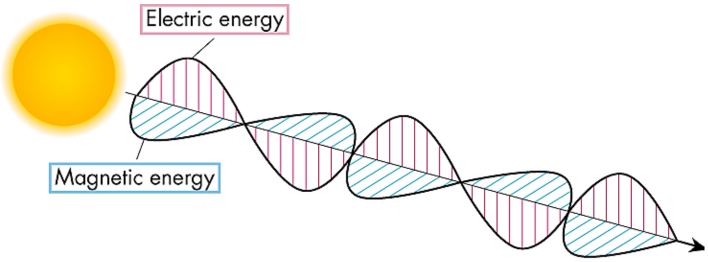
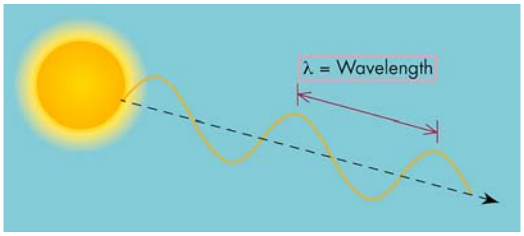
Introduction to Light and the E/M Spectrum

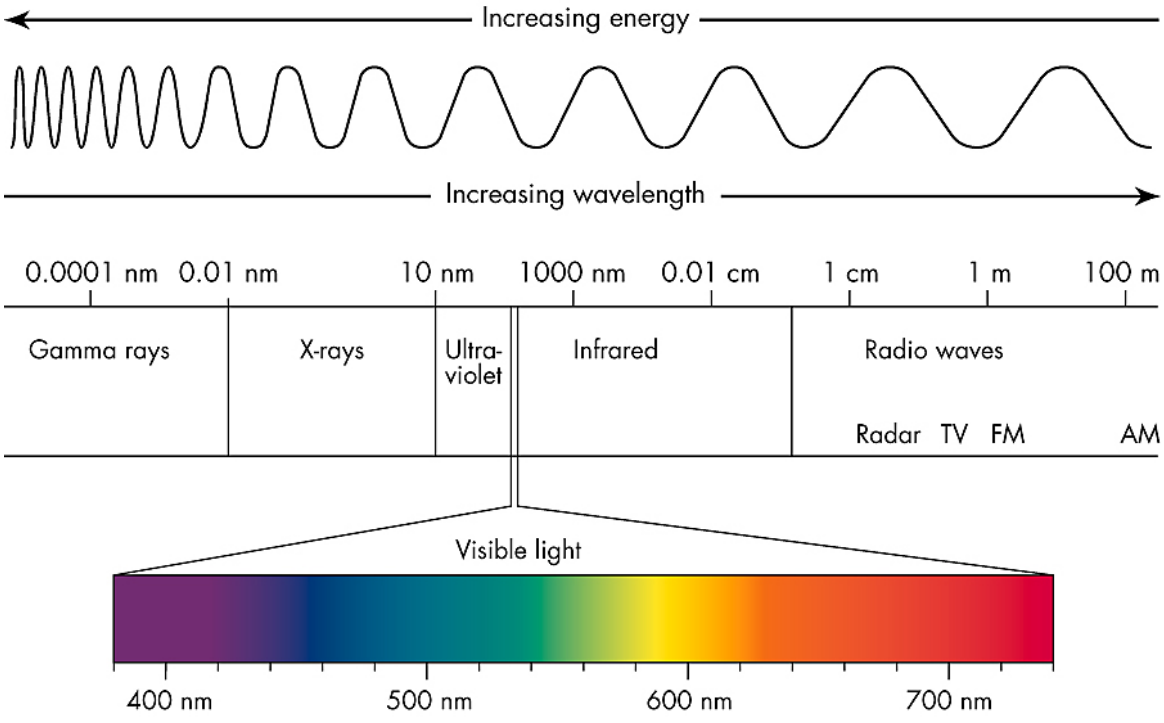
(all uncreditied figures courtesy of Thomas Arny)

The Electromagnetic Spectrum/Electromagnetic Waves

- E/M waves are disturbances that travel through space
- E/M waves are transverse, and harmonic
- E/M waves have particle and wave properties
- E/M waves travel at 3×10^8 m/s in free space, c
- E/M waves transport energy (electromagnetic energy)
- $c = \lambda f$
- $E = hf = h\frac{c}{\lambda}$
- decreasing wavelengths → increasing energy
- longer wavelengths → lower frequency
- E/M waves reflect, refract, scatter, diffract, interfere, and may be polarized





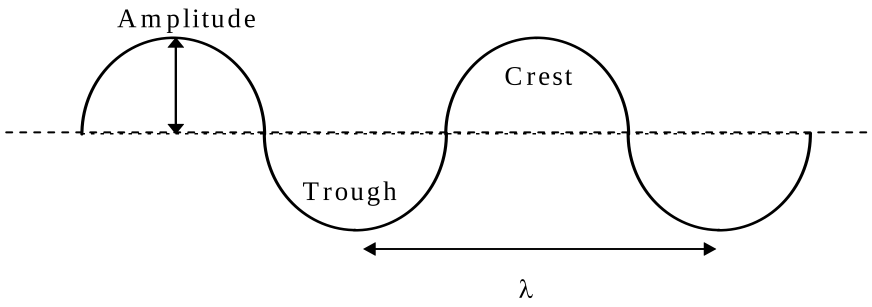


Historical Figures and Their Contributions to Theories of Light

- Newton – corpuscular theory
- Huygens – wave theory
- Young – interference
- Maxwell – E/M equations
- Michaelson & Morley – luminiferous ether
- Einstein – photons
- Bose – photons are indistinguishable particles

Physical Attributes of Light

- Visible light exists between 400 – 700 nanometers (10^{-9})
- White light consists of all frequencies in the visible range combined
- Wavelength – peak to peak distance
- Amplitude – ½ of the distance from peak to trough
- Frequency – number of oscillations per second



- Photons are indistinguishable particles known as *bosons* (not subject to the Pauli Exclusion Principle). Electrons, protons and neutrons are *fermions* (subject to the P.E.P.)
- The P.E.P. is very important in determining the fate of stars as we will see later.

Electromagnetic Waves and Maxwell's Equations

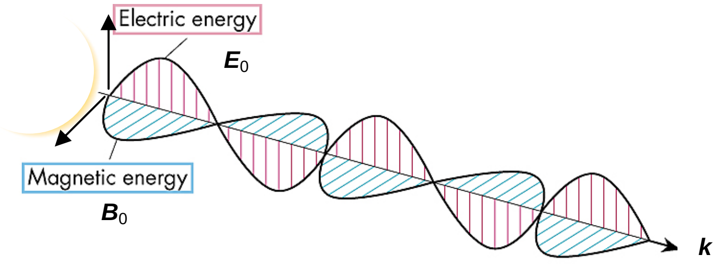
Maxwell's Equations describe all classical electromagnetic phenomena in much the same manner as Newton's Laws describe all classical mechanical phenomena

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$	Gauss' Law
$\oint \vec{B}\cdot d\vec{A} = 0$	Gauss' Law for Magnetism
$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_m}{dt}$	Faraday's Law
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$	Ampere's Law
$\vec{F} = q\vec{E} + qv\vec{B}$	The Lorentz Force Equation

For harmonic transverse waves traveling to the right along the *x*-axis of an arbitrary coordinate system (assuming that *y* = 0 when *x* = 0 and *t* = 0) the displacement of the medium with respect to time may be expressed as: $y = A\sin(kx \pm \omega t)$

An equivalent form of these equations is $\psi = \psi_0 \sin(kx \pm \omega t)$ where a "+" sign indicates the wave is moving to the left. In general the displacement of the medium is represented by a general coordinate, ψ, so that: $\psi = \psi_0 \sin(kx \pm \omega t)$ where: $\vec{k} = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$, $f = \frac{1}{T}$, $v = \lambda f = \frac{\omega}{k}$

- for e/m waves ψ is the amplitude of the ***E*** or ***B*** field
- ***E***, ***B*** and ***k*** are all perpendicular
- $\vec{E} = \vec{E}_0 \sin(kx \pm \omega t)$, $\vec{B} = \vec{B}_0 \sin(kx \times \omega t)$ where E_0 and B_0 are maximum amplitudes
- $E = cB$ in free space
- $E_0 = vB_0$ in general

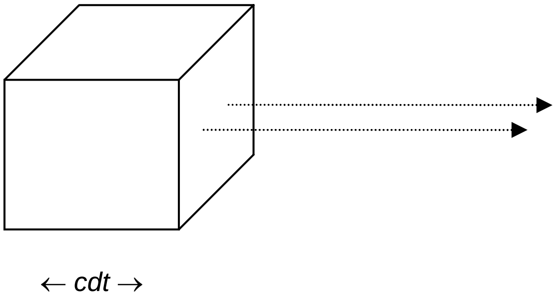


The Energy in E/M waves

The energy transported in an e/m wave is stored in the ***E*** and ***B*** fields.

