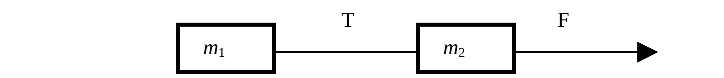
Vector Forces/Free Body Diagrams II – Dynamics

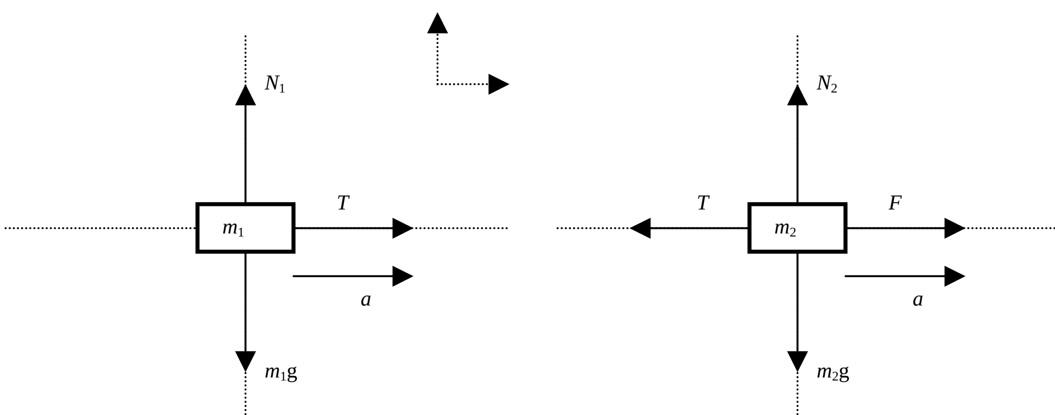
Now we'll have a look at systems where $\Sigma F = ma$

- Objectives will normally be to solve for either *F* or *a,* or both.
- FBD's will be used to generate enough equations to solve for the needed unknowns

Example 1 Consider a system consisting of two blocks (masses = m_1 & m_2) attached by a light cord on a smooth, flat table pulled to the right by a force *F*. Find the tension in the connecting cord and the acceleration of the system.



The FBD's for this system:



FBD₁

$$\sum F_x = T = m_1 a \qquad (1)$$
$$\sum F_y = N_1 - m_1 g = 0$$
$$\therefore N_1 = m_1 g$$

FBD₂

$$\sum F_x = F - T = m_2 a \qquad (2)$$
$$\sum F_y = N_2 - m_2 g = 0$$
$$\therefore N_2 = m_2 g$$

Now if we combine equations (1) and (2):

$$m_1 a = T = F - m_2 a \therefore m_1 a = F - m_2 a$$

$$(m_1 + m_2)a = F$$

$$a = \frac{F}{m_1 + m_2}$$

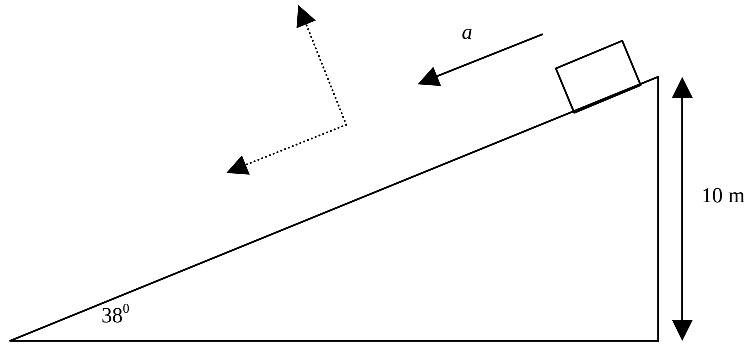
Note that this is in the form: $a = \frac{F}{\sum m}$

Plugging this value for acceleration back into equation (1) yields:

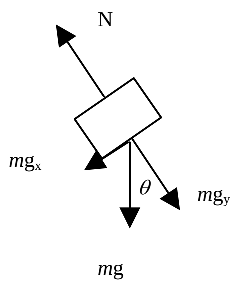
$$T = m_1 \left(\frac{F}{m_1 + m_2} \right)$$

Note that this is in the form: $F = ma$

Example 2 A 10 kg block slides down a smooth surface. It is released from rest at the top of an incline (38^0 to the horizontal) at a height 10 meters. What is its speed at the bottom of the incline?



