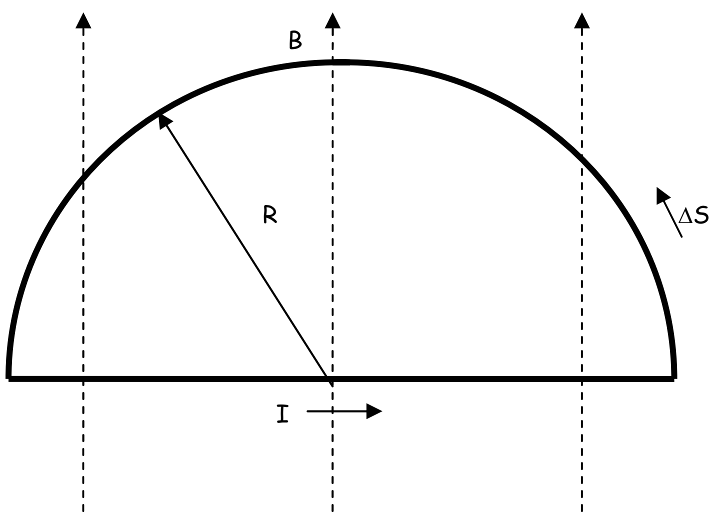
Force and Torque on a Current Loop

We can represent current carrying loops as a series of straight line segments. Even curved segments can be approximated as straight line segments. Recall that while the total force on any current carrying loop is zero, the force on any particular segment of such a loop is $F = I\ell B\sin\theta$

Force on a Semicircular Conducting Loop in a Uniform ***B*** Field

Consider:



We'll treat this as two connected conductors: the (1) straight current-carrying element along the bottom and the (2) curved element along the top.

1 $\vec{F}_1 = I\vec{\ell}\times\vec{B} \Rightarrow F_1 = I\ell B\sin\theta = I\ell B$ since $F \perp I$

$\ell = 2R \rightarrow F_1 = I2RB$

Notice that the R.H.R. yields a direction of •

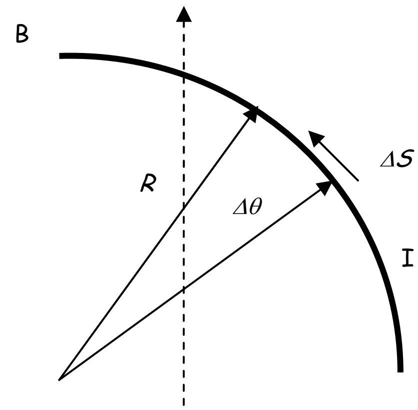
So $\vec{F}_1 = 2IRB\hat{k}$

2 $dF_2 = IB\sin\theta\Delta s$

$s = R\theta \rightarrow \Delta F_2 = IB\sin\theta R\Delta\theta$

it may be shown that:

$F_2 = 2IRB$



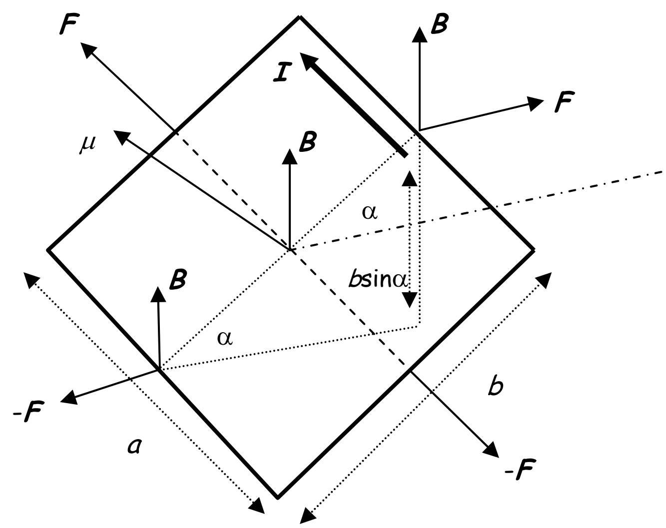
Notice that the R.H.R. yields a direction of $\times$

So $\vec{F}_2 = 2IRB - \hat{k}$

Given that $\vec{F}_1 = 2IRB\hat{k}$ and $\vec{F}_2 = 2IRB - \hat{k}$ the forces are equal and opposite but constitute a torque couple.