- The respective energy densities: $u_E = \frac{1}{2}\varepsilon_0 E^2$, $u_B = \frac{1}{2\mu_0}B^2$
- $u_B = \frac{1}{2\mu_0}\left(\frac{E}{c}\right)^2 = \frac{\varepsilon_0\mu_0}{2\mu_0}(E)^2 = u_E \rightarrow u_B = u_E$, hence the energy of an e/m wave is equally divided between the electric and magnetic fields
- Total energy $u = u_E + u_B = 2u_E = 2u_B = \varepsilon_0 E^2 = \left(\frac{1}{\mu_0}\right)B^2$

Power carried by E/M waves – The Poynting Vector



Consider a cube with sides of area *A* that encloses some flux of e/m energy in free space. The energy in this volume of space is:

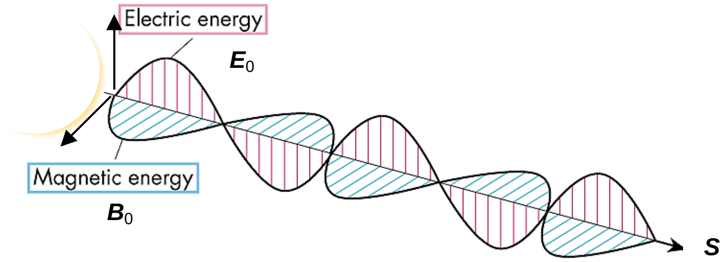
E = energy density × volume = $\left(\varepsilon_0 E^2\right)Acdt$

Note that power is energy per unit time: $\rightarrow P = \frac{\left(\varepsilon_0 E^2\right)Acdt}{dt} = \left(\varepsilon_0 E^2\right)Ac$

Define: $\vec{S} = \frac{Power}{Area} = \varepsilon_0 cE^2 = \varepsilon_0 cEcB = \varepsilon_0 c^2 \vec{E}\vec{B} = \varepsilon_0 c^2 \left|\vec{E} \times \vec{B}\right|$

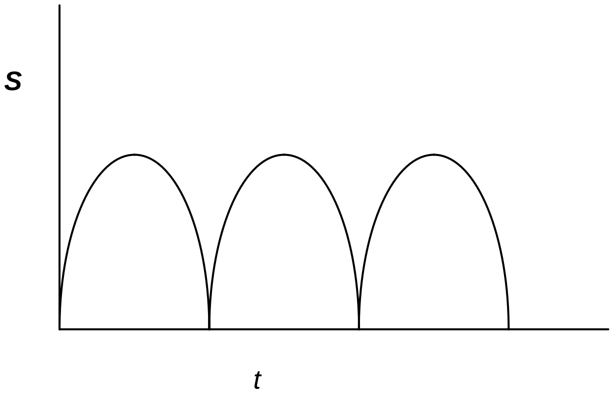
More commonly **The Poynting Vector** is written $\vec{S} \equiv \frac{1}{\mu_0}\vec{E} \times \vec{B}$. Note that the Poynting vector is a measure of the *intensity* of an e/m wave.

The ***E*** and ***B*** ,vectors are perpendicular to each other an the Poynting Vector points in the direction of the wave travel.



$$\vec{S} = \varepsilon_0 c E^2 = \varepsilon_0 c E_0^2 \sin^2(kx \pm \omega t)$$

S oscillates at twice the frequency of the wave and is always positive



Define *irradiance* E_e – the *time average value* of the absolute value of the Poynting vector.

$$E_e = \langle |\vec{s}| \rangle = \varepsilon_0 c E_0^2 \langle \sin^2(kx \pm \omega t) \rangle$$

To find the time average value of $f = \sin^2 \omega t$ over one full cycle of the function:

$$\langle f \rangle = \frac{\int_{t=0}^{t=\frac{2\pi}{\omega}} \sin^2 \omega t dt}{\int_{t=0}^{t=\frac{2\pi}{\omega}} dt} = \frac{1}{2}$$

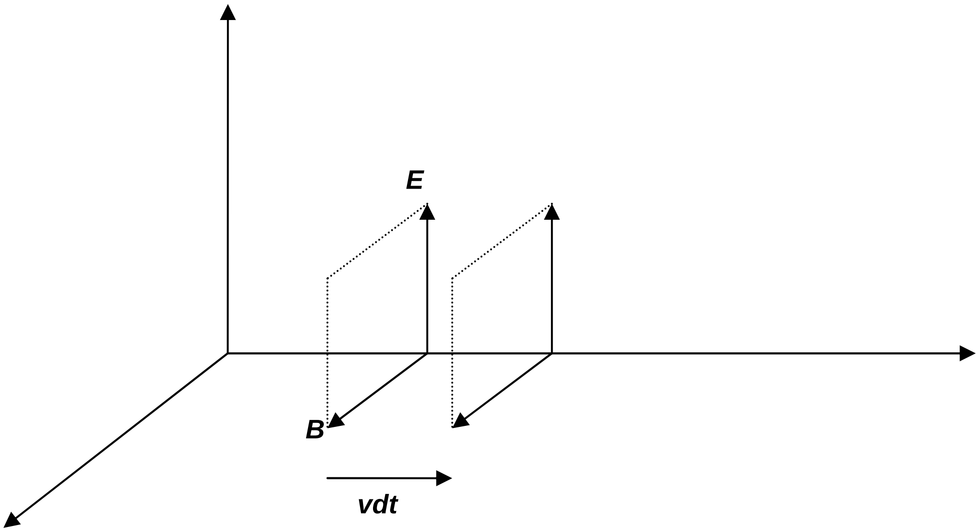
So: $E_e = \frac{1}{2}\varepsilon_0 c E_0{}^2 = \frac{1}{2}\varepsilon_0 c B_0 c c B_0 = \frac{1}{2}\varepsilon_0 c^3 B_0{}^2 = \frac{1}{2}\varepsilon_0 c \frac{1}{\varepsilon_0 \mu_0} B_0{}^2 = \frac{1}{2}\frac{c}{\mu_0} B_0^2$

The Speed of E/M waves

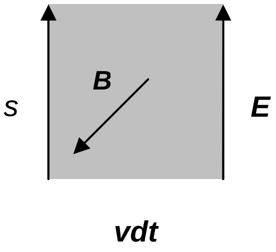
In free space the speed of an e/m wave is $c = \lambda f = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$,were ε_0 is the permittivity of free space (electricity) and μ_0 is the permeability of free space (magnetism).

In matter $v = \lambda f = \frac{1}{\sqrt{\varepsilon \mu}}$.

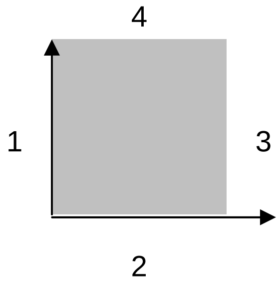
Derivation of the speed of light in free space from Maxwell's equations



Consider an e/m wavefront moving through space as shown above. As the wavefront moves through space the ***E*** vector sweeps out area *svdt*, where *s* is length of the ***E*** vector



This area swept out by the ***E*** field experiences a change in *magnetic flux*. Note that there is a simultaneous ***B*** field present with field lines that penetrate this changing area. By applying Faraday's law to this we may determine the relationship between ***E*** and ***B***.



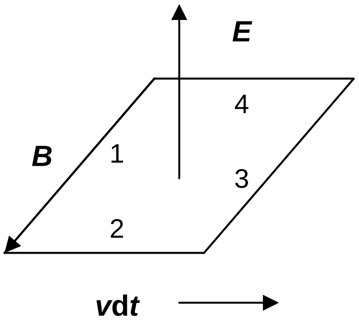
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$ Faraday's Law

Summing up the contributions of ***E·s*** from each segment of the loop, ccl:

1. -***Es***
2. 0
3. 0
4. 0

So from Faraday's Law: $-\vec{E}s = -\frac{d\Phi_m}{dt} = -\vec{B}sv \rightarrow \vec{E} = v\vec{B}$

By symmetry the ***B*** field sweeps out an area in the same time that is penetrated by electric flux from the ***E*** field. We may therefore apply Ampere's law to determine the relationship between ***E*** and ***B*** here.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ Ampere's Law

Once again, ccl around the loop:

1. ***Bs***
2. 0
3. 0
4. 0

So from Ampere's Law:

$$\vec{B}\cdot\vec{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 Evs$$
$$\vec{B} = \mu_0\varepsilon_0\vec{E}v$$
$$\vec{E} = \frac{1}{\mu_0\varepsilon_0}\frac{1}{v}\vec{B} \rightarrow \vec{E} = \frac{\vec{B}}{v\mu_0\varepsilon_0}$$

If Maxwell's equations are valid, then both Ampere's and Faraday's laws must be valid and both expressions must be true.

Hence:

$$\vec{E} = v\vec{B} = \frac{\vec{B}}{v\mu_0\varepsilon_0}$$
$$\rightarrow v^2 = \frac{1}{\mu_0\varepsilon_0} \rightarrow v = \frac{1}{\sqrt{\left(8.85\times10^{-12}C^2\cdot N^{-1}\cdot m^{-2}\right)\left(4\pi\times10^{-7}N\cdot A^2\right)}} = 2.998\times10^8 m\cdot s^{-1}$$

The quantity $\mu_0 c$ is known as the *impedance of free space*, has the SI unit of ohms and the value:

$$\mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

Radiation Pressure

E/M waves transport momentum as well as energy. This means that when an e/m wave impinges on a surface it exerts pressure on that surface.

If *p*, *U*, and *c* represent linear momentum, energy and wave speed respectively:

$$p = \frac{U}{c} \qquad \text{for an absorbing surface}$$

$$p = \frac{2U}{c} \qquad \text{for a reflecting surface}$$

Note that the first expression applies strictly only to blackbodies. In terms of the Poynting vector (*P* is the radiation pressure):

$$P = \frac{S}{c} \qquad \text{for an absorbing surface}$$

$$P = \frac{2S}{c} \qquad \text{for a reflecting surface}$$

Radiation pressures are small for sunlight on earth (about $5\times10^{-6}\,N\cdot m^{-2}$ in direct sunlight).