Note: $\sin 38^0 = \dfrac{10m}{hyp} \Rightarrow$ the length of the incline is 16.2 meters. In the coordinate system we've chosen, the FBD is:

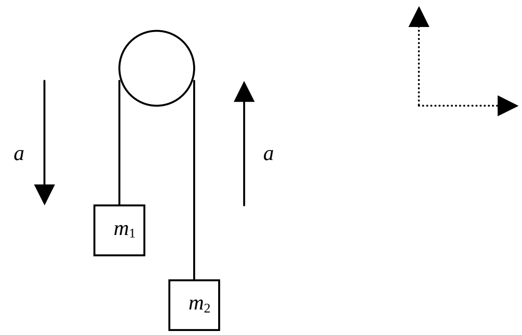


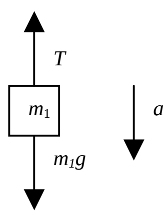
Based on this FBD: $\sum F_x = mg_x = ma = mg\sin\theta \therefore a = g\sin\theta = 6.03\text{m/s}^2$

By determining the acceleration we've solved the *dynamics* part of the problem and with this information we can proceed to solve the *kinematics* part of the problem.

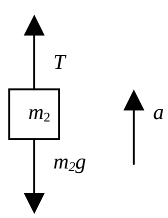
$$v^2 = v_0{}^2 + 2a(x - x_0) \rightarrow v = \sqrt{2a(x - x_0)} = \sqrt{(2)(6.03m \cdot s^{-2})(16.2m)} = 14m \cdot s^{-1}$$

Example 3 Consider a light, frictionless pulley with two masses m_1 & m_2 ($m_1 > m_2$) attached by a light cord as shown below (such a system is known as *Atwood's Machine*). Find the tension in the connecting cords and the acceleration of the system.





$$\sum F_x = 0$$
$$\sum F_y = T - m_1 g = -m_1 a$$
$$\therefore T = m_1 g - m_1 a \quad (1)$$



$$\sum F_x = 0$$
$$\sum F_y = T - m_2 g = m_2 a$$
$$\therefore T = m_2 g + m_2 a \quad (2)$$

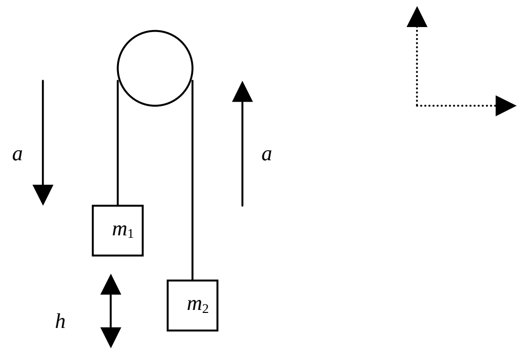
The tension in the connecting cords is the same everywhere since the pulley is massless. The magnitude of the acceleration is the same everywhere. This being the case, (1) and (2) may be set equal to each other.

$$m_1 g - m_1 a = m_2 g + m_2 a$$
$$m_1 a + m_2 a = m_1 g - m_2 g$$
$$(m_1 - m_2)g = (m_1 + m_2)a$$
$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

and

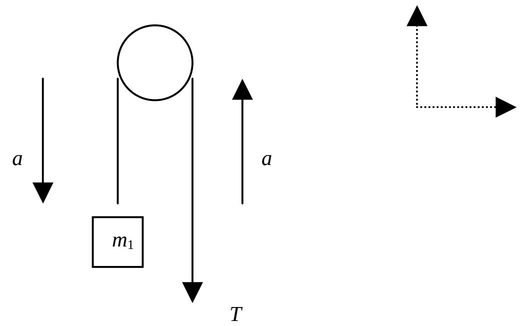
$$T = m_1 g - m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$
$$T = m_1 g - \left(\frac{m_1 m_1 g - m_1 m_2 g}{m_1 + m_2}\right)$$
$$T = \frac{m_1 m_1 g + m_1 m_2 g - m_1 m_1 g + m_1 m_2 g}{m_1 + m_2}$$
$$T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Example 4 Consider the Atwood Machine of the previous example. Find the velocity of the blocks after the system has moved, starting from rest, a distance of *h*.

