Kinematics Part 1: Motion in one dimension (horizontal) with constant acceleration

Three important kinematic equations:

1. $\vec{v} = \vec{v}_0 + \vec{a}t$
2. $\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$
3. $v^2 = {v_0}^2 + 2a(x - x_0)$

We will derive these equations from Newton's second law later. These three equations are common to most problems involving kinematics. One is usually asked to find time, distance or acceleration in terms of the other variables. Remember, our sign convention for vectors is ↑ and → are +. Assume all quantities are positive unless otherwise indicated.

Example 1

Suzuki claims that a Hayabusa will accelerate from 0 to 60 m/s in 10 seconds. Assuming constant acceleration (not a great assumption but one that will do for the present) find:

- *The acceleration*
- *The distance traveled*
- *The speed of the bike*
- *The speed of the bike after 15 seconds under constant acceleration*
- *The speed of the bike after 15 seconds if the acceleration stops after 10 seconds*

We can use the first of our kinematic equations to solve for acceleration in terms of initial velocity, final velocity and time, all of which we know. The acceleration is:

$$\frac{\vec{v} - \vec{v}_0}{t} = \vec{a} = \frac{60m/s}{10s} = +6m/s^2$$

From the second kinematic equation, which yields displacement from initial velocity, acceleration and time, the distance traveled is:

$$\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = 0 + \frac{1}{2}\left(6m/s^2\right)\left(10s\right)^2 = +300m$$

The speed of the bike after 10 seconds is:

$$\vec{v} = \vec{v}_0 + \vec{a}t = 0 + \left(6m/s^2\right)\left(10s\right) = +60m/s$$ (about 134 mph)

The speed of the bike after 15 seconds is:

$$\vec{v} = \vec{v}_0 + \vec{a}t = 0 + \left(6m/s^2\right)\left(15s\right) = +90m/s$$ (about 200 mph)

The speed of the bike after 15 seconds (if acceleration lasts only 10 seconds) is 134 mph.

Example 2

You are designing an airport for small private aircraft. A Cessna 182 has a stall speed of about 27.8 m/s and can accelerate at a rate of 2 m/s^2. How long must a runway be in order to accommodate this plane?

Given:

$\vec{v}_0 = 0m/s$

$\vec{a} = +2m/s^2$

$\vec{v} = 27.8m/s$ @ liftoff

You need to determine displacement: $\vec{x} - \vec{x}_0$

$$\frac{v^2 - {v_0}^2}{2a} = x - x_0 = \frac{(27.8m/s)^2}{4m/s^2} = 193.2m$$

Notice that vector notation is not used in this equation. The squaring of the velocity terms obscures their vector nature. The answer of 193.2m is the *magnitude* of the displacement vector. In cases like this it is up to us to determine the direction of the displacement vector from other data in the problem. In this case the direction is not necessary because the question asks only how *long* the runway must be.

Example 3

The steam catapult on the aircraft carrier Carl Vinson is able to hurl an F/A-18 Hornet into the air from rest in 1.2 seconds at a speed of 60 m/s (about 117 knots). How many "G's does the pilot experience?

A "*G*" is the acceleration due to gravity, 9.8 m/s^2.

$$\frac{\vec{v}-\vec{v}_0}{t} = a = \frac{60m/s}{1.2s} = 50m/s^2 \Rightarrow \frac{50m/s^2}{9.8m/s^2} = 5.1$$

So the pilot experiences an acceleration of 5.1 *G*'s. This is a lot! Most people black out somewhere between 6 and 7 *G*'s.

Example 4

Estimate the minimum stopping distance for a car on a dry road that can accelerate (slow down) at a rate of -6 m/s^2 if the initial velocity of the car is +28 m/s (about 62 mph) and the driver's reaction time is 0.5 seconds (a little below average).

- There are two intervals of interest in this problem.
- Since the problem asks for a distance, the intervals should be stated in terms of displacement vectors.
- The first interval spans the distance from the time the driver first reacts until they hit the brakes (reaction time). During this time the car is traveling at constant velocity.
- The second interval spans the distance from where the brakes are first applied to where the car comes to rest. An acceleration is present during this interval.
- Notice that the sign of the acceleration vector is opposite the sign of the velocity vector in the problem statement. Remember, when an object is gaining speed the signs of the velocity and acceleration vectors are the same, when an object is slowing down these signs must be opposite. In this case the choice of which was positive and negative was completely arbitrary. Without working the problem, can you predict the sign of the displacement vector?