Example The average intensity of light (power per unit area or the value of the Poynting vector) from the sun on the earth's surface worldwide is about 380 watts per square meter (the maximum amount is about 1000 watts per square meter near the equator). Calculate the power delivered to a solar panel with dimensions of 8 × 15 meters (about the size of the roof of a house), assuming complete conversion.

$$P = SA = \left(380W\cdot m^{-2}\right)\left(8\times15m^2\right) = 45.6kW$$

In practice most solar panels are about 15% efficient for a yield of about 7 kW. This is good for about a 60 ampere service which is less than that the 100 – 200 amp service required by most homes. This energy is also available only while the sun is shining.

What is the radiation pressure on the roof? Assuming complete absorption:

$$P = \frac{S}{c} = \frac{380W\cdot m^{-2}}{3\times10^{8}m\cdot s^{-1}} = 1.3\times10^{-6}\,N\cdot m^{-2}$$

A small fraction of the normal load any roof supports.

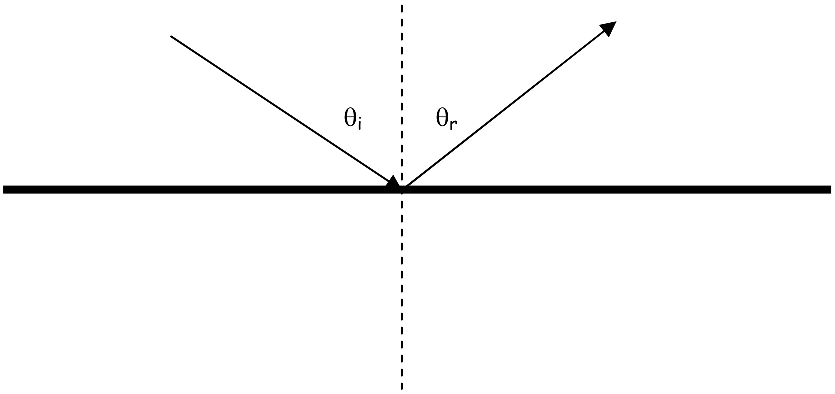
Computing the distances to stars

There are basically two methods of computing distances to stars: the *parallax method* and the method of *standard candles*.

- The parallax method uses trigonometry to compute the distance to a star based on a measuring a parallax angle formed due to the apparent shift of a stars position against the background of stars as the earth orbits the sun. This method works with stars up to 250 parsecs (800 ly) distant.
- The method of standard candles uses spectral data and Wien's law to determine a stars surface temperature, Stefan-Boltzman to determine its actual luminosity, and the inverse square law to determine the distance from its apparent luminosity. This method works well for stars greater than 250 parsecs (800 ly) distant.

Reflection and Refraction

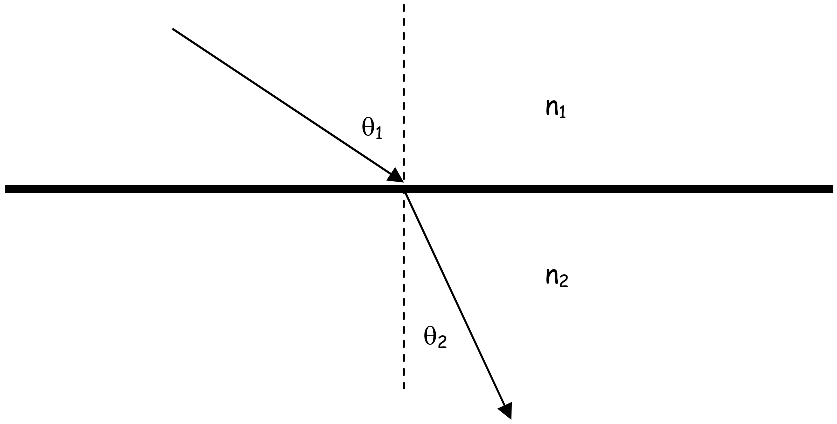
The Law of Reflection $\theta_i = \theta_r, coplanar$



The geometry and type of reflection depends on:

- The frequency/wavelength of the incoming wave
- The incident angle of the incoming wave
- The composition of the material off which the beam is being reflected. In general very good conductors are also very good reflectors
- The smoothness of the reflective surface
- Reflection may be thought of as a specific form of scattering that follows a specific geometric relationship.
- Some reflection occurs even at optical boundaries between transparent media.
- External reflections occur when the medium that contains both the incident and reflected rays has a lower index of refraction than the reflective medium.

The Law of Refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$



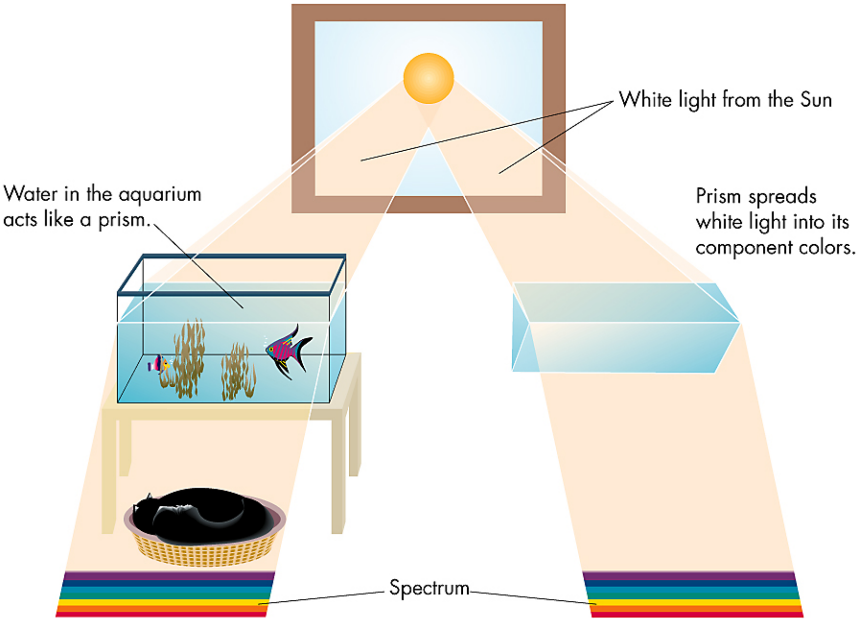
The geometry and type of refraction depends on:

- The frequency/wavelength of the incoming wave
- The incident angle of the incoming wave
- The opacity or transparency of the media
- The material on each side of the optical boundary, specifically a property known as the index of refraction of the material which is a measure of the speed with which light travels in the material as compared to the speed of light in free space

$$n = \frac{c}{v}$$

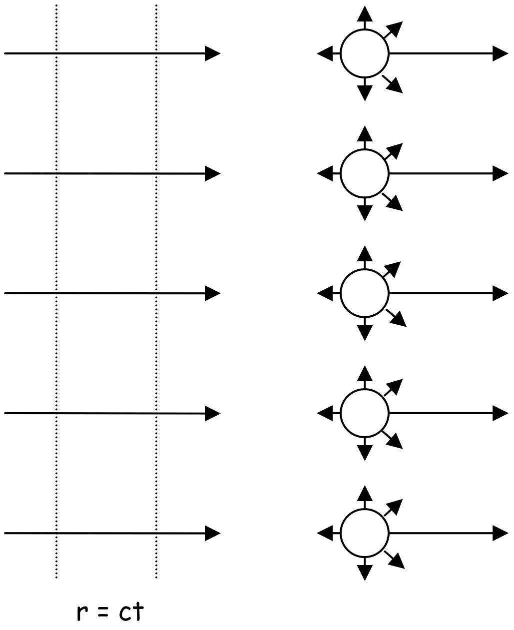
- Refraction or bending of a light beam as it crosses an optical boundary occurs due to the beam slowing down as to goes from a less to more dense medium or speeding up as it goes from a more dense to a less dense medium. *Frequency remains unchanged.*
- Since $v = \lambda f$, a monochromatic beam of light has a shorter wavelength in a dense medium than it does in free space: $\lambda_n = \dfrac{\lambda_0}{n}$
- The amount of refraction depends both on the frequency/wavelength of the light beam and the material in which it is being refracted.
- Light beams bend towards the normal when $n_1 < n_2$. Light beams bend away from the normal when $n_1 > n_2$.
- Broad spectrum, *polychromatic* light (such as white light) has components that bend at different angles in a given media

- Red light bends the least and blue the most as white light enters a dense transparent medium.
- The differential bending of each element of polychromatic or white light is known as dispersion.
- Dispersion is common in raindrops, lenses and prisms.
- Dispersion separates any beam of polychromatic light into its component colors.
- In the case of white light all colors of the visible spectrum result from dispersion.