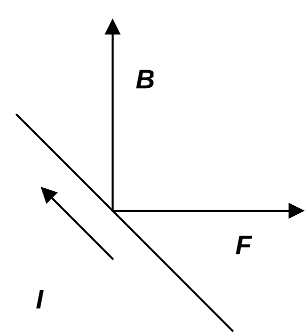
In this case it might be slightly problematic to easily choose an axis to convert this torque into smooth rotation. But it we were more judicious in choosing the shape of our current loop this would be easier.

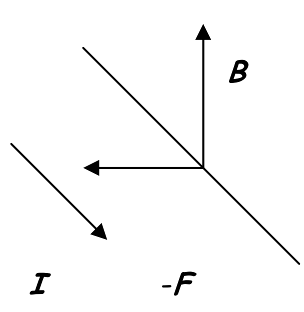
Torque on a current loop in a Uniform Magnetic Field



Consider a rectangular loop with a uniform magnetic field as shown above. Let's first examine the force on right side (ℓ = *a*) of the loop due to the current moving ccl around the loop.

The RHR gives the direction. The magnitude is $F_a = IaB\sin\theta = IaB$





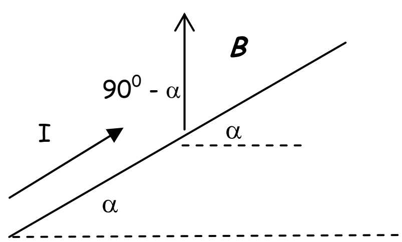
On the left side of the loop, $F_a = IaB$ but in the opposite direction

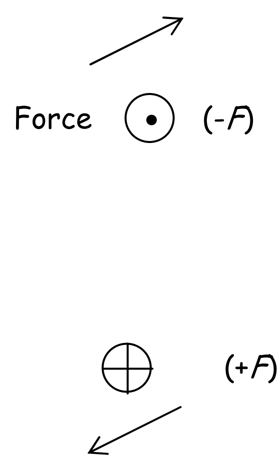
It is clear from looking at both of the *a* sides of the loop that $-F_a + F_a = 0 = F_{a\,net}$.

Again even though the forces are equal an opposite in the situation shown they constitute a torque couple with the given axis around which the loop is free to rotate. We shall exploit this presently.

Now let's consider the *b* sides of the loop (top and bottom where *ℓ* = *b*)

This is a view of a *b* side from the bottom of the loop. A "side" view.