Given:

$$\vec{v}_0 = +28m/s$$
$$\vec{a} = -6m/s^2$$
$$t_r = 0.5s$$

Implied:

$$\vec{v} = 0m/s$$

To be determined:

$$\vec{x} - \vec{x}_0$$

For the first phase:

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = (+28m/s)(0.5s) = +14m$$

For the second phase:

$$v^2 = v_0^{\ 2} + 2a(x - x_0) \rightarrow \frac{v^2 - v_0^{\ 2}}{2a} = x - x_0 = \frac{-(28m/s)^2}{-12m/s^2} = 65.3m$$

Notice that the vector nature of the displacement is obscured by the squares in this equation. **Total stopping distance is 79.3 meters.**

Example 5

A motorcyclist traveling 33.3 m/s (75 mph) passes a constable in a 55 mph zone. The constable immediately begins pursuit at a constant acceleration of 2.8 m/s^2. How much time will it take for the constable to overtake the motorcyclist (who does not know that he is being pursued). How fast will the constable be traveling?

- What is actually being asked here? We seek *time* and *velocity* when the *displacement* is the same for both moving objects.
- Note that some important information is implied rather than stated directly, i.e., $\boldsymbol{v} = \boldsymbol{v}_0$ for the motorcyclist.
- Once we understand what the problem actually asks the next step is to translate the worded statement into mathematics.
- Notice that it is not necessary to actually determine the value of the displacement, just the time at which it is the same for both objects.

Hence:

Motorcyclist:

$$\vec{x} - \vec{x}_0 = (33.3m/s)(t) + \frac{1}{2}(0)(t^2)$$

Constable:

$$\vec{x} - \vec{x}_0 = (0)(t) + \frac{1}{2}(2.8m/s^2)(t^2)$$

Setting the two equations equal to each other:

$$(33.3m/s)(t) = \vec{x} - \vec{x}_0 = \frac{1}{2}(2.8m/s^2)(t^2)$$

$$t = 23.8s$$

and

$$\vec{v} = \vec{v}_0 + \vec{a}t = 0 + (2.8m/s^2)(24s) = 67.2m/s$$

Example 6

A motorcyclist traveling 40 m/s (89 mph) is caught in a speed trap. One second after the motorcycle passes the hidden officer pursuit begins with an acceleration of 3 m/s^2. How much time will it take for the officer to overtake the motorcyclist?

This is a variant of Example 5. The difference is that reaction time is factored into this example making it a bit more like the real world. The motorcycle is traveling at a constant velocity of +40 m/s. The officer begins from rest accelerating at a rate of 3 m/s^2. We seek the time when the displacement is the same for both.

Motorcyclist:

$$\vec{x} - \vec{x}_0 = (40m/s)(t) + \frac{1}{2}(0)(t^2)$$

Constable:

$$\vec{x} - \vec{x}_0 = (0)(t) + \frac{1}{2}(3m/s^2)(t^2)$$

But the pursuit begins after the motorcycle has traveled 40 meters (40 m/s @ 1 second).

Motorcyclist

$$\vec{x} - \vec{x}_0 = (40m) + 40m/s(t)$$

Constable:

$$\vec{x} - \vec{x}_0 = \frac{1}{2}(3m/s^2)(t^2)$$

Setting the two equations equal to each other:

$$(40m) + 40m/s(t) = 1.5m/s^2(t^2)$$

This is a quadratic equation in *t* (a*t*2 + b*t* + c = 0) with:

a = 1.5 m/s^2
b = -40 m/s
c = -40 m

The physically meaningful root is *t* = 27.6 seconds.

Example 7

A runner hopes to complete a 10k race in less than 30 minutes. After 27 minutes they still have 1100 meters to go. If the runner has enough strength left to accelerate at a rate of 0.20 m/s^2, how many seconds must they do so to complete the race in less than 30 minutes?

What is being asked? The interval of interest is the remaining 180 seconds of the race. At the beginning of this interval:

$$\vec{x} - \vec{x}_0 = +1100m$$
$$\vec{a} = +0.20m/s^2$$

Note that the runner has completed 8900 meters of the race in 1620 seconds.

$$\frac{8900m}{1620s} = +5.49m/s = \vec{v}_0$$

The runner will finish the race by attempting to accelerate for *t* seconds. They may not have to accelerate the entire distance to the finish line (unlikely!) so after a brief acceleration to increase their speed they will run at a constant velocity for 180s - *t* at rate given by $\vec{v} = \vec{v}_0 + \vec{a}t$. Hence:

$$\vec{x} - \vec{x}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 + \vec{v}t\left[= \left(\vec{v}_0 + \vec{a}t_a\right)t_c\right]$$

where t_a are the seconds during the acceleration phase and t_c are the seconds during the final constant speed phase.

1100m = [acceleration phase] + [constant speed phase]

$$1100m = \left[\left(5.49m/s\right)t + \frac{1}{2}\left(0.20m/s^2\right)t^2\right] + \left[\left(5.49m/s + \left(0.20m/s^2\right)t\right)\left(180s - t\right)\right]$$

$$111.8m = \left(-0.1m/s^2\right)t^2 + \left(36m/s\right)t$$

This is a quadratic. Solving the equation for *t* and eliminating the physically meaningless root yields *t* = 3.1 seconds. So the runner must accelerate for 3.1 seconds in order to finish the race in 30 minutes.