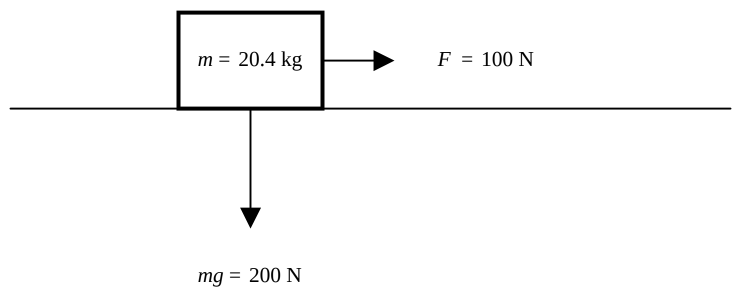
Vector Forces/Free Body Diagrams I

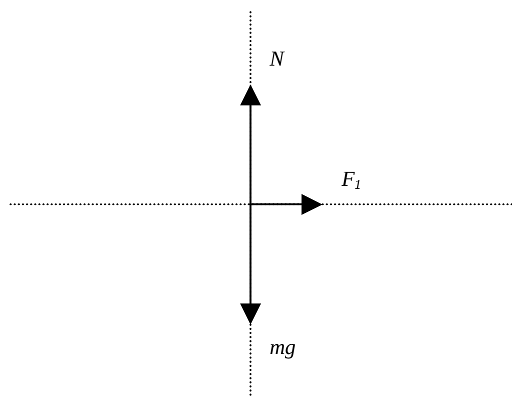
Newton's Laws:

- The Law of Inertia
- $F = ma$
- Forces occur in pairs

Consider a system consisting of a block (mass = 20.4 kg) on a smooth, flat table pulled to the right by a force of 100 Newtons.



The *F*ree *B*ody *D*iagram (FBD) for this system:



Note:

- the forces in the *y* direction are balanced (i.e., *mg* = *N*) while the forces in the *x* direction are not.
- the forces in this example lie along the *x* and *y*-axes.

Let's write Newton's second law for the sum of the forces along each axis.

$$\sum F_x = +F_1 = ma$$
$$\sum F_y = +N - mg = 0 \therefore N = mg$$

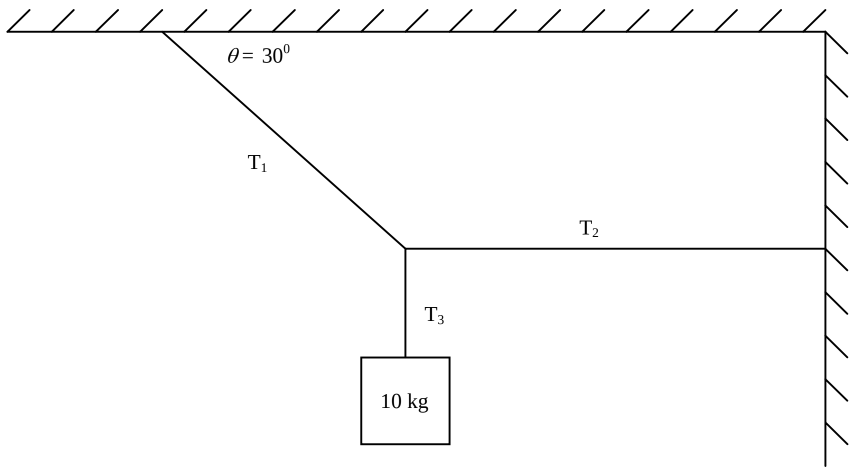
- The result of the vector analysis based on our FBD shows a net acceleration in the +*x* direction caused by the unbalanced force ($\boldsymbol{F}_1$).
- Can you compute the acceleration based on this data (the answer is $\boldsymbol{a}$ = +4.9 m/s^2)?
- How would you describe the motion of this block in words?

In general one chooses the portion of a system where most or all of the forces involved act directly to generate a FBD. In some cases it may be necessary to produce more than one FBD to completely analyze a system.