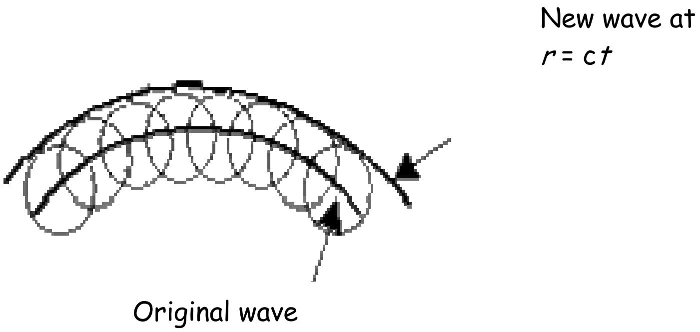


Huygens Model of Light Wave Propagation

A plane wave



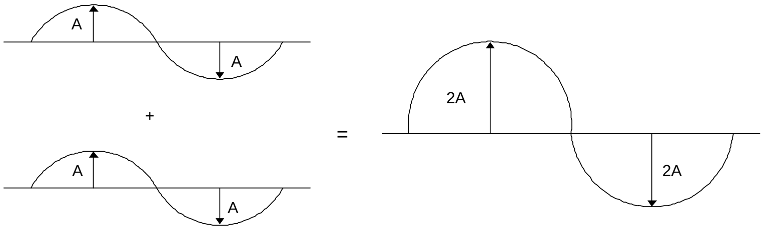
A spherical wave



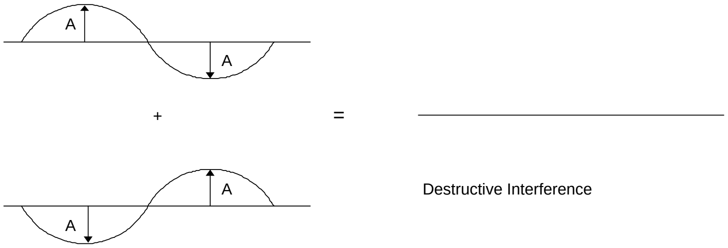
- Christian Huygens was the first to propose that light was a wave.
- Light waves traveling together through space or any dense medium form *wavefronts*.
- Any point on a wavefront is capable of acting as a new source of the wave.
- In general waves self-propagate through space by this method.
- This is known as ***Huygens Theory*** and may be used to explain *reflection, refraction, diffraction* and *interference*.

Interference

Just as with sound waves, light waves, when combined, may interfere constructively, destructively or some combination of both.

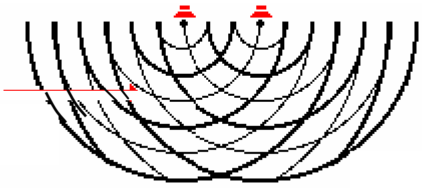


Constructive Interference



Destructive Interference

- In order for an interference pattern to be stable the waves must be emitted from *coherent* and *monochromatic* sources.
- Most natural light sources are both non-coherent and polychromatic so interference is not widely observed in nature.
- In order to create a stable interference pattern waves from different sources must maintain a constant phase relationship with each other.
- It is much easier to arrange coherent sound sources than light sources.
- Because the frequency of light waves (around 10^{15} Hz) is so high coherence between separate sources is difficult to arrange.
- Two loudspeakers driven by the same amp in mono are responding to the same inputs and will produce coherent waves.
- It is easy to introduce phase differences between the loudspeakers and constructive or destructive interference merely by either moving the loudspeakers around or moving around in the field produced by the loudspeakers.



Two Beam Interference

We'll add the electric fields of two light beams of identical frequency and wavelength that differ only by some initial phase difference:

$$E_1 = E_{01}\sin(\omega t + \alpha_1)$$
$$E_1 = E_{01}\sin(\omega t + \alpha_2)$$

where α is a constant that contains the phase difference between the waves.

$$E_R = E_1 + E_2 = E_1 = E_{01}\sin(\omega t + \alpha_1) + E_1 = E_{01}\sin(\omega t + \alpha_2)$$

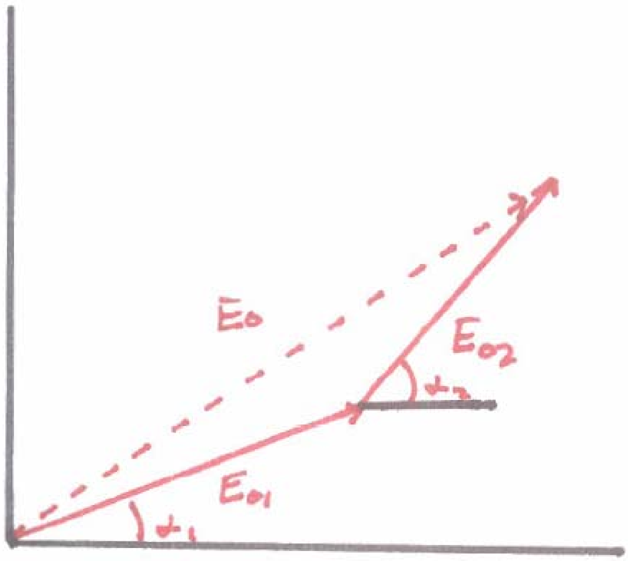
We'll invoke the trig identity:

$$\sin(\alpha + \beta) \equiv \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

so that:

$$E_R = (E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)\sin(\omega t) + (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)\cos(\omega t)$$

Notice that we can plot each of these component waves as *phasors* as shown at right by plotting the magnitude and phase angle of each.



$$E_R = E_{01} + E_{02} = E_0\cos\alpha + E_0\sin\alpha$$

$$E_0\cos\alpha = E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2$$

$$E_0\sin\alpha = E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2$$

$$E_R = E_0\cos\alpha\sin\omega t + E_0\sin\alpha\cos\omega t$$

and it is easily shown that this yields:

$$E_R = E_0\sin(\omega t + \alpha)$$

The resultant wave is another harmonic wave of the same frequency with an amplitude E_0 and a phase α that is related to the original waves as shown in the phasor diagram above.

The law of cosines may be applied to yield:

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$

and the phase angle may determined by:

$$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2}$$

Now recall that irradiance (the time average value of the Poynting vector) of a light beam is:

$$E_e = \varepsilon_0 c\left\langle E_0\right\rangle^2$$

So the irradiance at a point *P* due to two beam interference is:

$$E_e = \varepsilon_0 c\left\langle E_p\right\rangle^2 = \varepsilon_0 c\left\langle E_p \cdot E_p\right\rangle = \varepsilon_0 c\left\langle (E_1 + E_2)\cdot(E_1 + E_2)\right\rangle$$

which may be recast in the form:

$$E_e = \varepsilon_0 c\left\langle E_1^2 + E_2^2 + 2E_1 \cdot E_2\right\rangle$$

It is apparent that the first two quantities in the bracketed term are the individual contributions to the sum at *P* while the third term (the *interference* term) is the simultaneous contribution from both beams due to their interaction with each other.

- The interference term is indicative of the wave nature of the beams of light.
- If light behaved like classical particles there would be no interference and the irradiance would be: $E_e = \varepsilon_0 c\left\langle E_1^2 + E_2^2\right\rangle$, i.e. solely dependant on the separate contributions of the individual waves.
- The interference term varies between zero and $2E_1E_2$ depending on the orthogonality of the two beams.

It may be shown that:

$$E_e = E_{e1} + E_{e2} + 2\sqrt{E_{e1}E_{e2}}\cos\delta$$

where δ is the phase difference between the two waves due to either a path length difference or an actual phase difference (for our purposes here they are functionally the same.)

When $\cos\delta = +1$ constructive interference yields the maximum irradiance:

$$E_e = E_{e1} + E_{e2} + 2\sqrt{E_{e1}E_{e2}} \quad \text{(maximum irradiance)}$$

a condition that occurs whenever the phase difference is $\delta = 2m\pi$, where $m = 0, \pm1, 2, 3, \ldots$

When $\cos\delta = -1$ destructive interference yields the minimum irradiance:

$$E_e = E_{e1} + E_{e2} - 2\sqrt{E_{e1}E_{e2}} \quad \text{(minimum irradiance)}$$

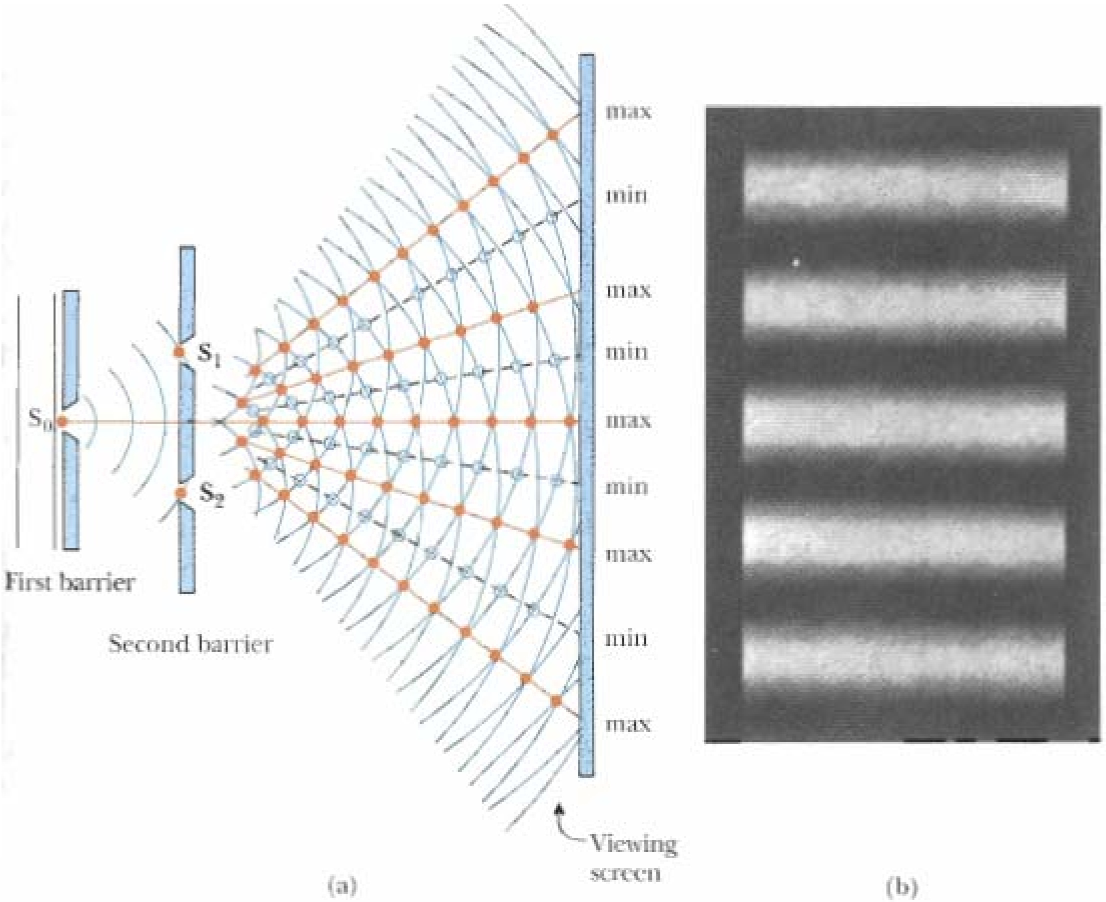
a condition that occurs whenever the phase difference is $\delta = (2m+1)\pi$ where $m = 0, \pm1, 2, 3, \ldots$

