Name: ____________________ Student ID #: KEY

Examination 2 Spring 2008 Physics 211 Professor Philip L. Cole

Date March 3, 2008

Test Time: 50 minutes

Write your name on each page. Do the easier problems first. If a problem seems too difficult, skip it, and return to it once you have completed all of the other problems first. There are five pages to this test. You will find a formula sheet on the last page. An extra sheet is provided for scratch.

You must show all work. No credit will be assigned for answers without the appropriate work. Be careful with significant figures.

Problem 1	____________	**(40 pts)**
Problem 2	____________	**(20 pts)**
Problem 3	____________	**(35 pts)**
Problem 4	____________	**(15 pts)**
Maximum	____________	**(100 pts)**

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Problem 1. **Kepler's Third Law**

The Moon's period about the Earth is 27.3 days and the distance from the Earth to the Moon is 3.84×10^8 m. Assume the orbit is circular, i.e. the speed is constant. We may take the constant of universal gravitation to be $G = 6.67 \times 10^{-11}$ N m^2/ kg^2. You may note that the magnitude of the force from Newton's law of universal gravity can be written as

$$F_{grav} = G \frac{Mm}{r^2}$$

a) Which pulls harder gravitationally, the Moon on the Earth or the Earth on the Moon? Explain.
b) What work does the Earth do on the Moon for it to orbit the Earth with uniform circular motion?
c) Draw a free-body diagram of the force on the Moon from the Earth. Indicate whether the force is toward the Earth, away from the Earth and/or tangent to the orbit.
d) Using Newton's second law, relate F_{grav} and ma_r and given that $vT = 2\pi r$, derive Kepler's third law, which states $T^2 = Kr^3$. That is find K in terms of the M and G. You must derive, not just state.

Answer either e) or f) or both. You can answer part f) without knowing K.

e) From what you determined for K, and using the Moon's period and mean distance from the center of the Earth, estimate the mass of the Earth. (SI units)

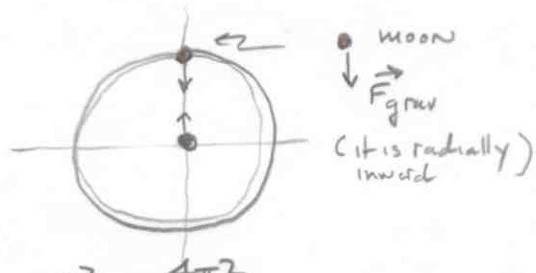
f) Now given that the mean distance between the centers of the Earth and the Moon is roughly equal to 60 Earth radii, quickly estimate for a satellite that orbits the earth in one day, how far the satellite must be from the surface of the earth in terms of Earth radii. (Hint: Think ratios to cancel K). This is the altitude for a *geosynchronous orbit.*

[NEITHER]

a) Equal and opposite: Newton's 3rd Law.

b) $\Delta V = 0 \Rightarrow \Delta K = 0 \Rightarrow$ Work is zero

c)



d) $\sum F_r = ma_r = m v^2/r$; $\sum F_{rad} = G \frac{Mm}{r^2}$

$$G\frac{Mm}{r^2} = mv^2/r = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = 4\pi^2 \frac{r}{T^2} \Rightarrow T^2 = \underbrace{\frac{4\pi^2}{GM}}_{K} r^3$$

e) $M = \frac{4\pi^2}{G}\frac{r^3}{T^2}$; $T = 27.3\,d \times \frac{24\,h}{1\,d} \times \frac{(60)^2\,s}{1\,h}$

$T = 2.36 \times 10^6$ s.

$$M_E = \frac{4\pi^2}{(6.67\times10^{-11}\ \frac{N\,m^2}{kg^2})}\frac{(3.84\times10^8\,m)^3}{(2.36\times10^6\,s)^2} \Rightarrow \boxed{M_E = 6.02\times10^{24}\ kg}$$

f) $T_1^2 = Kr_1^3$ & $T_2^2 = Kr_2^2 \Rightarrow$ Taking the ratio $\Rightarrow r_2 = \left(\frac{T_2}{T_1}\right)^{2/3} r_1$

