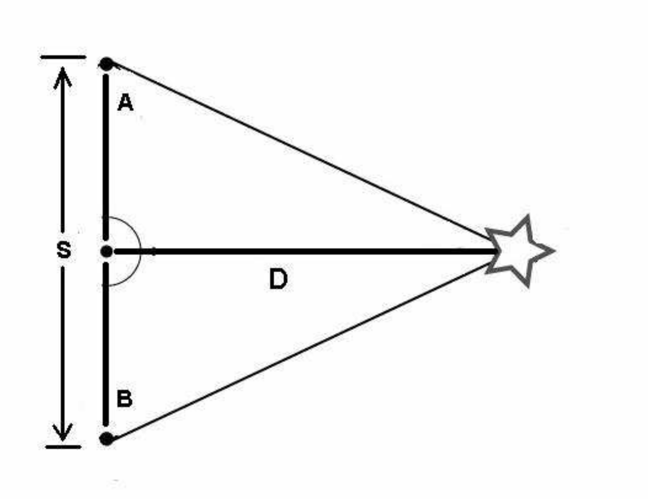
Lecture 20

1. Measuring a star's distance

Suppose you were trying to measure the width of a canyon from your side to the distant canyon wall. You couldn't pace the distance. Could you come up with a way to make this measurement safely? Surveyors and geologists encounter this kind of problem measurement problem all the time. Over the course of centuries they have found a simple way to solve this problem. They use a method called 'triangulation'. Here's how it works:



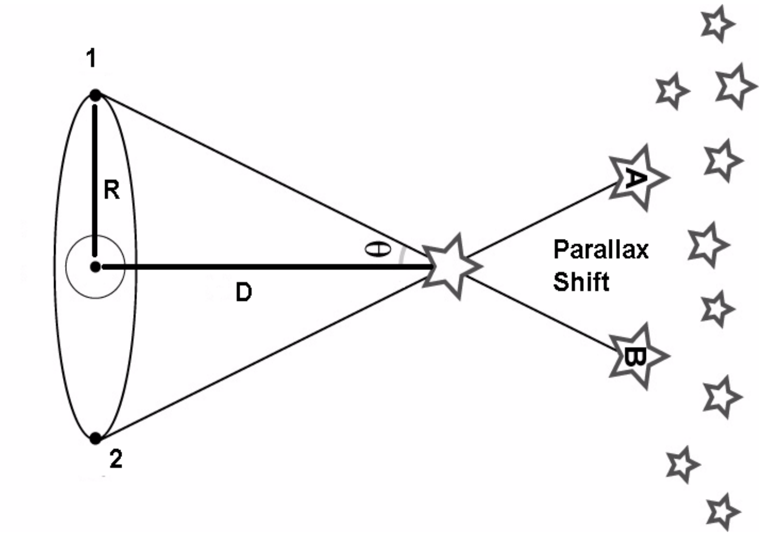
In ordinary land surveying, imagine a distant mountain peak (the star in the figure above) and two observers are located at 'A' and 'B' separated by a few miles (the length 'S'). The base angles at A and B can be measured with an instrument called a theodolite. By knowing the base distance A to B, and the baseline distance S, the distance to the peak can be worked out with a simple scaled drawing or with trigonometry.

2. Parallax

Suppose it was hard for you to measure the two base angles in the triangulation method. This could easily happen if the object were so far away that your instrument could not accurately discern that these angles were different than 90 degrees. For example, if the object is 10 miles away, and your baseline is only 5 feet long, the two base angles would have a measure of 89.9946 degrees. This angle differs from 90 degrees by only 0.0044 degrees which equals 16 seconds of arc (there are 60 minutes or arc/degree x 60 seconds of arc/minute of arc = 3600 seconds of arc per degree!) This would be a very difficult angle to measure even with very expensive modern surveying equipment!

To solve this problem, astronomers don't bother measuring the base angles at all. Instead, they measure the vertex angle in the triangle. It turns out that this angle is very easily measured using photographic techniques. The method is called trigonometric parallax or just 'parallax' for short. Here's how it works:

Extend your arm in front of you, hold your thumb up, and alternately open and close your eyes. You will see your thumb's position move against the more distant background in front of you. Astronomers call this the parallax shift as the figure below illustrates:



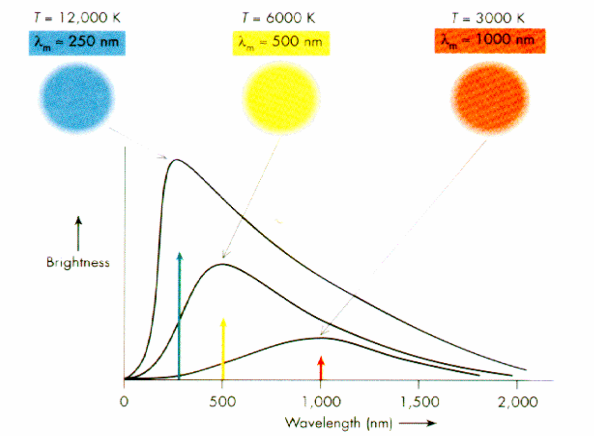
By knowing the distance between your eyes (2 x R) and how much this shift measures in degrees (twice the measure of the parallax angle q), you can calculate the distance to your thumb (D)! The formula that you use is:

Tan (q) = R / D

But this same principle applies to measuring distance to objects far away from you too...like the planets. Today, astronomers use photographs of stars taken 6 months apart. During that time, Earth has traveled from one side of its orbit to the other, and the orbit baseline is twice 93 million miles (150 million kilometers). By measuring how far the image of a star has shifted relative to the far more distant stars in the background between, say, January and June, astronomers can accurately measure angles as small as 0.001 seconds of arc or 0.0000003 degrees.

