Kinematics Part 2: Motion in one dimension (vertical) with acceleration due to gravity

Example 1

A rock is dropped from the top of a building 100 meters tall. Find:

- *The acceleration*
- *Time of flight*
- *The velocity of the rock just before impact*

Implied information: $\boldsymbol{v}_0 = 0$ and $\boldsymbol{a} = -g$

$$\vec{y} - \vec{y}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = -100m = 0 - \frac{1}{2}\left(9.8m/s^2\right)t^2$$

Notice that the signs of the displacement and velocity vectors are negative. Why?

$$t = \sqrt{\frac{-100m}{-4.9m/s^2}} = 4.5s$$

$$\vec{v} = -gt = \left(-9.8m/s^2\right)\left(4.5s\right) = -44.1m/s$$

Instead of dropping the rock it is thrown downward with an initial velocity of -5 m/s. Determine the acceleration, time of flight and final velocity.

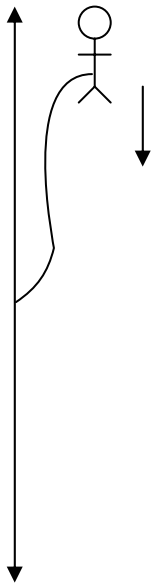
$\boldsymbol{a} = -g$

$$v^2 = v_0{}^2 + 2a(y - y_0) = \left(-5m/s\right)^2 + 2\left(-9.8m/s^2\right)\left(-100m\right) \therefore \vec{v} = -44.6m/s$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \rightarrow \frac{\vec{v} - \vec{v}_0}{g} = t = \frac{-44.6m/s + 5m/s}{-9.8m/s^2} = 4s$$

Example 2

A climber decides to see if UIAA specs are good and take a minimum fall factor roped fall on a climbing rope. The climber ascends to a point 60 meters above an anchor (and over 120 meters above the ground) and jumps. What is the total distance the climber falls during the 1st second, the 2nd second, and the 3rd second? How long does it take to reach the end of the rope? How fast is the climber moving when they reach the end of the rope.



For the 1st second (t = 1s):

$$\vec{y} - \vec{y}_0 = 0 - \frac{1}{2} g t^2 = \left(-4.9 m/s^2\right)(1s)^2 = -4.9m$$

For the 2nd second (t = 2s):

$$\vec{y} - \vec{y}_0 = 0 - \frac{1}{2} g t^2 = \left(-4.9 m/s^2\right)(2s)^2 = -19.6m$$

For the 3rd second (t = 3s):

$$\vec{y} - \vec{y}_0 = 0 - \frac{1}{2} g t^2 = \left(-4.9 m/s^2\right)(3s)^2 = -44.1m$$

Notice that the distance covered in 1st second is 4.9 meters, 14.7 meters in the 2nd second , and 24.5 meters in the 3rd second.

The end of the rope comes after falling a distance of 120 meters:

$$\vec{y} - \vec{y}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \;\therefore -120m = -4.9 m/s^2\left(t^2\right) \therefore t = 4.9s$$

And the falling climber is moving downward at a speed of:

$$v = v_0 + at = 0 - \left(9.8 m/s^2\right)(4.9s) = 48 m/s$$ (about 107 mph)