From center to center: $r_1 = 60 r_E$

$T_1 = 27.3\,d$

$T_2 = 1.0\,d$

$h = r_2 - r_E$

$\boxed{h = 5.6\, r_E}$

$$r_2 = \left(\frac{1.0}{27.3}\right)^{2/3} 60 r_E \rightarrow 6.6\, r_E$$

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Problem 2. Busting Loose

A car passenger is buckled tightly in with a seat belt and is holding a 10.-kg package on her lap.

a) What is the definition of work for a
   i. constant force in the direction of displacement
   ii. varying force making an angle of θ with respect to the direction of displacement. Hint: dot product and integral….
b) What is the Work-Energy Principle *mathematically*?
c) While traveling at 20. m/s, the driver has to make an emergency stop over a distance of 40. m. Assuming constant deceleration, how much force must the arms of the passenger exert to hold the package in place during this deceleration period.
d) Should the passenger's hold on the package slip during this deceleration period, describe and explain the motion of the package.

a) $W = \vec{F} \cdot \vec{s} = Fs$ $\quad \vec{F} \parallel \vec{s}$, $|\vec{F}|$ is constant.

$W = \int \vec{F} \cdot d\vec{s}$

These formulas are on the FORMULA SHEET.

b) $W_{net} = \Delta K$

c) $Fs = \frac{1}{2}mv^2 \Rightarrow F = \frac{mv^2}{2s} = \frac{(10.\,kg)(20.\,m/s)^2}{2(40.\,m)}$

$$\boxed{F = 50.\,N}$$

d) By Newton's 1st Law, the object will continue in its STATE OF MOTION unless there is a force holding it back. In simpler terms, the package will collide with the dashboard. The dashboard is changing speed in time & the package is not so constrained. Collision

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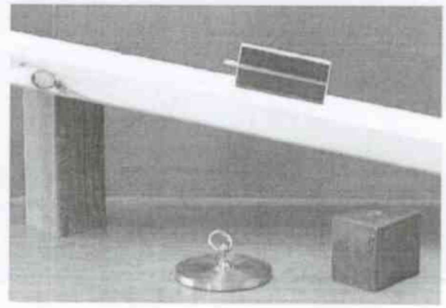
Problem 3. Slip/Sliding Away

A block of mass 1.0001 kg is released from rest on the inclined plane from a height *h* of 0.191 m above the table as depicted below. The incline makes an angle of 11.0° with respect to the horizontal. It slides a distance *d* of 1.00 m along the length of the incline, where the coefficient of kinetic friction μ_k is 0.1002.

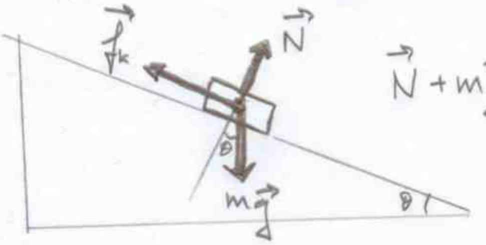
a) What is the potential energy of the block with respect to plane of the table?
b) Draw the free body diagram for the block as it is traveling down the incline. (Draw your *x-y* axes. Hint. The *y* direction should be parallel to the normal force). Resolve these forces into components and apply Newton's second law accordingly.
c) What is the work done by friction on the block? And is this work by friction due to a conservative force? Explain.
d) What is the work done by the normal force on the block?
e) Find the speed of the block after it has traveled 1.0 m down the incline.



a) $U_{grav} = E_i = mgh = (1.00\text{kg})(9.80\text{m/s}^2)(0.191\text{m})$

$\boxed{U_{grav} = 1.87\text{J}}$

b)



$\vec{N} + m\vec{g} + \vec{f}_k = m\vec{a}$

$\Sigma F_x: \; mg\sin\theta - f_k = ma$

$\Sigma F_y: \; N - mg\cos\theta = 0$

$f_k = \mu_k N = \mu_k mg\cos\theta.$

c) The work done by friction is $W_f = \vec{f}_k \cdot \vec{s} = -f_k s$, $= \boxed{-0.962\text{J}}$