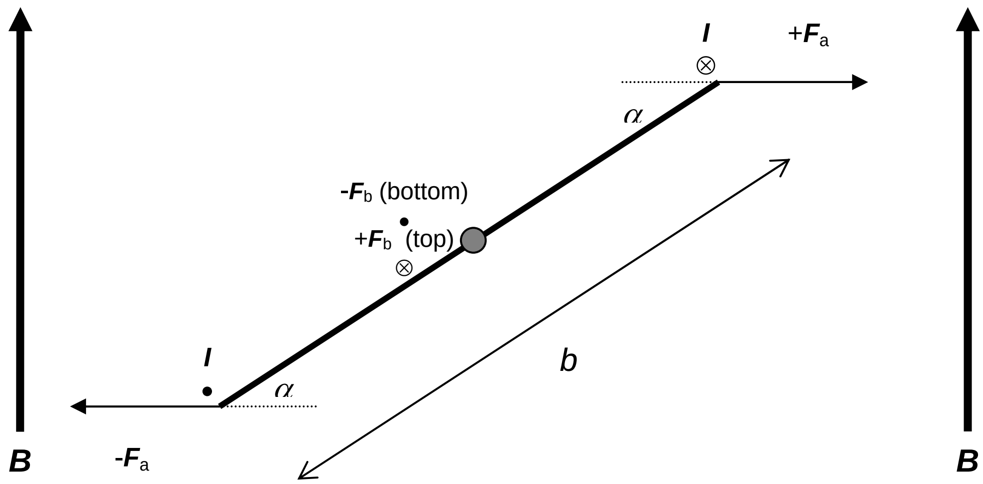
$$F_b = IBb\sin(90° - \alpha)$$

$$-F_b = IBb\cos\alpha$$

$$+F_b = IBb\cos\alpha$$

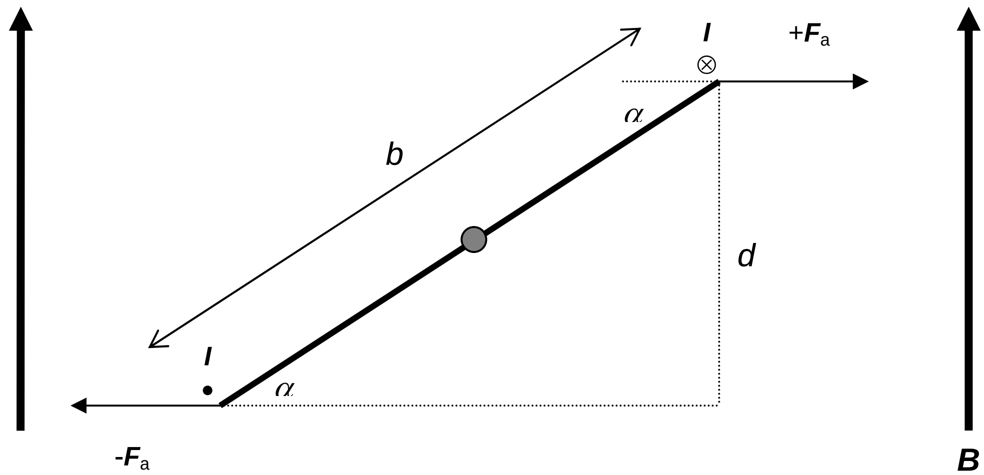
$$F_b + -F_b = 0 = F_{b\,net_b}$$

The first condition of equilibrium is met for both sides of the loop but the second condition of equilibrium is not because while $\boldsymbol{F}_b$ and $-\boldsymbol{F}_b$ lie along the same line $\boldsymbol{F}_a$ and $-\boldsymbol{F}_a$ do not.



Consider the sketch above. The perspective is the plane of the loop viewed from the bottom

- The current is coming out of the page on the lower left side of the loop
- The current goes from lower left to upper right along the bottom side of the loop (view)
- The current goes into the page at the upper right side of the loop
- The current goes from upper right to lower left along the top of the loop (hidden)
- $\boldsymbol{F}_a$ and $-\boldsymbol{F}_a$ are equal and opposite everywhere along side *a* but *do not* share a common line of action
- $\boldsymbol{F}_b$ and $-\boldsymbol{F}_b$ are equal and opposite everywhere along side *b* and *do* share a common line of action
- Recall that when the second condition of equilibrium is not met that a torque is produced
- $\Gamma = \vec{R} \times \vec{F}$ = $\boldsymbol{F} \times$ distance
- $\boldsymbol{F}_a$ and $-\boldsymbol{F}_a$ produce a couple that exerts a torque about the rotational axis of the loop