3. Measuring a star’s temperature

Wien's Law is an important formula that allows us to determine the temperature of a star. It is based on the fact that hotter objects have more energy than cooler objects and therefore emit more radiation at higher frequencies than at lower frequencies. Wien discovered that there was a direct relationship between the wavelength (or frequency) at which an object emits most of its energy and the temperature of that object.



$$\lambda\ T = 30{,}000{,}000$$

(Where λ is measured in Angstroms and T is measured in degrees Kelvin)

In this form, wavelength must be measured in Angstroms (one Angstrom is 10^{-10} meters) and temperature in degrees Kelvin. We can understand the logic of Wien's law by looking at the graph above. Notice that as the wavelength where the most energy is given off (maximum wavelength) goes from red to yellow to blue as the temperature increases. Now, we know that light with longer wavelengths has lower frequencies and therefore less energy than light with shorter wavelengths. Since the wavelength of red light is longer than green light (and green light is longer than blue), red light must have a lower frequency and less energy than green light (and green has less energy than blue). Therefore, we expect a star that emits mostly blue light to emit more energy (and thus be hotter) than a star that emits mostly red light (if they are the same size). We see from the graphs that our expectation is correct: the temperature for a star which emits most of its light at a maximum wavelength of 1000 nanometers is 3000 K, the temperature for a star with a maximum wavelength at about 500 nanometers is 6000 K, and the temperature for a star with a maximum wavelength at 250 nanometers is about 12,000 K. Hotter stars emit more energy per unit area than cooler stars.

4. Luminosity and brightness

The **Luminosity** of a star depends on both its temperature and its radius (surface area):

L is proportional to $R^2 T^4$.

- A hotter star is more luminous than a cooler one of the same radius.
- A bigger star is more luminous than a smaller one of the same temperature.

Thus a cool (red) giant star is more luminous than the Sun because, even though it is cooler, it is much larger than the Sun.

The **Brightness** of a star depends on its luminosity and how far away you are from the star:

$$b = \frac{L}{4\pi d^2} \qquad\qquad L = (4\pi d^2)\ b$$

5. The magnitude system

Astronomers usually measure brightness in *magnitudes*. The magnitude system stems from ancient Greece. A very bright star was called “first magnitude”, a pretty bright star is “second magnitude”, and a barely visible star is “sixth magnitude”. Now some stars have negative magnitudes as they are much brighter than “first magnitude” stars like Betelgeuse. Absolute magnitude tells you what a star’s brightness would be if you are 10 parsec away from it. (1 parsec is about 3.3 light-years)

6. Spectra of stars

Because the efficiencies of absorption depend on temperature, so do the appearances of the spectra of the stars. Stellar spectra were first observed in the middle of the 19th century. To the great confusion of the astronomers of the time, most spectra looked nothing like the solar spectrum. Some, like that of Vega, had powerful hydrogen lines, whereas others had none at all and displayed what were later shown to be molecular lines of titanium oxide. It looked as though different stars were made of different elements. As an aid to understanding, astronomers began classifying the spectra, the schemes culminating about 1890 in the one still used today when E.C. Pickering lettered the stars according to the strengths of their hydrogen lines, his assistants Annie Cannon, Antonia Maury, and Williamina Fleming aiding in development and observation. As observation improved, they dropped some letters, rearranged others according to different spectral criteria, and added decimalization. The result was the classic seven-group sequence OBAFGKM. A bit over a century later, as a result of new technologies, astronomers added another two classes whose spectra contained molecules, L and T. About the first thing any astronomer wants to know about a star is its class. The Sun is class G.

THE SPECTRAL SEQUENCE

Class	Spectrum	Color	Temperature
O	**ionized and neutral helium, weakened hydrogen**	**bluish**	**31,500-49,000 K**
B	**neutral helium, stronger hydrogen**	**blue-white**	**10,000-31,500 K**
A	**strong hydrogen, ionized metals**	**white**	**7500-10,000 K**
F	**weaker hydrogen, ionized metals**	**yellowish white**	**6000-7500 K**
G	**still weaker hydrogen, ionized and neutral metals**	**yellowish**	**5300-6000 K**
K	**weak hydrogen, neutral metals**	**orange**	**3800-5300 K**
M	**little or no hydrogen, neutral metals, molecules**	**reddish**	**2100-3800 K**
L	**no hydrogen, metallic hydrides, alkalai metals**	**red-infrared**	**1200-2100 K**
T	**methane bands**	**infrared**	**under 1200 K**

In the modern spectral sequence, OBAFGKMLT, the hydrogen absorption lines weaken in both directions away from class A. Various other absorptions round out the picture. It was noted very early that the spectral sequence in this form correlates with color, ranging from a blue tint for O and B stars to reddish for class M. Since color depends on surface temperature, so must the spectral class. Stars of class T and cool L radiate only in the infrared and are invisible to the eye. Class T contains only brown dwarfs, while class L (and even cool M) is a mixture of brown dwarfs and true dwarfs that run full hydrogen fusion.