FBD's are generally applied to problems in one of two categories: *statics* or *dynamics*. In statics the entire system is in equilibrium (no accelerations are present). In dynamics accelerations are present and generally must be determined based upon the information provided in the problem.

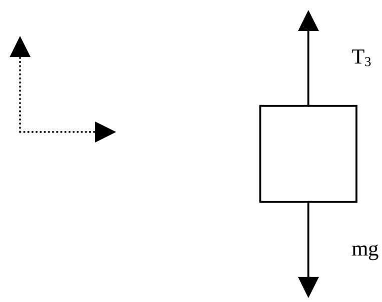
- In static systems the sum of all of the forces acting on a system is zero ($\Sigma F = 0$)
- In dynamic systems the sum of all the forces acting on a system produces acceleration ($\Sigma F = ma$).

Example 1.



- In this particular case, the system consists of the block and three light cords anchoring it to a ceiling and wall. The mass of the block is known.
- We seek the tension in each of the connecting cords.
- In this example all of the forces are balanced and the system is said to be in *equilibrium* ($\Sigma F = 0$), i.e., no accelerations are present.
- In order to perform a vector analysis of this system we will need to produce FBD's that will yield sufficient equations to solve for all the unknown's in the system, T_1, T_2, T_3 (three unknowns require three equations).

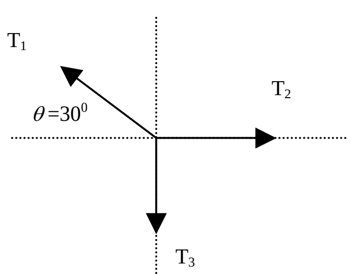
Let's look first at the hanging mass:



$$\sum F_x = 0$$
$$\sum F_y = T_3 - mg = 0 \qquad \textbf{(1)}$$
$$\therefore T_3 = mg = 98N$$

This FBD yields one useful equation (T_3 = 98*N*).

Let's try a FBD of the "knot" where the cords are joined. This will yield two of the three equations we need.

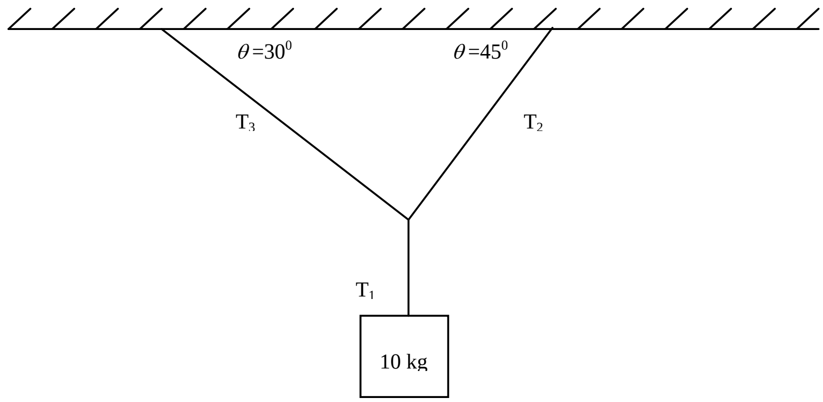


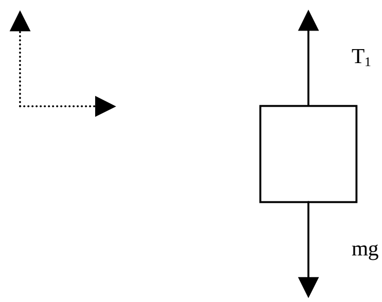
$$\sum F_x = T_2 - T_1\left(\cos 30^0\right) = 0 \qquad \textbf{(2)}$$
$$\therefore T_2 = T_1\left(\cos 30^0\right)$$

$$\sum F_y = T_1\left(\sin 30^0\right) - T_3 = 0$$
$$\therefore T_1\left(\sin 30^0\right) = T_3 = 98N \qquad \textbf{(3)}$$
$$\therefore T_1 = 196N$$