- The torque produced by F_a and $-F_a$ about the rotational axis of the loop is equal to either *both* forces multiplied by ½ *d* or either force multiplied by *d*.
- The torque will have a maximum value when the loop is vertical and a minimum value when the loop is horizontal (as $d \rightarrow 0$). Be sure to verify this for yourself.
- Recalling that the force on either side *a* of the loop is ***IℓB*** or in this case ***IaB***

$$\sin\alpha = \frac{d}{b} \rightarrow d = b\sin\alpha \rightarrow \Gamma = Fb\sin\alpha = IBab\sin\alpha$$

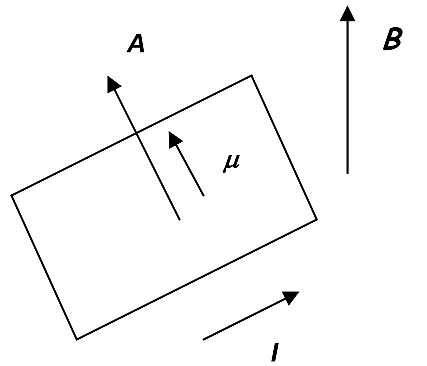
- The torque has a maximum value when $\alpha = 90^0$ (plane of the loop parallel to ***B***) and a minimum value when $\alpha = 0^0$ (plane of the loop perpendicular to ***B***)
- The torque tends to rotate the loop in the direction of decreasing α
- Since the area of the loop is *ab*, *A* = *ab* and

$$\Gamma = IAB\sin\alpha = IA \times B$$

- The quantity I***A*** is known at the *magnetic moment* of the loop,

$$\vec{\mu} = I\vec{A} \rightarrow \Gamma = \vec{\mu} \times \vec{B} = \vec{\mu}\vec{B}\sin\alpha \ .$$

The magnetic moment is a vector that is parallel to ***A***.

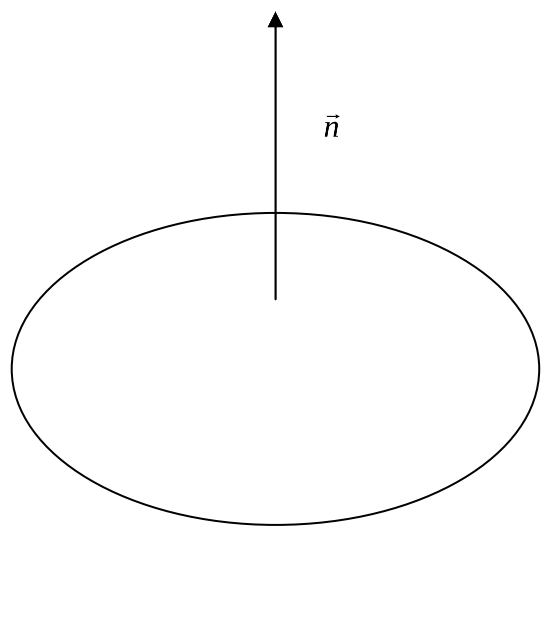


We define vector ***A*** perpendicular to the plane of the loop.

The magnitude of ***A*** is the area of the loop and direction is determined by RHR

For a circular loop of radius ***r*** it can be shown that:

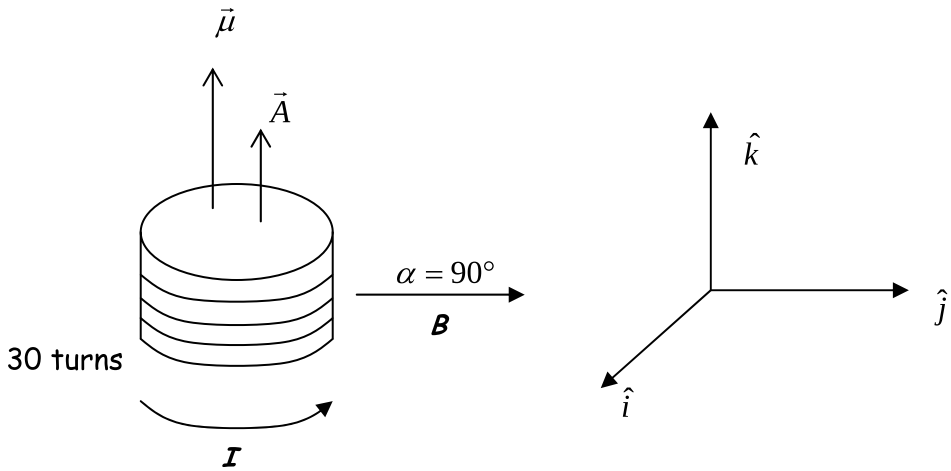
$$\Gamma = IAB\sin\alpha = I\pi r^2 B\sin\alpha \text{ or } \vec{\Gamma} = \left(I\pi R^2\right)\times\vec{B}$$