It is also apparent that complete destructive interference can occur when $E_{e1} = E_{e2} = E_{e0}$ and that the irradiance, in this case, varies between:

$$E_{e\max} = 4E_{e0} \text{ and } E_{e\min} = 0$$

Alternatively, it may be shown that the irradiance between two equal interfering beams may be written:

$$E_e = 4E_0 \cos^2\left(\frac{\delta}{2}\right)$$



(a) (b)

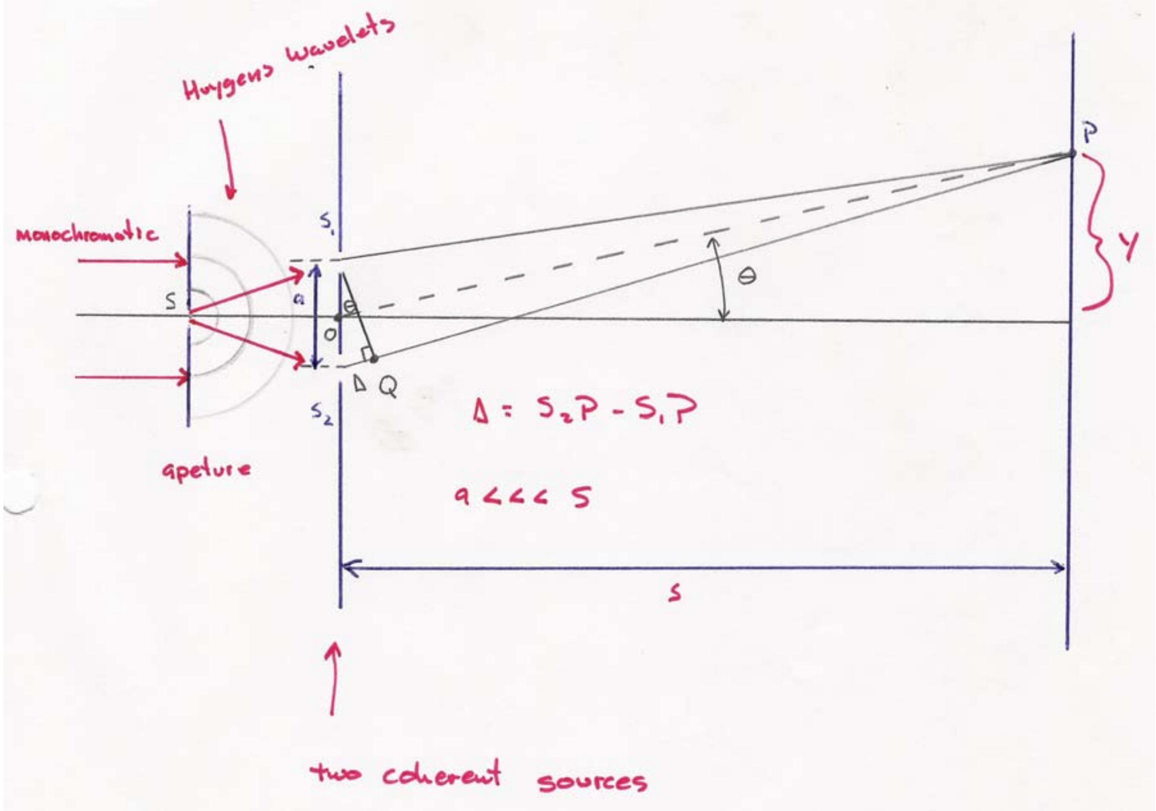
Figure 37.2 An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive (A) and destructive (B) interference. *(Richard Megna/Fundamental Photographs)*

Courtesy of *Physics for Scientists and Engineers*, Serway and Beichner, 5th Ed.

Young's Double Slit

Consider a beam of light incident upon two slits, S_1 and S_2, as shown below. This arrangement insures that the beams that leave the slits are coherent.

We'll choose an arbitrary point P on a distant screen and look at the pattern of interference which results from combining the two beams at this point. The path length difference between the two beams is $S_2P - S_1P = \Delta$.



- When $S_2P - S_1P = m\lambda$, constructive interference occurs
- When $S_2P - S_1P = \left(m + \frac{1}{2}\right)\lambda$, destructive interference occurs

  $m = 0, \pm 1, \pm 2, \ldots.$

The conditions for interference are:

- $S_2P - S_1P = \Delta = m\lambda \approx a\sin\theta$ (constructive)
- $S_2P - S_1P = \Delta = \left(m + \frac{1}{2}\right)\lambda \approx a\sin\theta$ (destructive)

  $m = 0, \pm 1, \pm 2, \ldots.$

Notice that a phase difference between the two beams, δ, is equivalent to a path length difference between the two beams, Δ so that:

$$\delta = \frac{2\pi}{\lambda}\Delta$$

It then follows that the irradiance at the point of interference varies as a cosine function that also depends on λ and Δ:

$$E_e = 4E_0\cos^2\left(\frac{\delta}{2}\right) \rightarrow E_e = 4E_0\cos^2\left(\frac{\pi}{\lambda}\Delta\right) = 4E_0\cos^2\left(\frac{\pi}{\lambda}a\sin\theta\right)$$

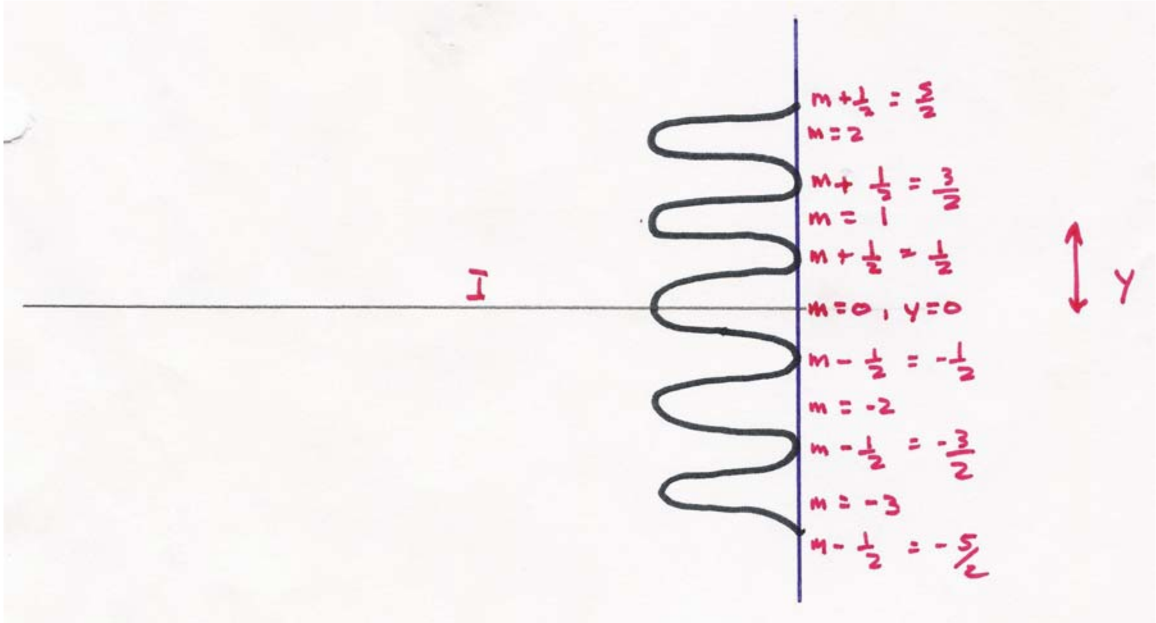
For points near the optical axis where y << s:

$$\sin\theta \approx \tan\theta \approx \frac{y}{s}$$

$$E_e = 4E_0\cos^2\left(\frac{\pi}{\lambda}a\frac{y}{s}\right)$$

As the cosine term varies from ±1 and 0 (usually as a function only of *y*), the intensity of the pattern on the screen varies from $4E_0$ to zero, i.e. constructive and destructive interference occur.

The pattern that results is an alternating series of bright and dark fringes.



The location of the bright fringes (intensity maxima): $y_m = m\frac{\lambda s}{a}, \quad m = 0, \pm 1, \pm 2 \ldots$

and the separation of the intensity maxima is: $\Delta y = \frac{\lambda s}{a}$

The Young's double slit experiment is very important historically because it is used to demonstrate the wave nature of light.

