Work/Kinetic Energy

Let's start with the definition of *work* in physics. The work done by a general applied force is:

$$W = \int_0^s \vec{F} \cdot d\vec{s} = Fs\cos\theta$$

- Notice that work is a *scalar* quantity and may be either positive or negative.
- Since we will generally deal with forces that are constant (at least initially) it usually won't be necessary for us to integrate this expression (since *F* is constant). Note, however, that when the force does vary it may do so with respect to either time of distance. It may be necessary to integrate with respect to either time, or distance, or perhaps both.
- The units of work are: $N \cdot m = \left(kg \cdot m \cdot s^{-2}\right)\!\left(m\right) = kg \cdot m^2 \cdot s^{-2}$, a.k.a. a *Joule*.

Now let's define *kinetic energy*:

$$KE = \frac{1}{2}mv^2$$

- This is also a scalar quantity, the units are: $kg \cdot m^2 \cdot s^{-2}$, a.k.a. a Joule.
- Kinetic Energy is the energy of motion. The higher an objects speed, the greater it's kinetic energy.
- Notice that work and energy have the same units.
- *Torque* and work also have the same base units. In that particular case each set of similar units represents a different physical quantity one, torque, being a vector and the other, work, being a scalar. But when two scalars have similar units they must be related. As we will see, work and energy are the same thing.
- The energy of an object changes if it exchanges energy with its environment. Energy transfer to an object is considered (+) and energy transfer from an object is considered (-).
- A relationship between work and change in kinetic energy applies when only *conservative* forces act on a system. This relationship is known as the *Work-Energy Principle*:

  $$\Delta KE = KE_f - KE_i = W$$

  So Work may be defined as a change in Kinetic Energy.
- Finally, of considerable interest in atomic and nuclear physics is a unit of energy known as the *electron-volt* (eV).

$$1eV = 1.60 \times 10^{-19}\,J$$

Work Done by Spring Forces

- Spring forces are fundamentally different than forces we've looked at before in that they vary with the distance that the spring is stretched or compressed from it's *equilibrium position* (it's unstretched position).
- The force exerted on a system attached to a spring is given by Hooke's Law:

$$\vec{F} = -k\vec{x}$$

  where k is known as the spring constant (a measure of a spring's stiffness) and x is the displacement of the spring from its equilibrium (unstretched or uncompressed) position.
- Notice that the magnitude of the force given in Hooke's Law varies with the parameter x (displacement), since k for a given spring is constant. What is the significance of the minus sign on the right side of the equation?
- The force exerted by a spring on an object varies *harmonically*, i.e., as a function of sines and cosines (harmonic functions). Harmonic systems are of considerable interest in physics, as we will see in the coming weeks.

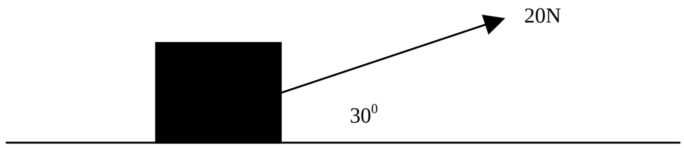
Work done by spring forces:

$$W_s = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (-kx) dx = -k \int_{x_i}^{x_f} x dx = \frac{1}{2} k x_i^{\,2} - \frac{1}{2} k x_f^{\,2}$$

- The work done by a spring force may be either positive or negative. In this case, a positive value for work implies that the spring ends up closer to its equilibrium position than it was before, and a negative value implies the opposite. Work is zero if the displacement is zero (common in cyclic harmonic processes). If the spring starts from its equilibrium position:

$$W = -\frac{1}{2} k x^2$$

Example 1. Compute the work done by a 20 N force applied to a 10 kg block as shown below if the block moves 20 meters as a result of the application of the force.



We simply use the definition of work and compute the necessary cosine. Before we begin the computation of work, let's examine what happens when this system is set into motion. The application of the 20N force causes the block to *accelerate* to the right since there is no balancing force to the left (the magnitude of this acceleration is 1.73 m/s^2).

$$W = Fs\cos\theta = 20N(20m)(\cos 30^0) = 346J$$

Notice that no FBD's are necessary here!

Example 2. What acceleration is required to stop a 1000kg car traveling 28 m/s in a distance of 100 meters?

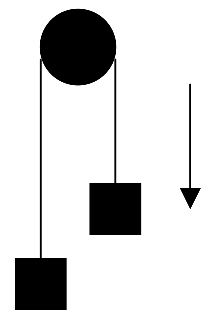
$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f^{\ 2} - \frac{1}{2}mv_i^{\ 2} = Fs\cos\theta$$

$$0 \text{ - } (500\text{kg})(28\text{m}\cdot\text{s}^{-1})^2 = F(100m)$$

$$\vec{F} = \text{-}3920N \Rightarrow \vec{a} = \text{-}3.92m\cdot s^{-2}$$

What is the significance of the minus sign in the answer?

Example 3. Use the *work-energy principle* to determine the final speed of a 5 kg mass (m_1) attached via a light cord over a massless, frictionless pulley to another mass of 3.5 kg (m_2), when the 5 kg mass has fallen (starting from rest) a distance of 2.5 meters.



$$W = \Delta KE_{system} = KE_{f_{sys}} - KE_{i_{sys}} = \sum \vec{F} \cdot \vec{s}$$

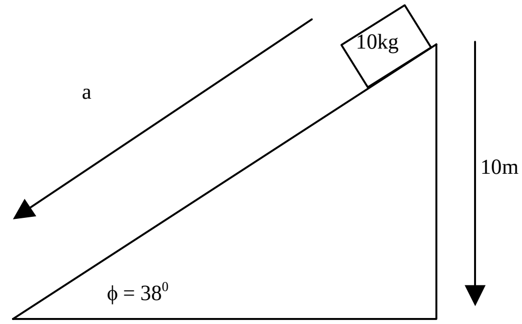
$$\sum \vec{F} \cdot \vec{s} = KE_{f_{sys}} - 0 = \frac{1}{2}(m_1 + m_2)v_f^{\ 2} - 0$$

$$v_f = \sqrt{\frac{2\left(\sum \vec{F} \cdot \vec{s}\right)}{m_1 + m_2}} = \sqrt{\frac{2[(m_1 g - m_2 g) \cdot s]}{m_1 + m_2}} = \sqrt{\frac{2[(m_1 - m_2)g \cdot s]}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2[(5kg - 3.5kg)(9.8m \cdot s^{-2}) \cdot 2.5m]}{8.5kg}} = 2.94m \cdot s^{-1}$$

Notice that FBD's are still required! Although this method is less complex that a straight Newton's 2nd law approach it is still somewhat involved.

Example 4. Consider the following.



If the surface is smooth what is the speed of the block at the bottom of the incline?

$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f^{\ 2} - \frac{1}{2}mv_i^{\ 2} = Fs\cos\theta$$

$$\frac{1}{2}mv_f^{\ 2} - 0 = (mg\sin\phi)(s)(\cos\theta)$$

$$\frac{1}{2}v_f^{\ 2} = (g\sin\phi)(s)$$

Notice that (*sinϕ*)(s) = 10 meters, or the initial height of the block above the ground.

$$v_f = \sqrt{2gh} = \sqrt{2(9.8m\cdot s^{-2})(10m)} = 14m\cdot s^{-1}$$

Not only does this agree with previous results but it is a much more compact method of solution.