Work/Kinetic Energy

Let's start with the definition of *work* in physics. The work done by a general applied force is:

$$W = \vec{F} \cdot \Delta\vec{s} = Fs\cos\theta$$

Notice that work is a *scalar* quantity and may be either positive or negative.

- Since we will generally deal with forces that are constant (at least initially) it usually won't be necessary for us to worry about any change in the force in this expression (since *F* is constant).
- The units of work are: $N \cdot m = (kg \cdot m \cdot s^{-2})(m) = kg \cdot m^2 \cdot s^{-2}$, a.k.a. a *Joule*.

Now let's define *kinetic energy*:

$$KE = \frac{1}{2}mv^2$$

This is also a scalar quantity, the units are: $kg \cdot m^2 \cdot s^{-2}$, a.k.a. a Joule! Kinetic Energy is the energy of motion. The higher an objects speed, the greater it's kinetic energy.

- There is another quantity in physics that has the same units as work: *torque*.
- Torque is a vector and work is a scalar.
- In the case of torque and work each set of similar units represents a different physical quantity.
- When two scalars have similar units they must be related.
- Work and energy are the same thing.
- The energy of an object changes if it exchanges energy with its environment.
- Energy transfer to an object is considered (+) and energy transfer from an object is considered (-).

The relationship between Work and Kinetic Energy when only *conservative* forces act on a system are (for the purposes of the present discussion, *conservative* forces are all forces except friction. is known as the *Work-Energy Principle*:

$$\Delta KE = KE_f - KE_i = W$$

So Work may be defined as a change in Kinetic Energy.

Finally, of considerable interest in atomic and nuclear physics is a unit of energy known as the *electron-volt* (eV). $1eV = 1.60 \times 10^{-19} J$

Work Done by Spring Forces

Spring forces are fundamentally different than forces we've looked at before in that they vary with the distance that the spring is stretched or compressed from it's *equilibrium position* (it's unstretched position).

The force exerted on a system attached to a spring is given by Hooke's Law: $\vec{F} = -k\vec{x}$ where *k* is known as the spring constant (a measure of a spring's stiffness) and *x* is the displacement of the spring from its equilibrium (unstretched or uncompressed) position.

Notice that the magnitude of the force given in Hooke's Law varies with the parameter *x* (displacement), since *k* for a given spring is constant. What is the significance of the minus sign on the right side of the equation?

The force exerted by a spring on an object varies harmonically, i.e., as a function of sines and cosines (harmonic functions). Harmonic systems are of considerable interest in physics, as we will see in the coming weeks.

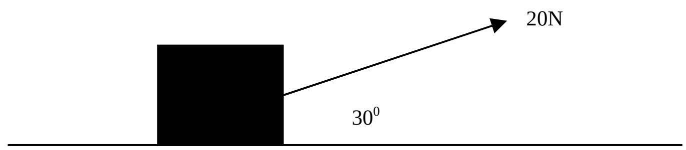
Work done by spring forces:

$$W_s = F\Delta x = (-kx)\Delta x = -kx\Delta x = \frac{1}{2}kx_i^{\,2} - \frac{1}{2}kx_f^{\,2}$$

The work done by a spring force may be either positive or negative. In this case, a positive value for work implies that the spring ends up closer to its equilibrium position than it was before, and a negative value implies the opposite. Work is zero if the displacement is zero (common in cyclic harmonic processes). If the spring starts from its equilibrium position:

$$W = -\frac{1}{2}kx^2$$

Example 1. Compute the work done by a 20 N force applied to a 10 kg block as shown below if the block moves 20 meters as a result of the application of the force. The surface is smooth.



Before we begin the computation of work, let's examine what happens when this system is set into motion. The application of the 20N force causes the block to *accelerate* to the right since there is no balancing force to the left (the magnitude of this acceleration is 1.73 m/s^2).

$$W = Fs\cos\theta = 20N(20m)(\cos 30^0) = 346J$$

Notice that no FBD's are necessary here!

How would this problem be different if the 20 N force were applied at an angle of zero dergrees with the horizontal? What about 90?

Example 2. What acceleration is required to stop a 1000kg car traveling 28 m/s in a distance of 100 meters?

$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2 = Fs\cos\theta$$

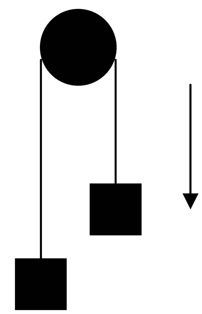
$$0 - (500\text{kg})(28\text{m}\cdot\text{s}^{-1})^2 = F(100m)$$

$$\vec{F} = -3920N \Rightarrow \vec{a} = -3.92m \cdot s^{-2}$$

This is known as the work-energy principle.

What is the significance of the minus sign in the answer?

Example 3. Use the *work-energy principle* to determine the final speed of a 5 kg mass (m_1) attached via a light cord over a massless, frictionless pulley to another mass of 3.5 kg (m_2), when the 5 kg mass has fallen (starting from rest) a distance of 2.5 meters.



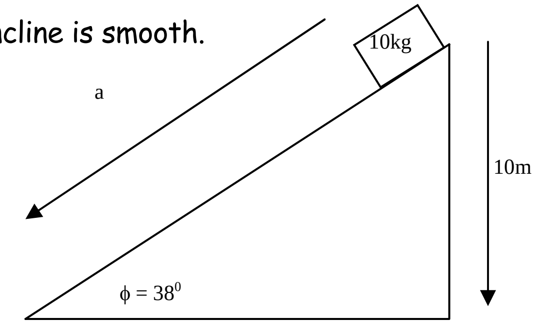
$$W = \Delta KE_{system} = KE_{f_{sys}} - KE_{i_{sys}} = \sum \vec{F} \cdot \vec{s}$$

$$\sum \vec{F} \cdot \vec{s} = KE_{f_{sys}} - 0 = \frac{1}{2}(m_1 + m_2)v_f^{\,2} - 0$$

$$v_f = \sqrt{\frac{2\left(\sum \vec{F} \cdot \vec{s}\right)}{m_1 + m_2}} = \sqrt{\frac{2[(m_1 g - m_2 g) \cdot s]}{m_1 + m_2}} = \sqrt{\frac{2[(m_1 - m_2)g \cdot s]}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2[(5kg - 3.5kg)(9.8m \cdot s^{-2}) \cdot 2.5m]}{8.5kg}} = 2.94 m \cdot s^{-1}$$

Example 4. Consider the following. The incline is smooth.



What is the speed of the block at the bottom of the incline?

$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f^{\ 2} - \frac{1}{2}mv_i^{\ 2} = Fs\cos\theta$$

$$\frac{1}{2}mv_f^{\ 2} - 0 = (mg\sin\phi)(s)(\cos\theta)$$

$$\frac{1}{2}v_f^{\ 2} = (g\sin\phi)(s)$$

Notice that (*sinϕ*)(s) = 10 meters, or the initial height of the block above the ground.

$$v_f = \sqrt{2gh} = \sqrt{2(9.8m\cdot s^{-2})(10m)} = 14m\cdot s^{-1}$$