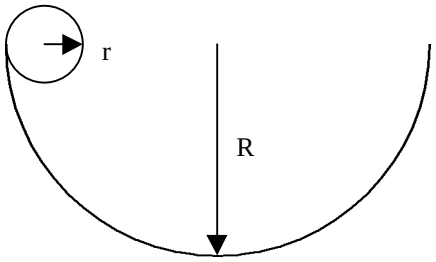
Example: A uniform solid sphere of radius *r* is placed on the inside surface of a hemispherical bowl of radius *R*. The sphere is released from rest at the edge of the bowl as shown below and rolls without slipping. Determine the translational speed of the sphere when it reaches the bottom of the bowl.



center of mass of the sphere is located at $R - r$

conserve energy: $E_i = E_f$

$$mg(R-r)_i + \frac{1}{2}mv_i^{\,2} = mg(R-r)_f + \frac{1}{2}mv_f^{\,2} + \frac{1}{2}Iw^2$$

$$mg(R-r) = +\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)w^2$$

$$v = rw \therefore mg(R-r) = +\frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

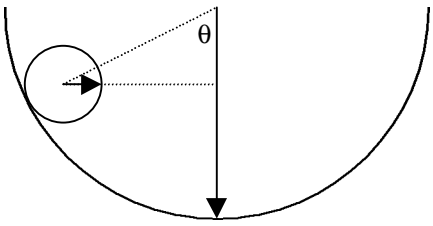
$$g(R-r) = \frac{7}{10}v^2 \therefore v = \sqrt{\frac{10}{7}g(R-r)}$$

What is the angular velocity of the sphere at the bottom of the bowl?

$$v = rw \therefore mg(R-r) = +\frac{1}{2}mr^2w^2 + \frac{1}{5}mr^2w^2 = \frac{7}{10}mr^2w^2$$

$$g(R-r) = \frac{7}{10}r^2w^2 \therefore w = \sqrt{\frac{10}{7}\frac{g(R-r)}{r^2}}$$

What is the angular velocity of the sphere if it is released at a random point along the edge of the bowl some angle *q* to the vertical?



$$h = (R-r) - \left[(R-r)(\cos q)\right]$$

$$\therefore w = \sqrt{\frac{10}{7}\frac{g(R-r)\left[(R-r)(\cos q)\right]}{r^2}}$$