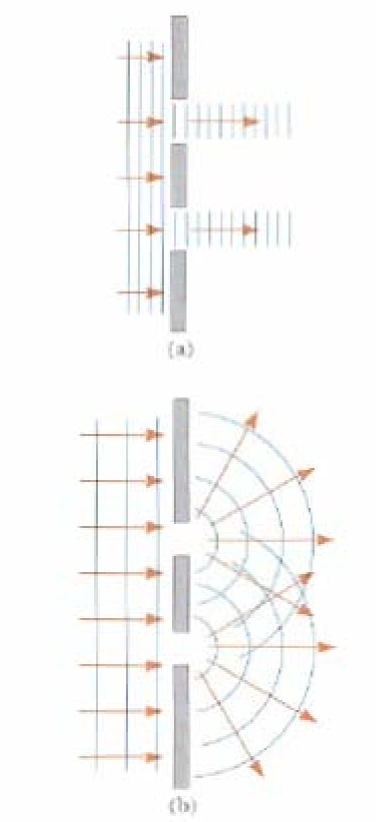
Young's double slit still has a lot of utility as an easy method of experimentally determining the wavelength of a beam of monochromatic light since:

$$\frac{\Delta y a}{s} = \lambda$$

Diffraction

Diffraction may be thought of as a deviation of a light beam caused by partial obstruction of a wave front

- Recall that interference effects depended on light spreading out after a wavefront passed through a slit.
- The spreading out is caused by partial obstruction of a wavefront by a mask or a sharp edge.
- Sound waves are diffracted in the same manner as light waves.
- Diffraction patterns are marked by a rapid decrease in intensity with increasing distance from the center of the pattern.

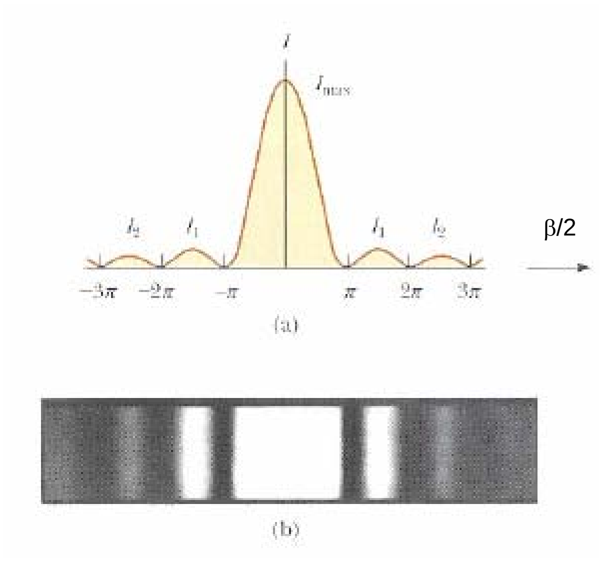


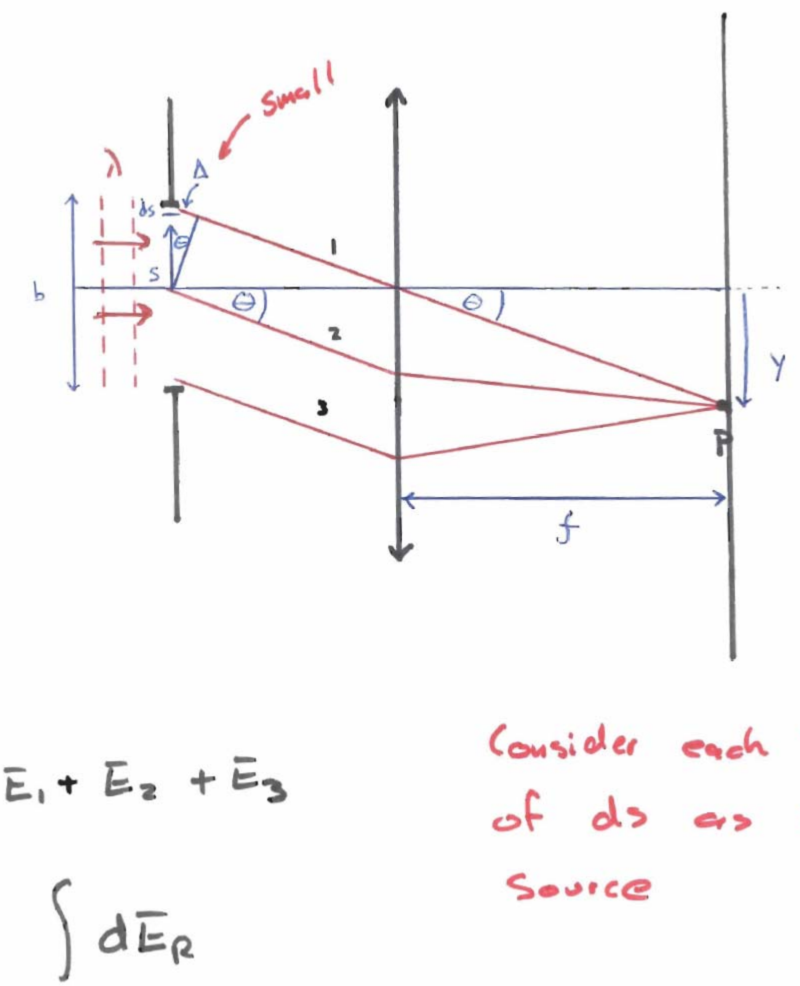
Fraunhofer diffraction

"far field" diffraction – assumption is that all light rays are approximately parallel

Single-Slit Diffraction

Consider Fraunhofer diffraction through a slit of width *b* as shown below. Note that the locations of relative maxima are about halfway between the minimum values (zero).





$$E_R = E_1 + E_2 + E_3$$

$$= \int dE_R$$

Consider each interval of ds as a source

It may be shown that the intensity at point *P* on the screen is given by:

$$I = I_0 \left[\frac{\sin\beta}{\beta} \right]^2$$

where $\beta = \frac{\pi}{\lambda} b \sin\theta$

(Note: some books use $\beta/2$ in the above expression in which case $\beta = \frac{2\pi}{\lambda} b \sin\theta$.

I_0 is the intensity at $\theta = 0$ or the position on the screen even with the center of the slit known as the *central maximum*.

Minima occur in the pattern at:

$$b \sin\theta = m\lambda$$

or

$$\sin\theta = m\frac{\lambda}{b} \qquad m = \pm 1,2,3,\ldots$$

Circular Apertures

Fraunhofer diffraction by a circular aperture is of great interest because most optical systems have round (rather than square) apertures.

- Maxima and minima form concentric rings.
- The central bright maxima is known as the *Airy Disk*.
- The mathematical expression for diffraction by a circular disk is more complicated:

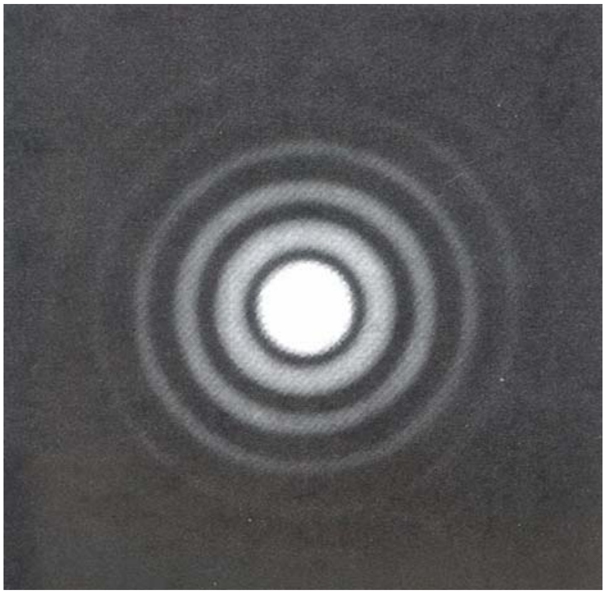
$b\sin\theta = m\lambda$ (slit) must be replaced by

$b\sin\theta = J\lambda$ where *J* is a first order Bessel Function.

Positions of minima in Fraunhofer Diffraction:

	Slit	Circular
First Order	m = 1	J = 1.220
Second Order	m = 2	J = 2.233
Third Order	m = 3	J = 3.238

For diffraction with a circular aperture the Bessel Functions also oscillate between maxima and minima but decrease in amplitude with increasing distance from the central axis.



Positions of maxima: $J_1 = 1.635$
$J_2 = 2.679$

Diffraction behind a circular aperture is nearly the same as diffraction behind a circular obstacle.

Rayleigh's Criterion

Diffraction at a circular aperture sets the limits of resolution for virtually any optical system.