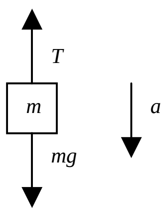


$$v^2 = v_0^{\ 2} + 2ah \Rightarrow v = \sqrt{2\frac{m_1 - m_2}{m_1 + m_2}gh}$$

Example 5 Consider the block-pulley system below. The pulley is light and frictionless, the cord is light, and *T* < *mg* (the block is being lowered). Determine the acceleration of the system in terms of *T*, *m* and *g*.



FBD

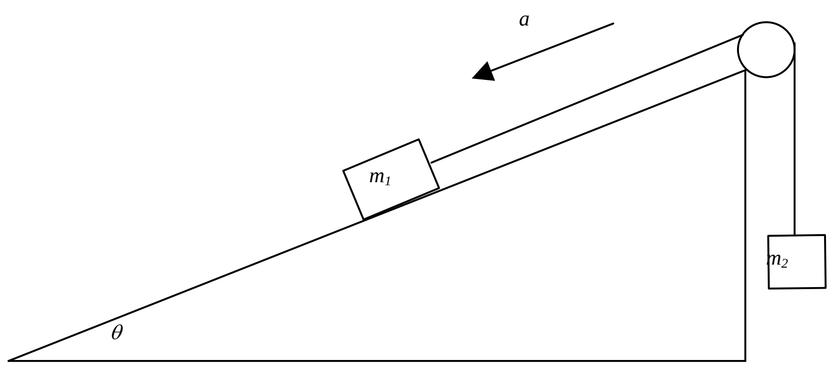


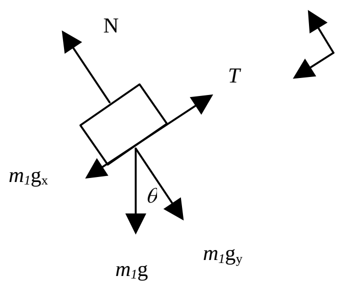
$$\sum F_x = 0$$
$$\sum F_y = T - mg = -ma$$
$$\therefore T = mg - ma \therefore a = \frac{mg - T}{m}$$

Compare with $T = \frac{2m_1 m_2 g}{m_1 + m_2}$ and $a = \frac{m_1 - m_2}{m_1 + m_2}$ for the two-block system.

Why are these quantities different from their counterparts in the two-block system? How are these systems different (or are they really)?

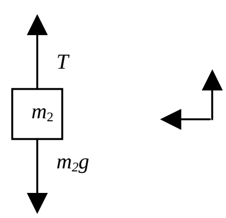
Example 6 Consider the system below. If $m_1 > m_2$ find the tension in the connecting cord and the acceleration of the system.





$$\sum F_x = m_1 g\sin\theta - T = m_1 a \quad (1)$$
$$\sum F_y = N - m_1 g\cos\theta = 0$$

FBD #1



$$\sum F_x = 0$$
$$\sum F_y = T - m_2 g = m_2 a \quad (2)$$

FBD #2

Only equations (1) and (2) yield useful information. Notice that *T* and *a* are the same throughout the system. Hence:

$$m_1 g\sin\theta - m_1 a = T = m_2 a + m_2 g$$

$$(m_1 + m_2)a = (m_1\sin\theta - m_2)g$$

$$a = \frac{(m_1\sin\theta - m_2)g}{(m_1 + m_2)}$$

It follows that:

$$T - m_2 g = m_2 a = m_2 \frac{(m_1 \sin\theta - m_2)g}{(m_1 + m_2)}$$

$$T = \frac{m_2 m_1 g \sin\theta - m_2 m_2 g}{m_1 + m_2} + m_2 g$$

$$T = \frac{m_2 m_1 g \sin\theta - m_2 m_2 g + m_1 m_2 g + m_2 m_2 g}{m_1 + m_2}$$

$$T = \frac{m_2 m_1 g \sin\theta + m_1 m_2 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g (1 + \sin\theta)}{m_1 + m_2}$$