Linear Momentum/Conservation of Linear Momentum

Linear Momentum

The definition of linear momentum is: $\vec{p} = m\vec{v}$

Notice that since mass is an intrinsically positive quantity, ***p*** and ***v*** always point in the same direction. The S.I. unit of momentum is a kg · m · s^{-1}.

Newton's second law may be expressed (and was originally expressed) in terms of the time rate change of momentum or:

$$\sum F = \frac{d\vec{p}}{dt}$$

The equivalence of the two expressions may be easily shown:

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} \therefore \sum F = m\vec{a}$$

For a system consisting of more than one particle, the total momentum, ***P***, of the system is:

$$\vec{P} = M_{total}\vec{v}_{centerofmass}$$

Conservation of Linear Momentum

Consider our definition of linear momentum, $\sum F = \frac{d\vec{p}}{dt}$. Suppose that the sum of all external forces acting on a particle or system of particles is zero. In this case, the momentum of the system cannot be changing in time since $\frac{d\vec{p}}{dt} = 0$. When the sum of all external forces acting on an isolated system is zero, the total momentum of a system cannot change and momentum is said to be *conserved*, i.e.,

$$\vec{P}_i = \vec{P}_f$$

or

$$m\vec{v}_i = m\vec{v}_f$$

Caveat! Linear momentum is a vector quantity. What do you suppose is the consequence of external forces acting on a system that have no components acting in the direction of the velocity of the particle(s) in the system?

Conservation of linear momentum (or not)

- A ball bouncing off a wall?
- Firing a gun?
- A collision between two billiard balls?

Example 1. A 4.5 kg shotgun fires a 0.1 kg slug with a muzzle velocity of +150 m/s. What is the recoil velocity of the shotgun?

There is an external force that acts on this system, gravity. But in the scale of this problem gravity acts only perpendicularly to the velocities of various parts of the system so it has no discernable effect on the momentum along a horizontal axis.

$$\vec{P}_i = \vec{P}_f$$

$$m\vec{v}_i = m\vec{v}_f$$

$$m_{shotgun} v_{shotgun_i} + m_{slug} v_{slug_i} = m_{shotgun} v_{shotgun_f} + m_{slug} v_{slug_f}$$

$$0 + 0 = m_{shotgun} v_{shotgun_f} + m_{slug} v_{slug_f}$$

$$m_{slug} v_{slug_f} = -m_{shotgun} v_{shotgun_f}$$

$$\frac{(0.1kg)(150m \cdot s^{-1})}{4.5kg} = -v_{shotgun} = -3.3m \cdot s^{-1}$$

The shotgun recoils with an initial velocity of about 3 m/s (≈ 7.5 mph). Depending upon how rapidly the stock of the gun comes to rest against the shooter's shoulder, this could be a heck of a wallop. Unless the gun has a padded stock or the shooter is wearing a padded jacked, this would be like getting hit in the shoulder by a 93 mph fastball (an official major league ball has a mass of 5 oz troy or about 0.16 kg).