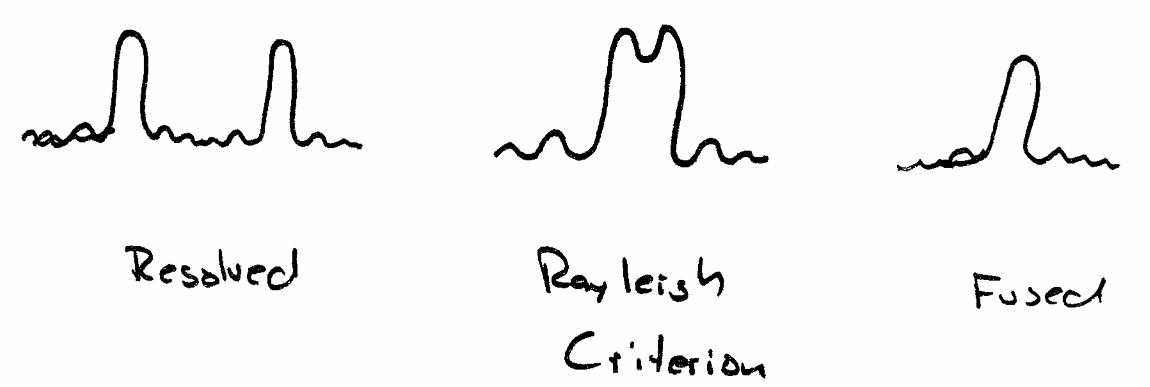
The Bessel functions for circular diffraction yield the diffraction limit for circular apertures:

$$\theta_{\min} \approx 1.22\frac{\lambda}{d}$$

Where $\theta_{\min}$ is the angular separation and *d* is the diameter of the circular opening.

Consider a telescope aimed at two stars next to each other of approximately equal magnitude. In order to "resolve" the two stars, the diffraction patters of each in the focal plan of the telescope must be separate.

- When the central maxima fuse, the two stars appear as one.
- When the central max of one star coincides with the minimum of the other, the resolution is marginal. This condition is known as Rayleigh's Criterion.



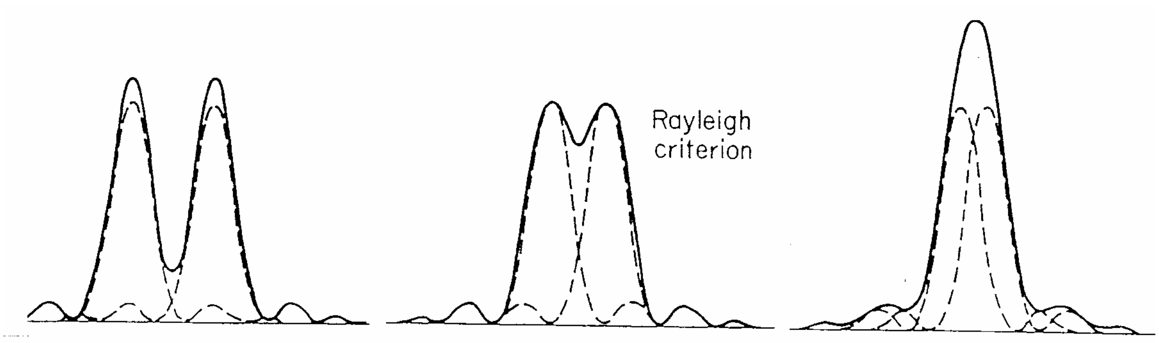
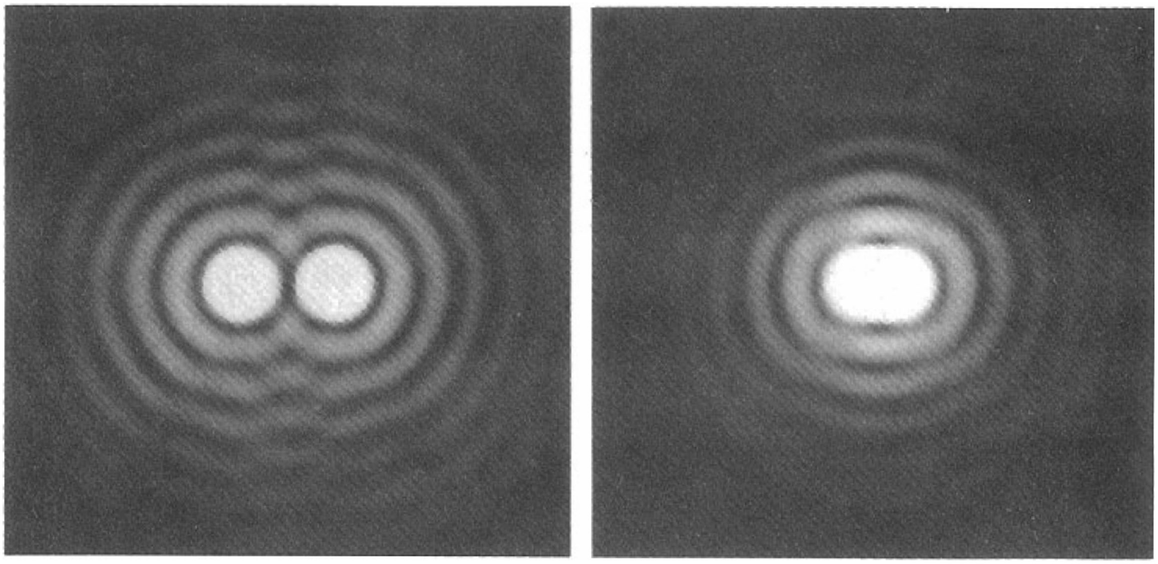


Figure 14-11 Rayleigh's criterion. Note the separation of the maxima in the left-hand plot and the close overlap on the right.



A well resolved image and an image at the limit of resolution (Rayleigh criterion)

Example Resolution of two stars 30,000 L.Y. distant in yellow light, with the eye.

$$\theta_{\min} = \frac{(1.22)(550nm)}{8\times10^{-3}m}$$

(diameter of a night adapted pupil)

$$\theta_{\min} = 8.39\times10^{-5}\,radians$$

Note: $s = r\theta$

$$= (30{,}000)(8.39\times10^{-5}\,rad)$$

$$s \approx 2.5\ L.Y.$$

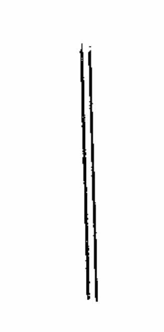
So, two stars must be 2.5 L.Y. apart to be resolved by the eye.

Diffraction Limit

- A lens, even one free of all aberrations, is still diffraction-limited.
- Diffraction limits apply to microscopes as well as telescopes.
- UV, x-ray and electron microscopes are used to extend the diffraction limit.
- Radio telescopes must be very large to have very good resolution for radio sources. Why?

The human eye is also diffraction limited

A test for visual Acuity



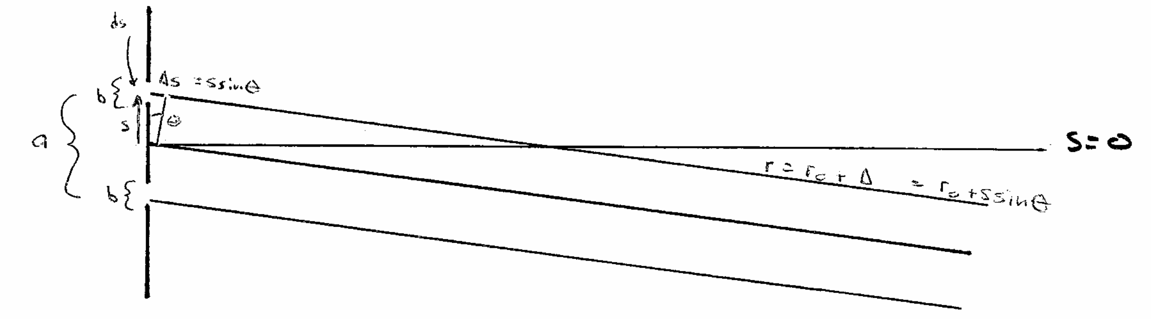
Can you resolve these two lines 1 mil apart at a distance of 4 meters? If so you have normal visual acuity.

At 12 inches, the normal visual acuity of the human eye is 0.00349 inch.

Double Slit Diffraction

Wavefronts are obstructed everywhere except at the slits

When we looked at two slit obstruction of a wavefront previously we assumed that the apertures (slits) were point sources. In reality, we must take into account the finite size of the slits.



It may be shown that the intensity at any random point on a far screen is determined by both interference and diffractive effects.

Recall that for two beam interference:

$$I = 4I_0 \cos^2\left(\frac{\pi}{\lambda} a \sin\theta\right) = 4I_0 \cos^2\left(\frac{\pi}{\lambda} a \frac{y}{s}\right)$$

and for single slit diffraction:

$$I = I_0 \left[\frac{\sin\beta}{\beta}\right]^2$$

For double slit diffraction it may be shown that:

$$I = 4I_0 \cos^2(\alpha)\left(\frac{\sin\beta}{\beta}\right)^2$$

where:

$$\alpha = \frac{\pi d \sin\theta}{\lambda} \text{ and } \beta = \frac{\pi}{\lambda} b \sin\theta$$

Hence:

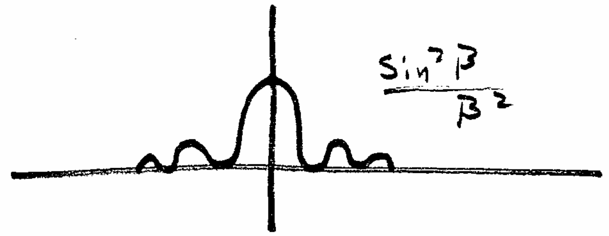
$$I = 4I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left(\frac{\sin\left(\pi b \sin\frac{\theta}{\lambda}\right)}{\pi b \sin\frac{\theta}{\lambda}}\right)^2$$

where *b* is the slit width and *d* is the distance between the slits.

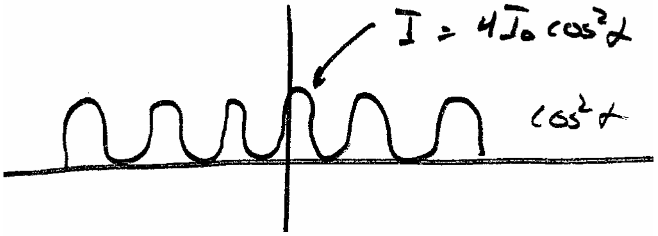
Note that the intensity is 4 times (in the center maximum) that of a single slit.