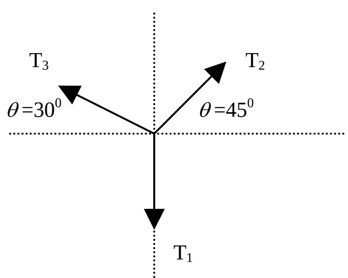
Equation (2) now yields $T_2 = 169.7N$ and **T_1 = 196*N*, T_2 = 169.7*N*, T_3 = 98*N***

Example 2. The system is in equilibrium, ($\Sigma F = 0$). Find the tension in all of the cords.





$$\sum F_x = 0$$
$$\sum F_y = T_1 - mg = 0 \qquad \textbf{(1)}$$
$$\therefore T_1 = mg = 98N$$



$$\sum F_x = T_2\left(\cos 45^0\right) - T_3\left(\cos 30^0\right) = 0 \qquad \textbf{(2)}$$
$$\therefore T_2\left(\cos 45^0\right) = T_3\left(\cos 30^0\right)$$

$$\sum F_y = T_2\left(\sin 45^0\right) + T_3\left(\sin 30^0\right) - T_1 = 0 \qquad \textbf{(3)}$$
$$\therefore T_2\left(\sin 45^0\right) + T_3\left(\sin 30^0\right) = T_1 = 98N$$

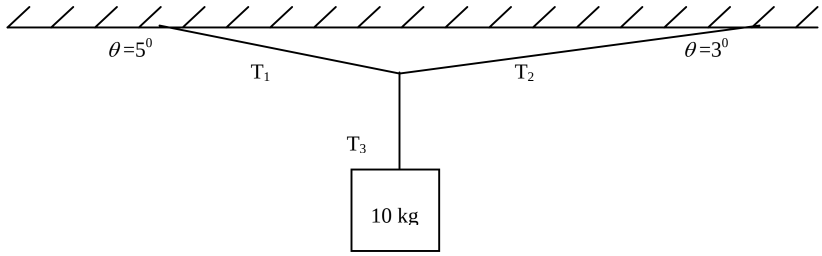
Equation (1) yields a solution for T_1 directly. Equations (2) and (3) must be solved simultaneously to obtain the values for T_2 and T_3.

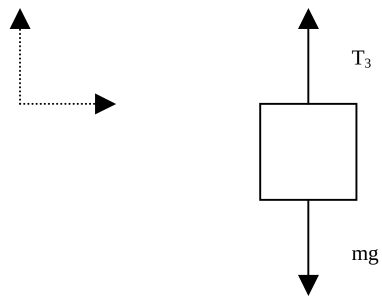
$$T_2(.707) - T_3(.866) = 0$$
$$T_2(.707) + T_3(.5) = 98N$$

Notice that if the first equation in the pair above is multiplied through by (-1) and the two equations are then added together the variable T_2 is eliminated.

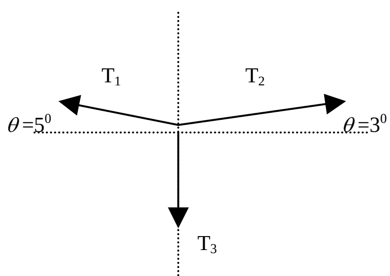
$$-T_2(.707) + T_3(.866) = 0$$
$$+T_2(.707) + T_3(.5) \quad = 98N \qquad \therefore T_3 = 71.7N \Rightarrow T_2 = 87.8N$$
$$0 \quad + T_3(1.366) = 98N$$

Example 3. The system is in equilibrium, ($\Sigma F = 0$); find the tension in all of the cords.





$$\sum F_x = 0$$
$$\sum F_y = T_3 - mg = 0 \quad \textbf{(1)}$$
$$\therefore T_3 = mg = 98N$$



$$\sum F_x = T_2\left(\cos 3^0\right) - T_1\left(\cos 5^0\right) = 0 \quad \textbf{(2)}$$

$$\sum F_y = T_1\left(\sin 5^0\right) + T_2\left(\sin 3^0\right) = T_3 \quad \textbf{(3)}$$

