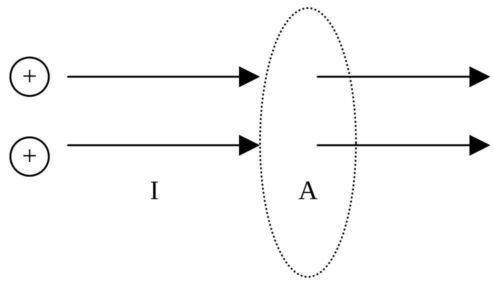
Electrodynamics I

The Classical Model of Electric Current Flow

Electric current consists of charge in motion



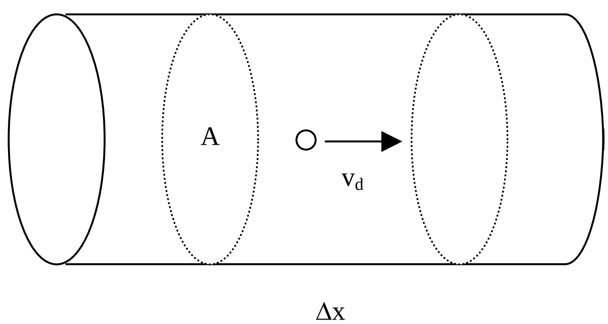
Current is defined as the amount of charge moving past a fixed point per unit time.

$$I_{av} = \Delta Q/\Delta t$$

And in terms of differentials.

$$I_{av} = dQ/dt$$

Consider a charge moving through a wire conductor



Where

- n - number of mobile charge carriers
- $A\Delta x$ - volume of the region of interest
- $nA\Delta x$ - number of charge carriers in the volume of interest
- ΔQ - number of charge carriers × charge per carrier ($nA\Delta x \times q$)

If the charge carriers move with a drift velocity v_d,

$$\Delta x = v_d \Delta t$$

$$\Delta Q = (nA\, v_d \Delta t)q$$

$$\Rightarrow I = \Delta Q/\Delta t = nA\, v_d q$$

The S.I. unit of current is the *Ampere* (A). One ampere is the equivalent to one coulomb of charge per second moving past some point of reference.

Resistance

Define: $\vec{J}$, current density:

$$J = \frac{I}{A} = nqv_d \quad (\text{A/m}^2)$$

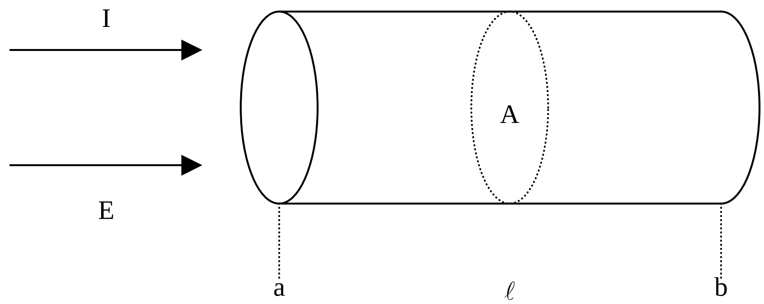
where ***J*** is parallel to ***A***. This yields:

$$\vec{J} = nq\vec{v}_d = \sigma\vec{E}$$

where σ is a constant of proportionality known as *conductivitity.*

- a current density ***J*** and an electric field ***E*** are established through any conductor across which a potential difference exists
- σ is a constant in ohmic materials that is a characteristic of the material.
- σ = ***J/E***
- $\vec{J} = \sigma\vec{E}$ is known as *Ohm's Law.*

Ohm's Law



Recall:

$$V = V_b - V_a = \int_a^b \vec{E} \cdot d\vec{s} \Rightarrow V = \int_0^\ell \vec{E} \cdot d\vec{\ell} = E\ell \therefore E = \frac{V}{\ell}$$

$$\vec{J} = \sigma\vec{E} = \sigma\frac{V}{\ell} = \frac{I}{A}$$

$$V = \frac{I\ell}{\sigma A}$$

The quantity $\frac{\ell}{\sigma A}$ is known as *R*, resistance, where *R* is a constant that is specific to different materials. So Ohm's Law may be rewritten:

$$V = IR$$

Note: $R = \frac{V}{I} = \frac{\ell}{\sigma A} = \rho\frac{\ell}{A}$ where ρ is known as *resistivitity*.