This expression looks very complicated but is fairly simple to dissect consisting of:

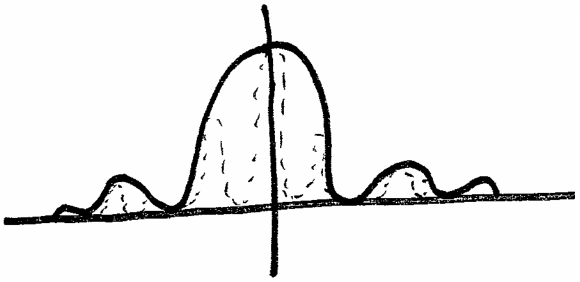
a diffraction term: $\left(\frac{\sin\beta}{\beta}\right)^2$

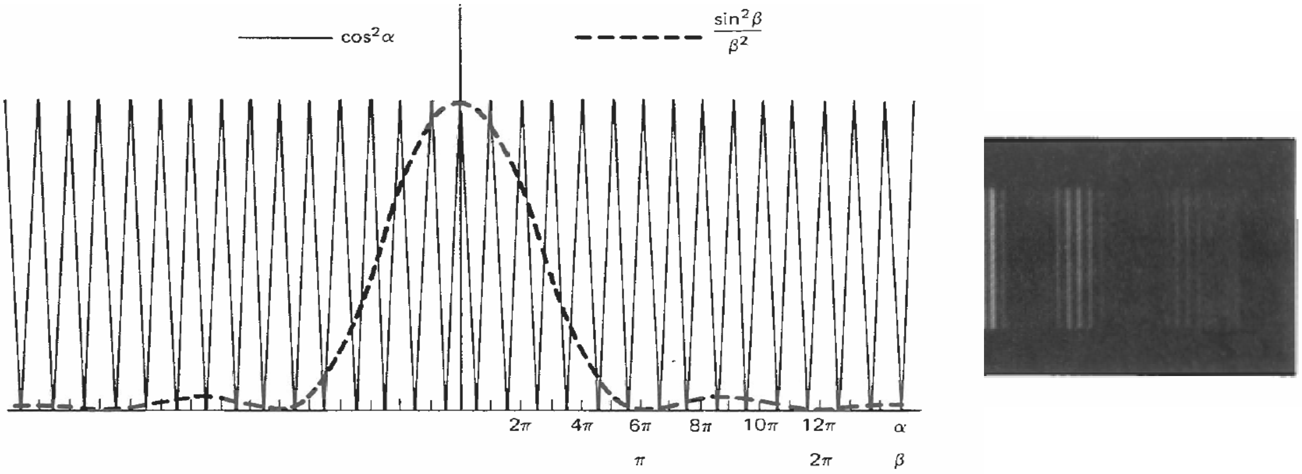


an interference term: $\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

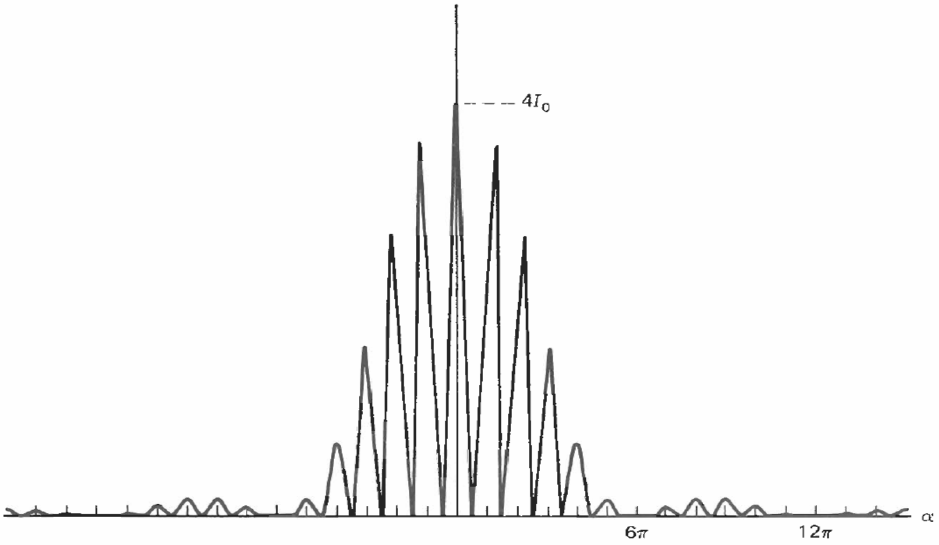


which combine to produce:

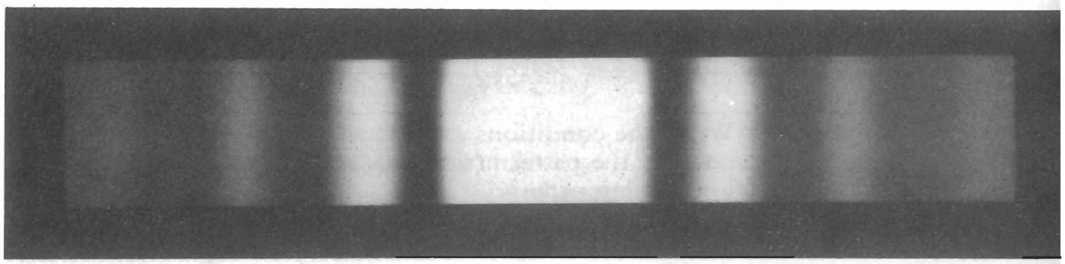




(a)



(b)



(c)

The diffraction envelope has minima when:

$$\beta = m\pi \qquad m = \pm 1, \pm 2, \ldots$$

or in terms of θ:

$$m\lambda = b\sin\theta \qquad \text{(diffraction minima)}$$

Notice that when these diffraction minima correspond to interference maxima the fringe is missing from the pattern.

Interference maxima occur when

$$\alpha = n\pi \qquad n = \pm 1, \pm 2, \ldots$$

or in terms of θ:

$$n\lambda = d\sin\theta \qquad \text{(interference maxima)}$$

When: $m\lambda = b\sin\theta$ and $n\lambda = d\sin\theta$ are satisfied for the same value of θ:

$$d = \frac{n}{m}b \text{ or } \alpha = \frac{n}{m}\beta$$

When d (slit separation) is an integer multiple of b (slit width) this condition is met exactly, e.g. $d = 2b \rightarrow n = 2m = \pm 2, 4, 6 \ldots$

- n must have an integer value so not all d/b rations work
- d/b must be > 1 or the physical implication is that the slit width is greater then the separation between them (impossible)
- when $\frac{d}{b} = \frac{4}{3} \rightarrow n = 4\frac{m}{3}$ so every 4th order will be missing coinciding with every 3rd envelope in the diffraction pattern

Multiple Slit Diffraction

Combination of previous methods (with lots of work) yields (for *N* identical equally spaced slits):

$$I = I_0 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2 \left(\frac{\sin \beta}{\beta} \right)^2$$

We are generally interested in very large values of *N* (*diffraction gratings*) where maxima are bright, distinct, and spatially well separated.

In this case it may be shown that intensity maxima are given by:

$$m\lambda = d \sin \theta \qquad \text{(the grating equation)}$$

- Many spectrometers use diffraction gratings instead of prisms to separate light into various spectral components.
- The resolving power, ***R***, of a grating is given by:

$$R = \frac{\lambda_{ave}}{\Delta\lambda}$$

- Gratings with high resolving powers can separate spectral components that are very close together.
- If *N* number of slits in a grating are illuminated it can be shown that for the *m*th order diffraction the resolving power is:

$$\boldsymbol{R} = Nm$$

- Resolving power increases (larger is better) with both the number of slits and order number (in a spectrometer the orders more distant from the central maximum have greater separation).
- Note that for *m* = 0, ***R*** = 0 which is consistent for the central maximum of any diffraction pattern.

Example Diffraction gratings have an equally spaced number of slits and are rated in terms of slits per cm. A common number of slits per cm is 6000. If such a grating is used to view a beam of pure blue light (440 nm) where will the 1st, 2nd, and 3rd order maxima be located?

$d = \frac{1}{6000} = cm = 1.667 \times 10^{-4} cm = 1667nm$ (why did we take the inverse here?)

$$\sin\theta_1 = \frac{1\lambda}{d} = \frac{440nm}{1667nm} = 0.264 \rightarrow \theta = 15.3^0$$

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{880nm}{1667nm} = 0.528 \rightarrow \theta = 31.9^0$$

$$\sin\theta_3 = \frac{3\lambda}{d} = \frac{1320nm}{1667nm} = 0.792 \rightarrow \theta = 52.3^0$$

Notice that if we try to computer the 4th order:

$$\sin\theta_4 = \frac{4\lambda}{d} = \frac{1760nm}{1667nm} = 1.06$$

so the sine is undefined and the 4th order maximum does not exist.

What is the resolving power of this grating in the 2nd order?

$$\boldsymbol{R} = Nm = 6000(2) = 12{,}000$$

Will this grating be capable of resolving the sodium doublet (589.00nm and 589.59nm) in the second order?

$$R = \frac{\lambda_{ave}}{\Delta\lambda} = \frac{589.30nm}{0.59nm} = 999$$

So yes, the grating has plenty of resolving power.