For multiple loops of any geometry the total torque is equal to the sum of the torque contributions from the individual loops, hence $\Gamma = nIAB\sin\alpha = nI\pi r^2 B\sin\alpha$ or $\vec{\Gamma} = n\mu\times\vec{B}$ where n is the number of loops.

Example 5

A *solenoid* of 30 turns with a radius ***r*** =0 .05m has a 5 amp current in its coils. It is placed in a ***B*** field of 1.2T as shown below. Find the magnetic moment and the torque generated by the ***B*** field on this coil.



To find the magnetic moment

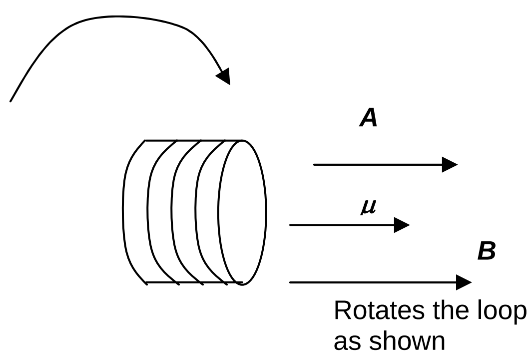
$$A = \pi r^2 = 7.85 \times 10^{-3} m^2$$

$$\mu = IA = 3.93 \times 10^{-2} A \cdot m^2$$ 1 loop

$$\mu = nIA = 1.18 A \cdot m^2$$ 30 loops

$\vec{\mu} = 1.18 A \cdot m^2$ in the $+\hat{k}$ direction

To find the torque



$$\alpha = 90° \qquad (\vec{B} \perp Area)$$

$$\Gamma = nIBA \sin \alpha$$

$$\Gamma = (30)(5A)(1.2T)(7.85 \times 10^{-3} m^2)(1) = 1.41 N \cdot m$$

or we can use the magnetic moment

$$\Gamma = \mu B \sin\alpha$$

$$\Gamma = \left(1.18A \cdot m^2\right)\left(1.2T\right)\left(1\right) = 1.41N \cdot m$$

What force would be required to hold the coils ⊥ to the field if applied to opposite edges of the coil?

$$\Gamma = \left(2\right)\left(0.05m\right)\left(F\right) \rightarrow 1.41N \cdot m = \left(.1\right)\left(F\right) \rightarrow F = 14.1N$$

An interesting way of looking at this current carrying loop in a magnetic field is from the perspective of energy. Is there any change in kinetic or potential energy here? Even though magnetic forces do no work in displacing particles traveling in ***B*** fields, here work is obviously done because the loop has a higher potential energy when $\alpha = 90^0$ than it does when $\alpha = 0^0$.

We will define the potential energy of any current carrying loop in a magnetic field to be:

$$u = -\vec{\mu} \cdot \vec{B} - \mu B \cos\alpha$$

So for this particular loop

$$u_i = -\left(1.18A \cdot m^2\right)\left(1.2T\right)\left(\cos 90°\right) = 0$$

$$u_f = \left(-1.18A \cdot m^2\right)\left(1.2T\right)\left(\cos 0°\right) = -1.41J$$

$$\therefore \Delta u = -1.41J$$