Note: $\rho \equiv \frac{1}{\sigma} \equiv \frac{\vec{E}}{\vec{J}} \Rightarrow R = \rho\frac{\ell}{A}$.

The unit of *R* is the Volt/Ampere, aka, the Ohm (Ω).

Resistance is basically a measure of the difficulty that charge carriers have moving through a material under the influence of an electric field.

Electrical Energy and Power

- The *current* present in any simple circuit is related to the voltage and resistance by Ohm's Law: $I = \frac{V}{R}$.
- A resistor uses the potential energy in an electrical circuit (either as heat or some form of work). Electrical potential energy (voltage) is used up in electrical circuits while current is conserved.
- The power consumed by a circuit is the energy per unit time used in Joules/sec. The SI unit of power is the Watt.
- The *power* used by a circuit is the energy consumed by the *resistance* or *load.*
- Pure resistors dissipate energy in the form of heat (a.k.a. Joule heating)
- A load dissipates energy in the form of work.
- Power is expressed in terms of current and voltage as $P = IV$ and in terms of current and resistance as $P = I^2 R$.

Example

A portable electric heater is plugged into a duplex Edison wall outlet (110V) If the elements on the inside of the heater have a total resistance of 16Ω, find the current drawn by the heater and the power that the heater is capable of producing.

$$I = \frac{V}{R} = \frac{110V}{16\Omega} = 6.9A$$

So the heater draws 6.9 amperes. The power rating is:

$$P = I^2 R = (6.9A)^2 16\Omega = 762W$$

This is the typical "low range" for a portable electric heater. Generally such devices have two ranges, high/low. The high range is accessed by a switch which connects a second set of elements in parallel with the first. This *increases* the load by decreasing the *resistance* of the circuit, in this application generally by a half. Let's assume this heater has a high range and the resistance is 8Ω.

$$I = \frac{V}{R} = \frac{110V}{8\Omega} = 13.8A \qquad P = I^2 R = (13.8A)^2 8\Omega = 1.52kW$$

This is the principle behind the ubiquitous 800/1500 watt floor heater.

One pays for electricity in terms of kilowatt-hours. A kilowatt-hour is the energy consumed in an hour at a rate of 1 kilowatt.

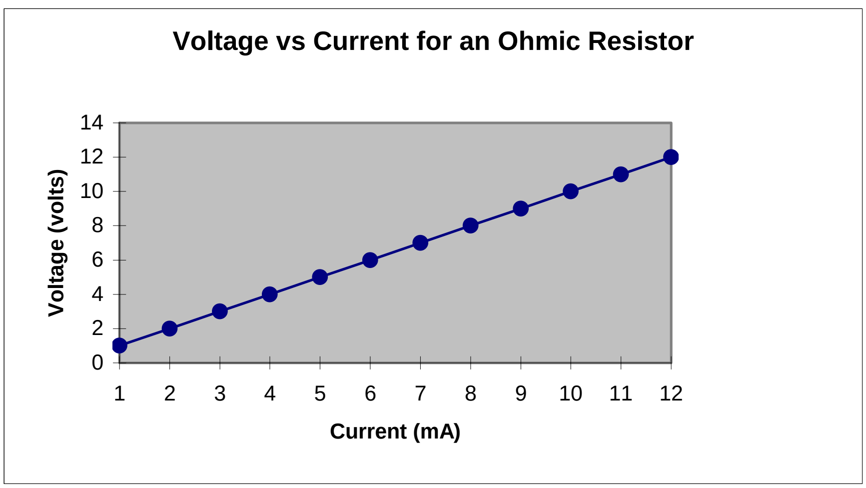
$$1kWh = (10^3 W)(3600s) = 3.6 \times 10^6 J$$

Your electricity usage is normally billed in multiples of kilowatt-hours.

Resistors

Resistors are physically either of the wire wound or semiconductor type

Ohmic resistors display constant resistance over a range of voltages and currents.



Non-Ohmic Resistors display varying resistance over a range of voltages and currents