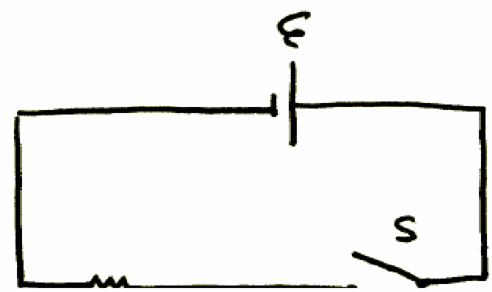
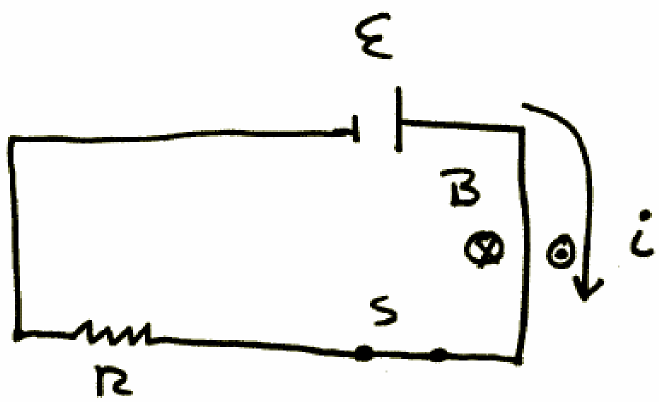
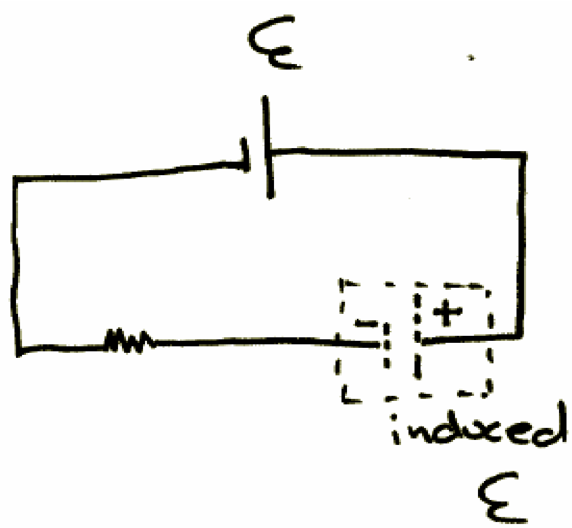
Self-Inductance

When *any* circuit is energized the current does not immediately rise to its final value of ξ/R. Electromagnetic induction prevents this (Recall: $\xi = -\frac{d\Phi_m}{dt} = \frac{d\vec{B}\cdot\vec{A}}{dt}$).

Consider the circuits at right. When *s* is closed, *i* begins to increase from *i* = 0 to *i* = ξ/R.







- As *i* increases the magnetic flux $\frac{d\Phi_m}{dt}$ increases due to the increasing ***B*** field produced by current flowing through the wires.

- The changing magnetic flux produces an induced EMF, $\xi = -\frac{d\Phi_m}{dt}$, which *opposes* the original EMF. This opposing EMF is a necessary consequence of conservation of energy and is required to keep the current in the circuit from increasing without limits.

- The presence of this induced EMF slows the rise of current in the circuit. This phenomenon is known as *self-induction*.

- For a loop of *n* turns, it may be shown that self-inductance is: $L = \frac{n\Phi_m}{i}$

- If Φ_m and i are time varying: $\xi_I = -n\frac{d\Phi_m}{dt} = -L\frac{di}{dt}$. This is know as a self - induced EMF

Although some amount of self-inductance exists in all current loops, some devices exploit inductance. These are known as *inductors, coils or chokes*.


the symbol for an inductor

- The S.I. unit of inductance is a Henry (H) . $1H = 1V \cdot s \cdot A^{-1}$
- Inductance depends upon geometry and may be increased or decreased by:

  - The size of inductor
  - The shape of inductor
  - The number of loops
  - The magnetic properties of inductor

- Inductors act like resistors in any circuit where current is changing.