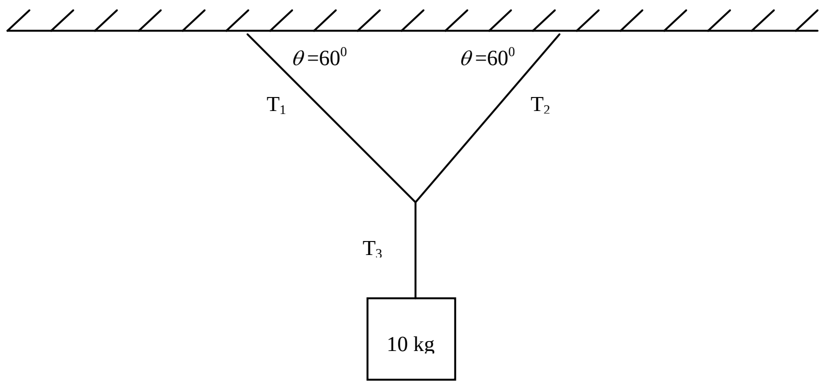
Using Cramer's Rule on equations (2) and (3):

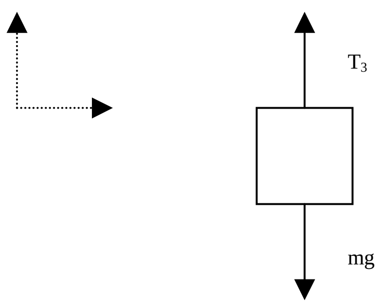
$$\begin{matrix} -T_1(.996) + T_2(.999) = 0 \\ T_1(.087) + T_2(.052) = 98N \end{matrix} \rightarrow \begin{matrix} 703N \\ 701N \end{matrix}$$

$\therefore$ T_1 = 703N, T_2 = 701N

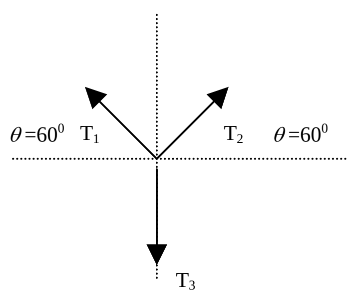
Notice that the tension in the cords is many times greater than the direct force of gravity on the hanging mass! This is what happens when an object is suspended from a taut cord. The effect is to multiply the forces on the anchors instead of lessening them. In the case above it would have been better to suspend the block singly from either of the anchors. That is why the cables in suspension bridges have large dips in them. Bear this in mind the next time you hang a painting in your home from on a taut wire. In this instance, where would the failure be most likely to occur?

Example 4. Let's fix the problem in the previous example. The system is in equilibrium, ($\Sigma F = 0$); find the tension in all of the connecting cords when the angle between cord 1 and 2 is lessened.





$$\sum F_x = 0$$
$$\sum F_y = T_3 - mg = 0 \qquad \textbf{(1)}$$
$$\therefore T_3 = mg = 98N$$



$$\sum F_x = T_2\left(\cos 60^0\right) - T_1\left(\cos 60^0\right) = 0 \qquad \textbf{(2)}$$

$$\sum F_y = T_1\left(\sin 60^0\right) + T_2\left(\sin 60^0\right) = T_3 \qquad \textbf{(3)}$$

Using Cramer's Rule on equations (2) and (3):

$$\begin{matrix} -T_1(.5) & +T_2(.5) = 0 \\ T_1(.866) & +T_2(.866) = 98N \end{matrix} \;\rightarrow\; \begin{matrix} 56.6N \\ 56.6N \end{matrix}$$

$\therefore$ T_1 = 56.6N, T_2 = 56.6N

An order of magnitude less! Now the anchor system works as intended.