

$a = \left(\frac{5.00\text{ kg}}{100.0\text{ kg}}\right)(9.80\text{ m/s}^2)$

$\boxed{a = 0.49\text{ m/s}^2}$

4. What is the tension in the cord on the m_A and m_B sides of the pulley, numerically.

From (1)

$T_A = m_A(g + a) = (45.00\text{ kg})(9.80\text{ m/s}^2 + 0.49\text{ m/s}^2)$ $\boxed{T_A = 463\text{ N}}$

From (2)

$T_B = m_B(g - a) = (50.00\text{ kg})(9.80\text{ m/s}^2 - 0.49\text{ m/s}^2)$ $\boxed{T_B = 466\text{ N}}$

Name: ______________________

Problem 3 <u>**Spinning About**</u>

A uniform stick 2.000 m long with a total mass of 500. g is pivoted at its center. A 10.0-g bullet is shot through the stick midway between the pivot and one end. The bullet approaches at 250. m/s and exits at 140. m/s.

1. Write down the mathematical expression for angular momentum in terms of the vector cross product.

$$\vec{L} = \vec{r} \times \vec{p}$$

2. What is the perpendicular distance of the bullet with respect to the pivot point, i.e. you may assume that the axis of rotation goes right through the pivot point directed into the page of this paper.

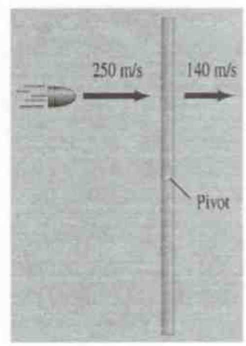
$$r_\perp = \ell/4\ ;\ \ell = 2.000\,\text{m} \Rightarrow r_\perp = 0.5000\,\text{m}$$

3. What is the angular momentum of the system before impact?

$$L_i = m r_\perp V_0$$

4. What is the angular momentum of the system right after the bullet has shot through the stick?

$$L_f = m r_\perp V_1 + I\omega$$



5. Calculate the angular speed of the stick right after the bullet is passes through the stick.

$$\Delta L = 0 \Rightarrow L_i = L_f$$

$$m r_\perp V_0 = I\omega + m r_\perp V_1$$

$$\omega = \frac{m r_\perp}{I}(V_0 - V_1) = \frac{m\left(\frac{\ell}{4}\right)}{\frac{1}{12}M\ell^2}(V_0 - V_1)$$

$$\omega = 3\left(\frac{m}{M}\right)\frac{V_0 - V_1}{\ell} = 3\left(\frac{10.0}{500.}\right)\frac{250.\,\text{m/s} - 140.\,\text{m/s}}{2.000\,\text{m}}$$

$$\boxed{\omega = 3.30\ \text{rad/s}}$$

6. If the stick makes 159 rotations before coming to a complete stop, find its angular acceleration.

$$\overset{0}{\omega_f^2} = \omega_0^2 + 2\alpha\Delta\theta \Rightarrow \alpha = -\frac{\omega_0^2}{2\Delta\theta}$$

$$\Delta\theta = 159\ \text{rotations} \times \frac{2\pi\ \text{rad}}{1\ \text{rotation}} = 999\ \text{rad}$$

$$\alpha = -\frac{(3.30\ \text{rad/s})^2}{2(999\ \text{rad})} \Rightarrow \boxed{\alpha = -5.45\times10^{-3}\ \text{rad/s}^2}$$

Name: ______________________

Problem 4 Halley's Comet

Halley's Comet approaches the Sun to within 0.570 A.U. and its orbital period is 75.6 years. Here A.U. is the abbreviation for astronomical unit, which is the mean Earth-Sun distance (1 A.U. = 1.50×10^{11} m).

1. Show that in the absence of any external torques, angular momentum must be conserved.
2. The distance of closest (farthest) approach for Halley's comet is 0.570 A.U. (35.2 A.U.). If the comet's speed at closest approach is 54 km/s, what is its speed when it is farthest from the Sun. You may assume that the mass of the comet does not change appreciably.

1. $\sum \vec{\tau}_{\phi} = \frac{d\vec{L}_{\phi}}{dt}$. If $\sum \vec{\tau}_{\phi} = 0 \Rightarrow \Delta \vec{L} = 0$

$\therefore \vec{L}_i = \vec{L}_f$

2. $m v_1 r_1 = m v_2 r_2$

$$V_2 = \frac{r_1}{r_2} V_1 = \left(\frac{0.570 \text{ A.U.}}{35.2 \text{ A.U.}}\right)(54 \text{ km/s})$$

$$\boxed{V_2 = 0.87 \text{ km/s}}$$

This expresses Kepler's Second Law in a nice compact form: Equal Areas are swept out in equal times or $\Delta L = 0 \Rightarrow$ KII