Energy in an Inductor

Since an inductor acts as a source of EMF it must have the ability to store energy.

Recall: $\xi = L\frac{di}{dt}$, $P = VI = i\xi \to P = iL\frac{di}{dt}$

The increment of work, *dw* done in time *dt* is P*dt*

$$dw = Pdt = iLdi$$

$$W = L\int_0^I idi = \frac{1}{2}LI^2$$

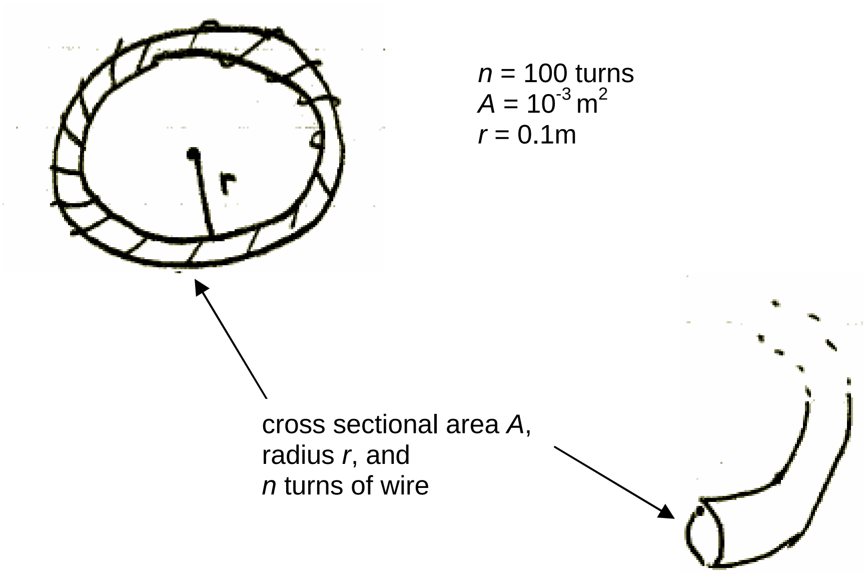
Note that when $\frac{di}{dt} = 0$, *i* = I and the *steady state* has been reached.

In the steady state, $\xi = L\frac{di}{dt} = 0$ so there is no induced EMF in the steady state condition of a current loop.

The potential energy of an inductor is $\frac{1}{2}LI^2$. The reference level here is I = 0 which is analogous to h = 0 in gravity fields.

Example

Find the self-inductance in an air core, toroidal solenoid of the following dimensions



$\Phi_m = BA = \dfrac{\mu_o niA}{2\pi r}$ for a toroidial solenoid. So the self-inductance of a toroidal solenoid is:

$$L = \frac{n\Phi_m}{i} = \frac{n^2\mu_o iA}{i2\pi r} = \frac{\mu_o n^2 A}{2\pi r}$$

For n = 100, A = 10^{-3} m^2, r = 0.1m

$$L = \frac{(4\pi x 10^{-7} Wb\cdot A^{-1}\cdot m^{-1})(100)(10^{-3}m^2)}{(2\pi)(0.1m)} = 20\times 10^{-6} H = 20\mu H$$

If the current in the inductor increases from 0 to 1 amps in 0.1s, $\dfrac{di}{dt} = 10A\cdot s^{-1}$

$\xi = -L\dfrac{di}{dt} = (20uH)\left(10A\cdot s^{-1}\right) = 2\times 10^{-4} Volts$. This EMF opposes the original EMF.

What is the energy stored in this inductor?

$U = \dfrac{1}{2}LI^2 \rightarrow U = \left(\dfrac{1}{2}\right)(20\mu H)(1A)^2 = 1\times 10^{-5} Joules$ or 10μ*J*