NO. Friction is not a CONSERVATIVE FORCE. It is not reversible and it depends on path. $[W_f = (0.1002)(1.001\text{kg})(9.80\text{m/s}^2)\cos 11° \times (1.00\text{m}) =$

d) $\vec{N} \perp \vec{s} \Rightarrow \vec{N} \cdot \vec{s} = 0$. The work done by the Normal Force IS ZERO.

e) $W_{other} = \Delta E$; $-f_k s = K_f - E_i = \frac{1}{2}mv^2 - mgh$

$\frac{1}{2}mv^2 = -f_k s + mgh = mgh - \mu_k mg\, s\cos\theta$

$v^2 = 2g(h - \mu_k s\cos\theta)$

$v = \left\{2(9.80\text{m/s}^2)\left[0.191\text{m} - (0.100)(1.00\text{m})\cos 11°\right]\right\}^{1/2}$

$\boxed{v = 1.35\text{m/s}}$

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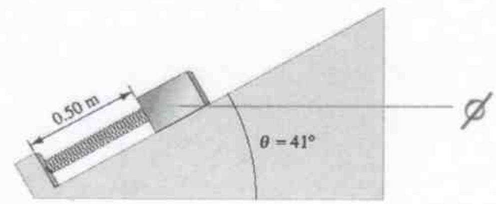
Problem 4. Spring Loaded.

A spring (k = 100. N/m) has an equilibrium length of 1.00 m. The spring is compressed to a length of 0.500 m. A mass of m of 2.0001 kg is placed at its free end on a frictionless slope which makes an angle of 41.0° with respect to the horizontal. The spring is then released. The mass is not attached to the spring.

a) How much energy is stored in the spring when compressed to 0.500 m?
b) How far up along the length of the slope will the mass move before coming to a complete stop?
c) What is the speed of the block once the spring travels 0.500 m or to its equilibrium length of 1.00 m?

We take the ref. pt (Ø) at its "compressed" pt.



a) $U_{el} = \frac{1}{2}k(\Delta s)^2$

$= \frac{1}{2}(100\,N/m)(0.500\,m)^2 = \underline{12.5\,J}$

b) $\Delta E = 0 \Rightarrow \cancel{K_f}^{0} + U_f = \cancel{K_i}^{0} + U_i \;;\; U_f = U_i$

$U_f = mgh = mg\,s\sin\theta = U_i = 12.5\,J$

$s = \dfrac{12.5\,J}{(2.00\,kg)(9.80\,m/s^2)\sin 41°}$ $\boxed{s = 0.972\,m}$

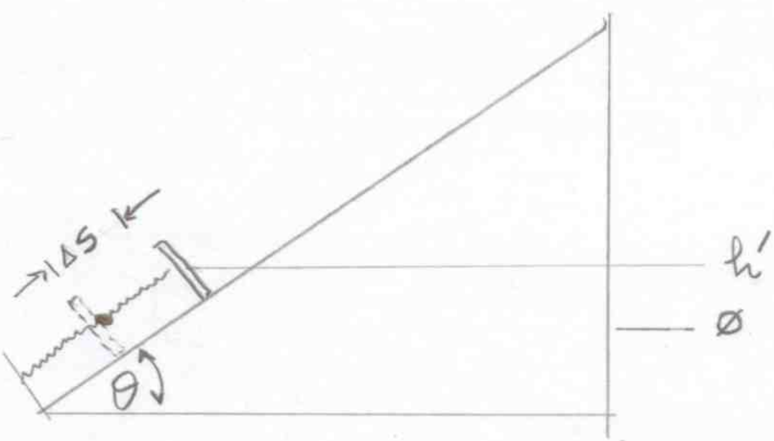
c)

$E_i = U_{el}$

$E_f = \frac{1}{2}mv^2 + mgh'$

$\Delta E = 0 \Rightarrow E_f = E_i$

$\frac{1}{2}mv^2 + mgh' = U_{el}$



$V^2 = \frac{2}{m}\left[U_{el} - mg(\Delta s)\sin\theta\right]$ $h' = \Delta s \sin\theta$

$V = \left\{\frac{2}{(2.00\,kg)}\left[12.5\,J - (2.00\,kg)(9.80\,m/s^2)(0.500\,m)(\sin 41°)\right]\right\}^{1/2}$

$\boxed{V = 2.46\,m/s}$

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